Q1
(a) Mass defect $\quad \Delta m=89.907152-89.904703=0.002449 \mathrm{u}$
$1 \mathrm{u}=931.5 \mathrm{MeV}$ (4M16 Data Sheet page 1)

$$
\therefore \text { Energy released }=0.002449 \times 931.5=2.281 \mathrm{MeV}
$$

(b)

$$
\begin{gathered}
A=\lambda N \\
\lambda=\frac{\ln 2}{T_{1 / 2}}=\frac{\ln 2}{64.05 \times 3600}=3.006 \times 10^{-6} \mathrm{~s}^{-1}
\end{gathered}
$$

$N=\frac{M L}{m}$ where $M$ is the mass ( 1 kg here), $m$ is the molar mass and $L$ is Avogadro's number.

$$
\begin{gathered}
\therefore \quad N=\frac{1 \times 6.022 \times 10^{26}}{89.9}=6.699 \times 10^{24} \mathrm{~kg}^{-1} \\
\therefore \quad a=3.006 \times 10^{-6} \times 6.699 \times 10^{24}=2.014 \times 10^{19} \mathrm{~Bq} \mathrm{~kg}^{-1}
\end{gathered}
$$

(c) The volume of a bead is

$$
V_{\mathrm{b}}=\frac{4}{3} \pi R_{\mathrm{b}}^{3}
$$

The volume of ${ }^{90} \mathrm{Y}$ contained within the bead is

$$
V_{\mathrm{Y}}=0.05 V_{\mathrm{b}}=\frac{0.2}{3} \pi R_{\mathrm{b}}^{3}
$$

The mass of ${ }^{90} \mathrm{Y}$ contained within the bead is therefore

$$
M_{\mathrm{Y}}=\rho V_{\mathrm{Y}}=\frac{0.2}{3} \pi \rho R_{\mathrm{b}}^{3}
$$

where $\rho$ is the density of yttrium.

$$
\therefore \quad M_{\mathrm{Y}}=\frac{0.2}{3} \pi \times 4.742 \times 10^{3} \times\left[\frac{25}{2} \times 10^{-6}\right]^{3}=1.940 \times 10^{-12} \mathrm{~kg}
$$

Hence the maximum possible activity (before any of the ${ }^{90} \mathrm{Y}$ has had a chance to decay) is

$$
A_{\max }=a M_{\mathrm{Y}}
$$

where $a$ is specific activity calculated in (b).

$$
\therefore \quad A_{\max }=2.014 \times 10^{19} \times 1.940 \times 10^{-12}=39.07 \times 10^{6} \mathrm{~Bq}
$$

(d) The half-life of ${ }^{90} \mathrm{Y}$ is comparatively short, so it is reasonable to assume that all the remaining (at the time of deployment) ${ }^{90} \mathrm{Y}$ will decay in situ. The total number of decays is therefore equal to the number of ${ }^{90} \mathrm{Y}$ atoms remaining at the time of deployment, which is $10 \%$ of the initial number, if the activity has reduced since manufacture as stated in the question.
Therefore, using the value of $M_{\mathrm{Y}}$ calculated in (c):

$$
N_{\mathrm{Y}-90}=0.1 \times \frac{M_{\mathrm{Y}} L}{\mathrm{~m}}=0.1 \times \frac{1.940 \times 10^{-12} \times 6.022 \times 10^{26}}{89.9}=1.30 \times 10^{12}
$$

Neglecting the (small) shielding effect of the bead (a conservative assumption, so the dose will be overestimated), assume that all the energy associated with these decays is absorbed in
surrounding tissue (a reasonable assumption given the short penetration range of $\beta$ radiation in tissue).
Using the average energy of ${ }^{90} \mathrm{Y} \beta$ radiation $(0.94 \mathrm{MeV})$, the energy deposited is

$$
J=1.30 \times 10^{12} \times 0.94 \times 1.602 \times 10^{-13}=0.196 \mathrm{~J}
$$

Assume that the radiation is emitted isotropically and therefore is absorbed within a sphere the radius of which is the maximum range of ${ }^{90} \mathrm{Y} \beta$ radiation in tissue ( 11 mm ).

