

Q1

$$(a) \quad \frac{dn}{dt} = -\nabla \cdot \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

$$\text{Steady-state:} \quad \therefore \frac{dn}{dt} = 0$$

$$\text{Source-free:} \quad \therefore S = 0$$

$$\text{Fick's Law:} \quad \underline{j} = -D\nabla\phi$$

$$\therefore 0 = -\nabla \cdot (-D\nabla\phi) + (\eta - 1)\Sigma_a \phi$$

If  $D$  is constant and the reactor is homogeneous

$$\therefore 0 = D\nabla^2\phi + (\eta - 1)\Sigma_a \phi$$

$$\therefore \nabla^2\phi + \frac{(\eta - 1)\Sigma_a}{D}\phi = 0$$

[15%]

$$(b) \quad \text{In Cartesian geometry} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\therefore \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} + B^2\phi = 0$$

Assuming that  $\phi(x, y, z) = X(x)Y(y)Z(z)$

$$\therefore YZ \frac{\partial^2 X}{\partial x^2} + XZ \frac{\partial^2 Y}{\partial y^2} + XY \frac{\partial^2 Z}{\partial z^2} + B^2 XYZ = 0$$

$$\therefore \frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + B^2 = 0$$

The first term is only a function of  $x$ , the second only a function of  $y$ , the third only a function of  $z$  and the last term is a constant,

$$\therefore \frac{1}{X} \frac{d^2 X}{dx^2} + \alpha^2 = 0; \quad \frac{1}{Y} \frac{d^2 Y}{dy^2} + \beta^2 = 0; \quad \frac{1}{Z} \frac{d^2 Z}{dz^2} + \gamma^2 = 0$$

$$\text{with} \quad \alpha^2 + \beta^2 + \gamma^2 = B^2$$

Considering the symmetry of a cubic reactor,

$$\alpha^2 = \beta^2 = \gamma^2 \Rightarrow \alpha^2 = \frac{B^2}{3}$$

Consider the equation in  $x$ :

$$\frac{1}{X} \frac{d^2 X}{dx^2} + \alpha^2 = 0 \Rightarrow \frac{d^2 X}{dx^2} + \alpha^2 X = 0$$

This is an SHM equation,

$$\therefore X = M \cos(\alpha x) + N \sin(\alpha x)$$

By symmetry  $X$  must be an even function  $\Rightarrow N = 0$

$$\therefore X = M \cos(\alpha x) = M \cos\left(\frac{B}{\sqrt{3}} x\right)$$

Similarly for the  $y$  and  $z$  variation. Thus:

$$\phi(x,y,z) = XYZ = C \cos\left(\frac{B}{\sqrt{3}} x\right) \cos\left(\frac{B}{\sqrt{3}} y\right) \cos\left(\frac{B}{\sqrt{3}} z\right)$$

where  $C$  is a constant

[35%]

- (c) The minimum volume bare core will be *spherical*.

$$\therefore \phi(r) = \frac{A}{r} \sin(Br)$$

Neglecting the extrapolation distance, the boundary condition is that  $\phi = 0$  at  $r = R$ , the physical edge of the reactor. The first (non-trivial) zero of  $\sin$  is at  $\pi$

$$\therefore BR = \pi$$

$$\therefore R = \frac{\pi}{B} = \frac{\pi}{\sqrt{0.25}} = 2\pi = 6.283 \text{ m}$$

$$\therefore V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi (2\pi)^3 = \frac{32}{3} \pi^4 = 1039 \text{ m}^3$$

[20%]

- (d) The minimum volume rectangular parallelepiped is cubic.

For a bare cubic reactor (of side length  $2X$ )

$$\phi(x,y,z) = C \cos\left(\frac{B}{\sqrt{3}} x\right) \cos\left(\frac{B}{\sqrt{3}} y\right) \cos\left(\frac{B}{\sqrt{3}} z\right)$$

With the boundary condition that  $\phi = 0$  at  $x = X$  and noting that the first zero of  $\cos$  is at  $\pi/2$

$$\therefore \frac{B}{\sqrt{3}} X = \frac{\pi}{2} \Rightarrow X = \frac{\sqrt{3}\pi}{2B} = \frac{\sqrt{3}\pi}{2\sqrt{0.25}} = 5.441 \text{ m}$$

(This means that the volume of the bare reactor is  $V = (2X)^3 = 8X^3 = 8(5.441)^3 = 1289 \text{ m}^3$ )

From the 4M16 data sheet,  $L$  for graphite is 54 cm (or 0.54 m). Therefore for the reflected reactor

$$X' = X - L = 5.441 - 0.54 = 4.901 \text{ m}$$

$$\therefore V' = (2X')^3 = 8X'^3 = 8(4.901)^3 = 942 \text{ m}^3$$

[30%]

#### Assessor's Comments:

*All candidates: 58 attempts, Average mark 15.7/20, Maximum 20, Minimum 7.*

A reasonably popular attempted by 77% of candidates, done very well by many of them.