Taking the density of tissue to be that of water $\left(10^{3} \mathrm{~kg} \mathrm{~m}^{-3}\right)$ and neglecting the (small) volume occupied by the bead, the mass of tissue irradiated is

$$
M_{\mathrm{t}}=\frac{4}{3} \pi \rho_{\mathrm{t}} R_{\mathrm{t}}^{3}=\frac{4}{3} \pi \times 10^{3} \times\left[11 \times 10^{-3}\right]^{3}=5.575 \times 10^{-3} \mathrm{~kg}
$$

The absorbed dose is therefore

$$
D=\frac{J}{M_{\mathrm{t}}}=\frac{0.196}{5.575 \times 10^{-3}}=35.2 \mathrm{Jkg}^{-1} \text { or } 35.2 \mathrm{~Gy}
$$

As $W_{R}=1$ for $\beta$ radiation, the equivalent dose

$$
H_{T}=W_{R} D=35.2 \mathrm{~Sv}
$$

(e) Using the energy deposited by one bead value from (d), the total energy deposited in the liver is

$$
\begin{align*}
& J_{\text {liver }}=N_{\text {beads }} J=10^{3} \times 0.196=196 \mathrm{~J} \\
& \therefore \quad D_{\text {liver }}=\frac{J_{\text {liver }}}{M_{\text {liver }}}=\frac{196}{1.35}=145.2 \mathrm{~Gy} \\
\therefore & H_{T \text { liver }}=W_{R} D_{\text {liver }}=1 \times 145.2=145.2 \mathrm{~Sv} \\
\therefore \quad & E_{\text {liver }}=W_{T} H_{T \text { liver }}=0.05 \times 145.2=7.26 \mathrm{~Sv}
\end{align*}
$$

(f) The effective dose calculated in (e) is significant. For comparison, the annual dose limit from occupational use of radiation for members of the UK general public is 1 mSv . A 7 Sv dose has the potential to do considerable harm.
The absorbed/equivalent dose calculations in (e) are higher than those in (d) but only by a factor of $\sim 4$, rather than 1000 (the number of beads). This shows how localised the dose given by an individual bead is. It is therefore very important that the beads are indeed delivered to the vicinity of the tumour, so that the vast majority of the dose is absorbed by the cancerous tissue and the healthy liver tissue receives a much lower dose.
${ }^{90} \mathrm{Y}$ is well suited to this application because:
(i) It is a pure $\beta$ emitter - no $\gamma$ radiation, which will penetrate much further through the body is emitted;
(ii) The maximum range of its $\beta$ radiation means that the dose is delivered in a reasonably small volume around the bead, making it suitable for targeting cancerous tumours;
(iii) The half-life of ${ }^{90} \mathrm{Y}$ is not so short as to be impractical but not so long that dose delivery takes an excessively long time.

## Assessor's Comments:

All candidates: 23 attempts, Average mark 10.9/20, Maximum 17, Minimum 6.
Comfortably the least popular and worst done question, probably because the context (radioembolization) was unfamiliar.
Except for mistakes due to lack of attention to detail, parts (a) to (c) were done well.
Part (d) was a struggle for many. Lots of candidates tried incorrectly to use the dose formula for an external gamma source. Others calculated the initial dose rate rather than the accumulated dose (integrated over time) as required.
Many candidates were unsure how to use the mass of the liver in the calculations for part (e). Answers to part (f) revealed that many candidates did not understand that a high, localized dose is not only acceptable but essential in radiotherapy.
(a) From the 4M16 data sheet, the general neutron diffusion equation is

$$
\frac{d n}{d t}=-\nabla \cdot \underline{j}+(\eta-1) \Sigma_{\mathrm{a}} \phi+S
$$

In steady state $\frac{d n}{d t}=0$; for a source-free system $S=0$; for a non-multiplying medium $\eta=0$.

$$
\therefore \quad 0=-\nabla \cdot \underline{j}-\Sigma_{\mathrm{a}} \phi
$$

Substituting for $\underline{j}$ using $\underline{j}=-D \nabla \phi$ (Fick's Law)

$$
\therefore \quad 0=\nabla \cdot(D \nabla \phi)-\Sigma_{\mathrm{a}} \phi
$$

As the medium is homogeneous, $D$ is constant

$$
\therefore \quad 0=D \nabla^{2} \phi-\Sigma_{\mathrm{a}} \phi
$$

Defining $L^{2}=\frac{D}{\Sigma_{\mathrm{a}}}$ where $L$ is the diffusion length

$$
\therefore \quad 0=\nabla^{2} \phi-\frac{\phi}{L^{2}}
$$

For spherical symmetry, from the 4M16 data sheet: $\nabla^{2}=\frac{1}{r^{2}} \frac{d}{d r}\left(r^{2} \frac{d}{d r}\right)$

$$
\therefore \quad 0=\frac{1}{r^{2}} \frac{d}{d r}\left(r^{2} \frac{d \phi}{d r}\right)-\frac{\phi}{L^{2}}
$$

Now $\frac{1}{r^{2}} \frac{d}{d r}\left(r^{2} \frac{d \phi}{d r}\right)=\frac{1}{r^{2}}\left(2 r \frac{d \phi}{d r}+r^{2} \frac{d^{2} \phi}{d r^{2}}\right)=\frac{1}{r}\left(2 \frac{d \phi}{d r}+r \frac{d^{2} \phi}{d r^{2}}\right)$
and $\frac{1}{r}\left(\frac{d^{2}(\phi r)}{d r^{2}}\right)=\frac{1}{r}\left(\frac{d}{d r}\left[\phi+r \frac{d \phi}{d r}\right]\right)=\frac{1}{r}\left(2 \frac{d \phi}{d r}+r \frac{d^{2} \phi}{d r^{2}}\right)$

$$
\therefore \quad 0=\frac{1}{r}\left(\frac{d^{2}(\phi r)}{d r^{2}}\right)-\frac{\phi}{L^{2}}
$$

(b) The result from (a) can be written as

$$
\frac{d^{2}(\phi r)}{d r^{2}}-\frac{\phi r}{L^{2}}=0
$$

By inspection, this differential equation has a general solution of the form

$$
\begin{aligned}
\phi r & =A \exp \left(\frac{r}{L}\right)+C \exp \left(-\frac{r}{L}\right) \\
\therefore \quad \phi & =\frac{A}{r} \exp \left(\frac{r}{L}\right)+\frac{C}{r} \exp \left(-\frac{r}{L}\right)
\end{aligned}
$$

From physical considerations, as $r \rightarrow \infty, \phi \rightarrow 0$. As $\exp \left(\frac{r}{L}\right)$ increases faster than $r$, this means that $A=0$.

$$
\begin{array}{ll} 
& \therefore \quad \phi=\frac{C}{r} \exp \left(-\frac{r}{L}\right) \\
\text { At } r=R_{1}, \phi=\phi_{1} & \therefore \phi_{1}=\frac{C}{R_{1}} \exp \left(-\frac{R_{1}}{L}\right) \\
& \therefore C=\phi_{1} R_{1} \exp \left(\frac{R_{1}}{L}\right) \\
\therefore \quad \phi=\frac{C}{r} \exp \left(-\frac{r}{L}\right)=\frac{\phi_{1} R_{1}}{r} \exp \left(\frac{R_{1}}{L}\right) \exp \left(-\frac{r}{L}\right)=\frac{\phi_{1} R_{1}}{r} \exp \left(\frac{R_{1}-r}{L}\right)
\end{array}
$$