The bookwork derivations were generally done well, although the fact that the reactor being homogeneous implied a spatially invariant diffusion coefficient was missed by many.

In part (c), a surprisingly large number of candidates did not recognise without evaluating alternatives that the minimum volume critical core would be spherical. Fortunately for them unnecessary calculations at this stage were often helpful in answering part (d).

In answering part (d), several candidates tried to solve the diffusion equation for the core and reflector configuration rather than just using the given result for the reflector saving.

Q2

- (a) The major simplifications of this model are that it assumes there is no spatial variation in behaviour, whereas in practice the reactor core is highly heterogeneous and the neutron population varies spatially, and it also assumes that there is only one type of precursor, whereas in reality there are a large number of them with widely varying production rates and half-lives. [10%]

- (b) In equilibrium (steady state)  $\frac{dn}{dt} = \frac{dc}{dt} = 0$

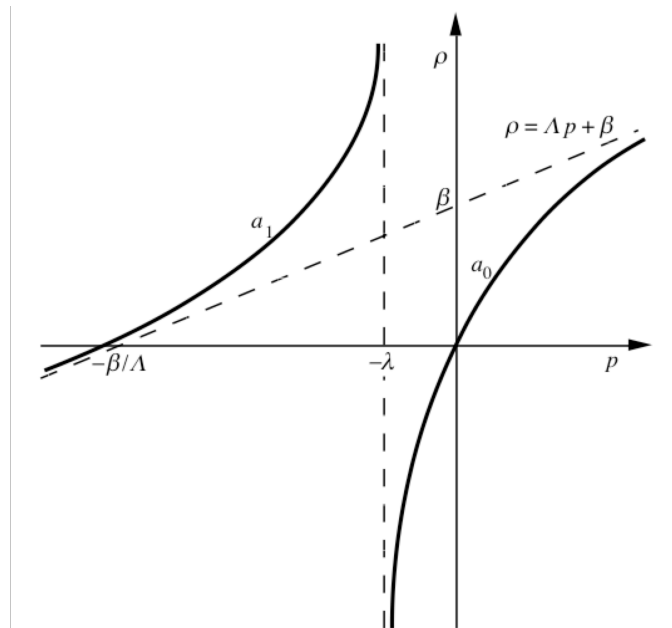
$$\frac{dc}{dt} = 0 \Rightarrow \frac{\beta}{\Lambda} n_0 - \lambda c_0 = 0 \Rightarrow \frac{\beta}{\Lambda} n_0 = \lambda c_0$$

where  $n_0$  and  $c_0$  are the equilibrium neutron and precursor populations.

$$\frac{dn}{dt} = 0 \Rightarrow \frac{\rho - \beta}{\Lambda} n_0 + \lambda c_0 = 0$$

$$\therefore \frac{\rho - \beta}{\Lambda} n_0 + \frac{\beta}{\Lambda} n_0 = 0 \Rightarrow \frac{\rho}{\Lambda} n_0 = 0 \Rightarrow \rho = 0 \quad [10\%]$$

- (c) (i)



[15%]

$$(ii) \quad \rho = p \left[ \Lambda + \frac{\beta}{(p + \lambda)} \right] \Rightarrow \rho(p + \lambda) = p \left[ \Lambda(p + \lambda) + \beta \right]$$

$$\therefore \frac{\rho}{\Lambda}(p + \lambda) = p^2 + p \left[ \lambda + \frac{\beta}{\Lambda} \right] \Rightarrow p^2 + p \left[ \lambda + \frac{\beta - \rho}{\Lambda} \right] - \frac{\rho \lambda}{\Lambda} = 0$$

$$\therefore p^2 + p \left[ 0.1 + \frac{0.0075 - 0.005}{10^{-4}} \right] - \frac{0.005 \times 0.1}{10^{-4}} = 0$$

$$\therefore p^2 + 25.1p - 5 = 0 \Rightarrow p = 0.19765 \text{ or } -25.29765 \text{ s}^{-1}$$

Therefore the dominant time constant (the positive one) is

$$T_+ = \frac{1}{p_+} = \frac{1}{0.19765} = 5.0595 \text{ s} \quad [15\%]$$

- (d) The prompt jump approximation (PJA) assumes that the neutron population remains in equilibrium with the precursor population, even when the latter is varying with time. Thus, for a source-free system:

$$\frac{dn}{dt} \approx 0 = \frac{\rho - \beta}{\Lambda} n + \lambda c \Rightarrow n = \frac{\Lambda \lambda c}{\beta - \rho} \quad (2.1)$$

Using this expression to substitute for  $n$  in the precursor equation gives:

$$\frac{dc}{dt} = \frac{\beta \lambda c}{\beta - \rho} - \lambda c = -\frac{\rho \lambda}{\beta - \rho} c$$

which, by inspection, has a solution:

$$c = c_0 \exp\left(\frac{\rho \lambda}{\beta - \rho} t\right) \quad (2.2)$$

if  $c = c_0$  at  $t = 0$ .

But, if the system was in equilibrium with  $n = n_0$  and  $\rho = 0$  before the change in  $\rho$ , then from part (b):

$$\beta n_0 = \Lambda \lambda c_0 \quad (2.3)$$

So, combining equations (2.1), (2.2) and (2.3), the variation in the neutron population after the change in  $\rho$  is given by:

$$n = \frac{\Lambda \lambda c}{\beta - \rho} = \frac{\Lambda \lambda}{\beta - \rho} c_0 \exp\left(\frac{\rho \lambda}{\beta - \rho} t\right) = \frac{\beta}{\beta - \rho} n_0 \exp\left(\frac{\rho \lambda}{\beta - \rho} t\right) \quad [40\%]$$

- (e) The time constant predicted by the PJA is

$$T_+ = \frac{\beta - \rho}{\rho \lambda} = \frac{0.0075 - 0.005}{0.005 \times 0.1} = 5.0 \text{ s}$$

Thus, the PJA gives a good estimate of the exact (according to the in-hour equation) time constant. Importantly, from a safety point of view, the PJA approximation underestimates  $T_+$ . Thus, the estimate is *conservative* – the real system will respond slower than predicted. [10%]

#### Assessor's Comments:

*All candidates: 66 attempts, Average mark 14.2/20, Maximum 20, Minimum 2.*

A popular question attempted by 88% of candidates, many of whom made good attempts. A quite common mistake in (c)(ii) was to assume that the fact that the locus of the dominant root passes through the origin in  $\rho$ - $p$  space could be used to estimate the value of time constant using gradient there. This mistake reveals a failure to appreciate that  $\rho = 0.005$  is a comparatively large reactivity change.

A surprising number of candidates could not solve the quadratic equation for  $p$  in (c)(ii). There were a number of attempts to solve the Prompt Jump Approximation equations by Laplace transforms in part (d). These were invariably unsuccessful.

Comments made in answer to part (e) often failed to address the implications of using the estimate of the time constant rather than the exact value from (c)(ii), focusing instead on the implications of the value of the time constant.

Q3

- (a) On-line refuelling increases availability (no shutdowns for refuelling) and also increases fuel utilisation (burn-up). The main disadvantage is the increased risk of accidents as the core configuration is changed during operation. There is also no scope for in-core fuel management. [10%]
- (b) Assume that the effect of a mixture of batches of different burn-ups  $\tau_i$  can be modelled by the *partial reactivity model*, so that the overall reactivity is given by

$$\rho = \sum_i \rho'_i(\tau_i)$$

where the partial reactivity of an individual batch is given by

$$\rho'_i = \frac{1}{M} \rho_0 \left( 1 - \frac{\tau_i}{T_1} \right)$$

Assume that the fuel is replaced on a “first-in, first-out” basis, and that the end-of-cycle (EOC) condition is that  $\rho = 0$ .

At the end of the first cycle, all  $M$  batches have accumulated burn-up  $\tau_1$ . The EOC condition therefore gives

$$\begin{aligned} \sum_{i=1}^M \frac{1}{M} \rho_0 \left( 1 - \frac{\tau_1}{T_1} \right) &= 0 \\ \therefore \frac{M}{M} \rho_0 \left( 1 - \frac{\tau_1}{T_1} \right) &= 0 \Rightarrow \tau_1 = T_1 \end{aligned}$$

(This result is obvious and it is acceptable to state it “by inspection”.)