(c) For a multiplying medium $\eta>0$, so the diffusion equation is now

$$
0=D \nabla^{2} \phi+(\eta-1) \Sigma_{\mathrm{a}} \phi
$$

(d) If $k_{\infty}>1$, then $\eta>1$

$$
\therefore \quad 0=\nabla^{2} \phi+B_{m}^{2} \phi
$$

where $B_{m}^{2}=\frac{(\eta-1) \Sigma_{\mathrm{a}}}{D}>0$ is the material buckling.
So, with spherical symmetry, $\quad \therefore \quad 0=\frac{1}{r}\left(\frac{d^{2}(\phi r)}{d r^{2}}\right)+B_{m}^{2} \phi$

$$
\therefore \quad \frac{d^{2}(\phi r)}{d r^{2}}+B_{m}^{2} \phi r=0
$$

This an SHM equation and therefore has the general solution

$$
\begin{gathered}
\phi r=A \sin \left(B_{m} r\right)+C \cos \left(B_{m} r\right) \\
\therefore \quad \phi=\frac{A}{r} \sin \left(B_{m} r\right)+\frac{C}{r} \cos \left(B_{m} r\right)
\end{gathered}
$$

When $r=0$, the flux $\phi$ must be finite

$$
\therefore \quad C=0
$$

$$
\therefore \quad \phi=\frac{A}{r} \sin \left(B_{m} r\right)
$$

The flux $\phi$ will not be zero at the physical edge of the reactor, $r=R_{2}$, say, because of neutron leakage. The standard boundary condition is to assume that the flux falls to zero at a small distance outside the reactor, the so-called extrapolation distance, at $r=R_{2}+\delta$, say. Using this boundary condition

$$
\begin{gather*}
\therefore \quad 0=\frac{A}{R_{2}+\delta} \sin \left(B_{m}\left[R_{2}+\delta\right]\right) \\
\therefore \quad B_{m}\left[R_{2}+\delta\right]=\pi \\
\\
\therefore \quad R_{2}=\frac{\pi}{B_{m}}-\delta
\end{gather*}
$$

## Assessor's Comments:

All candidates: 75 attempts, Average mark 12.9/20, Maximum 20, Minimum 2.
A popular question attempted by $89 \%$ of candidates, many of whom made good attempts. A general issue in many answers was insufficiently careful justification of steps.
Several answers to part (a) revealed that the candidate did not understand the meaning of the term 'non-multiplying'. The fact that the reactor being homogeneous implied a spatially invariant diffusion coefficient was also missed by many.
A number of candidates tried to apply a boundary condition at $r=0$ in part (b) despite this being outside the region of interest; others simply ignored the possible $\exp (r / L)$ solution entirely.
Many candidates had difficulties interpreting the significance of $k_{\infty}$ being greater than unity in part (d). Symmetry arguments were incorrectly invoked at $r=0$ to eliminate the sin term. Despite the requirement to carefully explain the boundary condition at the edge of the reactor, several candidates failed to discuss the extrapolation distance.

Q3
(a) AGR cladding is stainless steel. The coolant is carbon dioxide $\left(\mathrm{CO}_{2}\right)$.
(b)


Key features:
$\frac{d T}{d x} \neq 0$ at the ends as the power is not zero there (chopped cosine).
$T_{\text {coolant }}$ is symmetric about the channel centre.
$T_{\text {fuel }}>T_{\text {cladding }}>T_{\text {coolant }}$ throughout.
The temperature differences $T_{\text {fuel }}-T_{\text {cladding }}$ and $T_{\text {cladding }}-T_{\text {coolant }}$ are chopped cosines.
(c) Ginn's equation: $\quad \theta=\sin \left(\frac{\pi x}{2 L^{\prime}}\right)+Q \cos \left(\frac{\pi x}{2 L^{\prime}}\right)$
$\theta$ is a maximum when $\frac{d \theta}{d x}=0$

$$
\begin{gather*}
\therefore \frac{\pi}{2 L^{\prime}} \cos \left(\frac{\pi x}{2 L^{\prime}}\right)-\frac{\pi Q}{2 L^{\prime}} \sin \left(\frac{\pi x}{2 L^{\prime}}\right)=0 \\
\therefore \quad Q \sin \left(\frac{\pi x}{2 L^{\prime}}\right)=\cos \left(\frac{\pi x}{2 L^{\prime}}\right) \\
\therefore \quad \tan \left(\frac{\pi x}{2 L^{\prime}}\right)=\frac{1}{Q} \Rightarrow x=\frac{2 L^{\prime}}{\pi} \tan ^{-1}\left(\frac{1}{Q}\right)
\end{gather*}
$$

(d)

$$
Q=\frac{\pi \dot{m} c_{p}}{U A} \frac{L}{L^{\prime}}
$$

$$
A=4 \pi r_{o} L \quad \therefore \quad Q=\frac{\pi \dot{m} c_{p}}{U 4 \pi r_{o} L} \frac{L}{L^{\prime}}=\frac{\dot{m} c_{p}}{U 4 r_{o} L^{\prime}}
$$

From the information given in the question $r_{o}=0.4 \mathrm{~m}$ and $L^{\prime}=4.5 \mathrm{~m}$.
$\dot{m} c_{p}$ can be found from the channel power:

$$
\begin{gather*}
\dot{m} c_{p}\left(T_{\text {out }}-T_{\text {in }}\right)=P_{\text {channel }}  \tag{3.1}\\
\therefore \quad \dot{m} c_{p}=\frac{P_{\text {channel }}}{T_{\text {out }}-T_{\text {in }}}=\frac{10 \times 10^{6}}{635-335}=33.33 \times 10^{3} \mathrm{WK}^{-1}
\end{gather*}
$$

As there is no scale on the cladding, $U=h$.

$$
\begin{aligned}
& \therefore \\
& Q=\frac{\dot{m} c_{p}}{U 4 r_{o} L^{\prime}}=\frac{33.33 \times 10^{3}}{5 \times 10^{3} \times 4 \times 0.4 \times 4.5}=0.9259 \\
\therefore & x=\frac{2 L^{\prime}}{\pi} \tan ^{-1}\left(\frac{1}{Q}\right)=\frac{2 \times 4.5}{\pi} \tan ^{-1}\left(\frac{1}{0.9259}\right)=2.360 \mathrm{~m}
\end{aligned}
$$

i.e. 2.36 m past the channel centre.

$$
\begin{gathered}
\theta=\sin \left(\frac{\pi x}{2 L^{\prime}}\right)+Q \cos \left(\frac{\pi x}{2 L^{\prime}}\right) \\
\therefore \quad \theta_{\max }=\sin \left(\frac{\pi \times 2.36}{2 \times 4.5}\right)+0.9259 \cos \left(\frac{\pi \times 2.36}{2 \times 4.5}\right)=1.3628
\end{gathered}
$$

It is also acceptable to find $\theta_{\max }$ using the relationship $\theta_{\max }^{2}=1+Q^{2}$ (if remembered).

$$
\begin{aligned}
& \theta=\frac{T-T_{1 / 2}}{T_{\text {out }}-T_{1 / 2}} \sin \left(\frac{\pi L}{2 L^{\prime}}\right) \\
\therefore \quad & T=T_{1 / 2}+\frac{\theta}{\sin \left(\frac{\pi L}{2 L^{\prime}}\right)}\left(T_{\text {out }}-T_{1 / 2}\right)
\end{aligned}
$$

Here $T_{1 / 2}=485^{\circ} \mathrm{C}$ (the average of $T_{\text {in }}$ and $T_{\text {out }}$ ).