At the end of the second cycle,  $(M - 1)$  batches have accumulated burn-up  $(\tau_1 + \tau_2)$  and one batch has accumulated burn-up  $\tau_2$ . The EOC condition therefore gives

$$\begin{aligned} \frac{M-1}{M} \rho_0 \left( 1 - \frac{\tau_1 + \tau_2}{T_1} \right) + \frac{1}{M} \rho_0 \left( 1 - \frac{\tau_2}{T_1} \right) &= 0 \\ \therefore (M-1) \left( 1 - \frac{\tau_1 + \tau_2}{T_1} \right) + \left( 1 - \frac{\tau_2}{T_1} \right) &= 0 \\ \therefore M - \frac{(M-1)\tau_1 + M\tau_2}{T_1} &= 0 \\ \therefore M\tau_2 &= MT_1 - (M-1)\tau_1 \end{aligned}$$

Substituting for  $\tau_1$   $\therefore M\tau_2 = MT_1 - (M-1)T_1 = T_1 \Rightarrow \tau_2 = \frac{T_1}{M}$

At the end of the  $M$ -th cycle, the oldest batch has accumulated burn-up  $(\tau_1 + \tau_2 + \dots + \tau_M)$ , the second oldest  $(\tau_2 + \tau_3 + \dots + \tau_M)$  etc. and the most recently added batch has accumulated burn-up  $\tau_M$ . The EOC condition therefore gives

$$\frac{1}{M} \rho_0 \left( 1 - \frac{\tau_1 + \tau_2 + \dots + \tau_M}{T_1} \right) + \frac{1}{M} \rho_0 \left( 1 - \frac{\tau_2 + \tau_3 + \dots + \tau_M}{T_1} \right) + \dots + \frac{1}{M} \rho_0 \left( 1 - \frac{\tau_M}{T_1} \right) = 0$$

$$\therefore 1 - \frac{\tau_1 + 2\tau_2 + \dots + M\tau_M}{MT_1} = 0$$

$$\therefore \tau_M = T_1 - \frac{1}{M} \sum_{i=1}^{M-1} i\tau_i \quad [30\%]$$

(c) The last result in part (b) holds true for any  $M$  consecutive cycles. In equilibrium (steady-state) operation, all cycles are of length  $T_M$ , so setting  $\tau_i = \tau_M = T_M$

$$T_M + \frac{1}{M} \sum_{i=1}^{M-1} iT_M = T_1$$

$$\therefore MT_M + \sum_{i=1}^{M-1} iT_M = \sum_{i=1}^M iT_M = MT_1$$

$$\therefore \frac{1}{2}M(M+1)T_M = MT_1 \Rightarrow \frac{T_M}{T_1} = \frac{2}{M+1} \quad [15\%]$$

(d) Fuel utilisation  $B_M = MT_M = \frac{2M}{M+1}T_1$

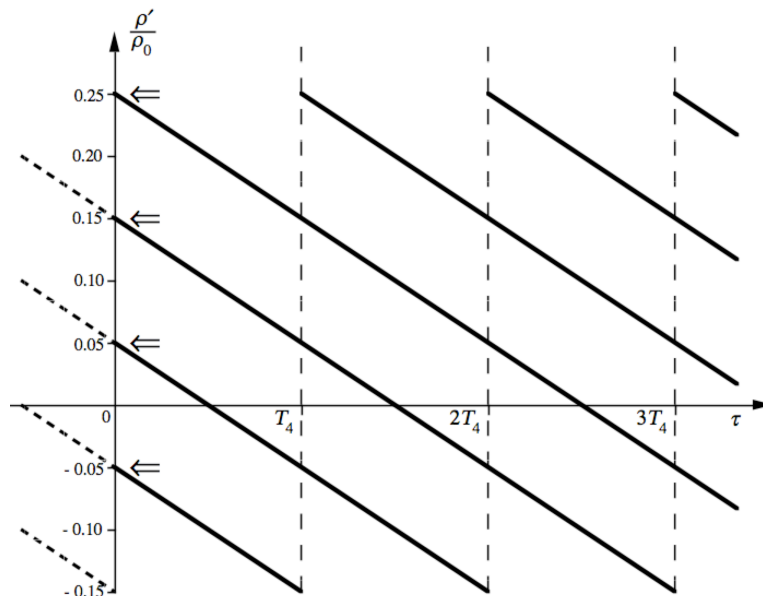
For  $M = 4$ ,  $B_4 = \frac{2 \times 4}{4+1}T_1 = 1.6T_1$

For on-line refuelling  $M =$  the number of fuel channels, which is typically  $> 100$ .