$$
\therefore \quad T_{\max }=485+\frac{1.3628}{\sin \left(\frac{\pi \times 4}{2 \times 4.5}\right)}(635-485)=692.6^{\circ} \mathrm{C}
$$

(e)
(i) From equation (3.1) for the same $T_{\text {out }}$ (given the same $T_{\text {in }}$ ) we need $P_{\text {channel }} / \dot{m} c_{p}$ to be constant. As $c_{p}$ is unchanged, if $P_{\text {channel }}$ is halved, we need to halve $\dot{m}$ ( $50 \%$ reduction).
(ii) If $T_{\text {in }}$ and $T_{\text {out }}$ are the same, $T_{\max }$ depends only on $\theta_{\max }$ (and therefore on $Q$ ). As $h$ is assumed to be unchanged, $U$ is unchanged. Therefore $Q$ depends only on $\dot{m} c_{p}$. As $\dot{m} c_{p}$ is halved, $Q$ is halved $\Rightarrow \theta_{\text {max }}$ is lower $\Rightarrow T_{\text {max }}$ is lower.
Alternatively: As a result of the gagging the axial coolant temperature distribution is the same in both channels. The local temperature difference between the coolant and the cladding surface depends on the power generated in the fuel at the axial location in question. Thus, these temperature differences are lower in the 5 MW channel, and hence the maximum cladding surface temperature will be lower.

## Assessor's Comments:

All candidates: 76 attempts, Average mark 13.7/20, Maximum 19, Minimum 5.
A popular question attempted by $90 \%$ of candidates, many of whom made good attempts. Disappointingly few candidates knew the materials used as cladding and coolant in AGRs. Most sketches provided in answer to part (b) were insufficiently detailed/accurate to gain full credit.
Several candidates provided proofs of $\theta_{\max }^{2}=1+Q^{2}$ as part of their answers to part (d), thereby wasting valuable time - the result was not even needed. Many attempts were undermined by errors of detail (confusing radius and diameter or length and half-length). Several completely implausible results were allowed to pass without comment. Several candidates overlooked the $\sin \left(\frac{\pi L}{2 L^{\prime}}\right)$ term in the equation for $\theta$; others failed to appreciate that $x$ is measured from the axial middle of the channel and/or used the fuel element crosssectional area rather than its surface area in calculating $Q$.

## Q4

(a) $\quad \gamma_{x} \Sigma_{f} \phi$ is the rate of production of $\mathrm{Xe}-135$ directly as a fission product
$\lambda_{i} I$ is the rate of production of $\mathrm{Xe}-135$ by the decay of I-135
$\lambda_{x} X$ is the rate of loss of Xe- 135 by radioactive decay
$\sigma X \phi$ is the rate of loss of $\mathrm{Xe}-135$ by transmutation to $\mathrm{Xe}-136$ through neutron capture
(b) In steady state, the equilibrium I-135 population is given by

$$
\frac{d I}{d t}=\gamma_{i} \Sigma_{f} \phi-\lambda_{i} I_{0}=0 \Rightarrow I_{0}=\frac{\gamma_{i} \Sigma_{f} \phi}{\lambda_{i}}
$$

and the equilibrium $\mathrm{Xe}-135$ population by

$$
\frac{d X}{d t}=\gamma_{x} \Sigma_{f} \phi+\lambda_{i} I_{0}-\lambda_{x} X_{0}-\sigma X_{0} \phi=0 \Rightarrow X_{0}=\frac{\gamma_{x} \Sigma_{f} \phi+\lambda_{i} I_{0}}{\lambda_{x}+\sigma \phi}=\frac{\left(\gamma_{x}+\gamma_{i}\right) \Sigma_{f} \phi}{\lambda_{x}+\sigma \phi}
$$