For large  $M$ ,  $B_M \rightarrow B_\infty = 2T_1$

$$\therefore \frac{B_4}{B_\infty} = \frac{1.6T_1}{2T_1} = 0.8 \quad [15\%]$$

(e) In equilibrium 4-batch operation the partial reactivity varies as follows:



To establish equilibrium operation immediately four different initial enrichments are needed with partial reactivities as indicated ( $\Leftarrow$ ). These correspond to the reactivities of standard (equilibrium operation) fuel when fresh and after one, two and three cycles of operation.

Using the result in part (c)  $T_4 = 0.4T_1$ . Therefore the initial batch reactivities required are:

Standard fresh fuel:  $\rho = \rho_0$

Once-burnt fuel:  $\rho = \rho_0 \left(1 - \frac{T_4}{T_1}\right) = \rho_0 \left(1 - \frac{0.4T_1}{T_1}\right) = 0.6\rho_0$

Twice-burnt fuel:  $\rho = \rho_0 \left(1 - \frac{2T_4}{T_1}\right) = \rho_0 \left(1 - \frac{0.8T_1}{T_1}\right) = 0.2\rho_0$

Thrice-burnt fuel:  $\rho = \rho_0 \left(1 - \frac{3T_4}{T_1}\right) = \rho_0 \left(1 - \frac{1.2T_1}{T_1}\right) = -0.2\rho_0$

As  $\rho \propto (e - 1.5)$  and  $e = 3.5$  corresponds to  $\rho = \rho_0$ , then  $\rho = \frac{1}{2}\rho_0(e - 1.5)$ .

The corresponding enrichments for the four initial batches are then:

Standard fresh fuel:  $\rho = \rho_0 \Rightarrow e = 3.5\%$

Once-burnt fuel:  $\rho = 0.6\rho_0 = \frac{1}{2}\rho_0(e - 1.5) \Rightarrow e = 2.7\%$

Twice-burnt fuel:  $\rho = 0.2\rho_0 = \frac{1}{2}\rho_0(e - 1.5) \Rightarrow e = 1.9\%$

Thrice-burnt fuel:  $\rho = -0.2\rho_0 = \frac{1}{2}\rho_0(e - 1.5) \Rightarrow e = 1.1\%$

[30%]

#### Assessor's Comments:

*All candidates: 56 attempts, Average mark 12.1/20, Maximum 19, Minimum 4.*

A reasonably popular attempted by 75% of candidates. There were some very good attempts but the overall standard was rather lower than for the first two questions on this paper.

A common failing in answers was the failure to state and justify assumptions.

Several answers were undermined by inconsistent or confusing notation.

Approaches to part (b) were all often rather unsystematic leading at best to unnecessary additional effort to reach the correct answer and at worst to complete confusion.

In part (d), a surprising number of candidates thought that on-line refuelling corresponded to the case  $M = 0$  rather than  $M \rightarrow \infty$ .

Although many candidates had some idea of how tackle part (e), attempts often failed to take advantage of results from earlier parts of the question in setting things up. Other answers failed to explain adequately what was being attempted, making it difficult to award much partial credit if the correct results were not obtained.

Q4

- (a) All fission reactors operate with a very sensitive neutron balance in which the number of neutrons generated per fission must equal those consumed by fission, capture and other losses. Nearly all civil nuclear power reactors use light water as a moderator and coolant and zircaloy cladding, both of which have significant thermal neutron absorption cross-sections, which means that, unlike the former Magnox graphite-moderated, CO<sub>2</sub>-cooled reactors with magnesium cladding, there are insufficient neutrons to maintain a controlled chain reaction with natural uranium. Normal civil enrichment values are between 2.5% and 4%. [10%]

- (b) Commercial enrichment processes use the difference in the size and mass of the <sup>235</sup>U and the <sup>238</sup>U atoms in the form of uranium hexafluoride (HEX). They use stage-wise contacting processes in which the enrichment takes place in a number of stages, each stage giving a small change in the concentration of the <sup>235</sup>U. In the earlier, now virtually obsolete, diffusion process, the separation is effected by a semi-porous membrane through which the slightly small <sup>235</sup>U HEX molecules are more likely to pass. Because the separation per stage is very low, the process requires a large number of stages, each with its own compressor and cooler. This process has the advantage of being relatively simple, but is very expensive in energy costs.

The current process uses very high speed gas centrifuges in which the heavier <sup>238</sup>U HEX is spun to the outside of the centrifuge whilst the slightly lighter <sup>235</sup>U remains in the centre. This process requires fewer stages and hence less power and space, but the construction of the centrifuges requires the use of very high grade materials to resist the enormous g forces. HEX has got to be used as it is the only uranium compound that is gaseous at relatively mild conditions.

Newer processes under development revolve around the use of specially tuned lasers that can excite one isotope without exciting the others. The excited isotope can then be separated by an electromagnetic field. Despite much research and various claims from time to time, industrial deployment of such processes is still well into the future, as it is very difficult to compete economically with the centrifuge process. [30%]

- (c) Separative work unit (SWU) is the unit in which enrichment is traded and is defined by:

$$S = E_w(2x_w - 1) \ln\left(\frac{x_w}{1-x_w}\right) + E_p(2x_p - 1) \ln\left(\frac{x_p}{1-x_p}\right) - E_f(2x_f - 1) \ln\left(\frac{x_f}{1-x_f}\right)$$

where  $E_i$  is the total mass of the feed ( $f$ ), product ( $p$ ) and waste ( $w$ ), respectively. The approximation to

$$S = E_w(-\ln x_w) + E_p(-\ln x_p) - E_f(-\ln x_f)$$

is generally valid for civil reactors where the enrichment levels are low.

SWU increases as the enrichment required increases and the tails concentration decreases. Since the enrichment of the fuel ( $x_p$ ) is dictated by reactor physics and the feed concentration ( $x_f$ ) by nature, the only variable that can be adjusted is the tails concentration ( $x_w$ ). The lower  $x_w$  the greater the SWU required but less feed will be needed so the optimal  $x_w$  value depends on the ratio of the uranium price and the cost of a SWU. When the uranium price is high it may be worth spending more on SWUs to reduce the feed requirements and the reverse is true if the SWU price is high. [20%]



(d) Thermal power is given by:  $P_{\text{th}} = \frac{1200}{0.3} = 4000 \text{ MW(th)}$

Annual thermal output is given by

$$4000 \text{ MW(th)} \times 0.85 \times 365 \text{ days/year} = 1.241 \times 10^6 \text{ MWd}$$

Annual fuel requirement (this is  $P$  as needed in calculations below) is therefore given by

$$\frac{1.241 \times 10^6 \text{ MWd}}{40 \times 10^3 \text{ MWdte}^{-1}} = 31.025 \text{ te}$$

Mass balance across enrichment plant requires

$$F = P + W \Rightarrow W = F - P$$

$$x_f F = x_p P + x_w W$$

$$\therefore x_f F = x_p P + x_w (F - P)$$

$$\therefore F(x_f - x_w) = P(x_p - x_w) \Rightarrow F = P \left( \frac{x_p - x_w}{x_f - x_w} \right)$$

Using the values given in the question and calculated above

$$\therefore F = 31.025 \left( \frac{0.04 - 0.003}{0.007 - 0.003} \right) = 286.98 \text{ te}$$

$$W = F - P = 286.98 - 31.025 = 255.96 \text{ te}$$

As UOC is 95% U, the total amount of UOC needed is  $\frac{286.98}{0.95} = 302.09 \text{ te}$

Using the approximate relationship, which is valid for these enrichment levels,

$$S = W(-\ln x_w) + P(-\ln x_p) - F(-\ln x_f)$$

$$\therefore S = 255.96[-\ln(0.003)] + 31.025[-\ln(0.04)] - 286.98[-\ln(0.007)]$$

$$\therefore S = 162.82 \text{ teSWU}$$

[40%]

### Assessor's Comments:

*All candidates: 46 attempts, Average mark 11.9/20, Maximum 18, Minimum 2.*

The least popular question, but nevertheless attempted by 61% of candidates.

Most answers to parts (a) and (b) were on the right lines but provided insufficient detail to gain full credit.

In answering part (b), some candidates showed insufficient attention to detail in reading the question, providing excessive detail about the historic diffusion separation process.

Some candidates were confused as to whether diffusion separation or centrifuge separation was the historic process.

Discussion of the criteria for determining the tails concentration in part (c) was very poor, with few candidates showing any appreciation that there is an economic trade-off between the cost of feed and the cost of separative work involved.

The calculations in part (d) were generally well done, although few justified their use of approximate form of value function.

There was some confusion as to the difference between the feed to the enrichment process and the amount of uranium ore concentrate.

Several candidates mistakenly believed that the mass units in measures of burnup refer only to the fissile component of the fuel rather than all heavy metal.