The poisoning effect of the Xe-135 is given by

$$
\rho_{\mathrm{Xe}}=\frac{\text { neutron loss rate due to xenon }}{\text { total neutron production rate }}=-\frac{\sigma X \phi}{v \Sigma_{f} \phi}=-\frac{\sigma X}{v \Sigma_{f}}
$$

So, the steady-state poisoning effect is

$$
\rho_{\mathrm{Xe} 0}=-\frac{\sigma}{v \Sigma_{f}} X_{0}=-\frac{\sigma}{v \Sigma_{f}} \frac{\left(\gamma_{x}+\gamma_{i}\right) \Sigma_{f} \phi}{\left(\lambda_{x}+\sigma \phi\right)}=-\frac{\sigma\left(\gamma_{x}+\gamma_{i}\right) \phi}{v\left(\lambda_{x}+\sigma \phi\right)}
$$

(c) If $\rho_{\mathrm{CR}}$ is the maximum excess reactivity available from the control rods, the maximum sustainable flux is reached when

$$
\begin{gather*}
\rho_{\mathrm{CR}}=\frac{\sigma\left(\gamma_{x}+\gamma_{i}\right) \phi_{\max }}{v\left(\lambda_{x}+\sigma \phi_{\max }\right)} \Rightarrow \phi_{\max }=\frac{v \rho_{\mathrm{CR}} \lambda_{x}}{\sigma\left(\gamma_{x}+\gamma_{i}-v \rho_{\mathrm{CR}}\right)} \\
\therefore \quad \phi_{\max }=\frac{2.43 \times 0.02 \times 2.093 \times 10^{-5}}{2.75 \times 10^{-22}(0.003+0.061-2.43 \times 0.02)}=2.402 \times 10^{17} \mathrm{~m}^{-2} \mathrm{~s}^{-1}
\end{gather*}
$$

(d) Post-shutdown the flux $\phi$ will be zero. With zero flux the I-135 and Xe-135 populations vary according to:

$$
\frac{d I}{d t}=-\lambda_{i} I
$$

and

$$
\frac{d X}{d t}+\lambda_{x} X=\lambda_{i} I
$$

Given that $I=I_{0}$ at $t=0$, the I-135 population will clearly just decline exponentially:

$$
\begin{gather*}
I=I_{0} \exp \left(-\lambda_{i} t\right) \\
\therefore \frac{d X}{d t}+\lambda_{x} X=\lambda_{i} I_{0} \exp \left(-\lambda_{i} t\right) \tag{4.1}
\end{gather*}
$$

The solution of equation (4.1) will be made up of a particular integral (PI) and a complementary function (CF). Looking at the right-hand side, the PI will be of the form:

$$
X_{\mathrm{PI}}=A \exp \left(-\lambda_{i} t\right)
$$

Substituting into the left-hand side:

$$
\begin{gathered}
-\lambda_{i} A \exp \left(-\lambda_{i} t\right)+\lambda_{x} A \exp \left(-\lambda_{i} t\right)=\lambda_{i} I_{0} \exp \left(-\lambda_{i} t\right) \\
\therefore A=\frac{\lambda_{i} I_{0}}{\lambda_{x}-\lambda_{i}}
\end{gathered}
$$

The CF is the solution of the homogeneous form of equation (4.1):

$$
\frac{d X}{d t}+\lambda_{x} X=0
$$

which is, by inspection, of the form:

$$
X_{\mathrm{CF}}=B \exp \left(-\lambda_{x} t\right)
$$

Thus, the general solution for $X$ is:

$$
X=X_{\mathrm{PI}}+X_{\mathrm{CF}}=\frac{\lambda_{i} I_{0}}{\lambda_{x}-\lambda_{i}} \exp \left(-\lambda_{i} t\right)+B \exp \left(-\lambda_{x} t\right)
$$

To find $B$, use the boundary condition $X=X_{0}$ at $t=0$ :

$$
\begin{gathered}
\therefore \quad X_{0}=\frac{\lambda_{i} I_{0}}{\lambda_{x}-\lambda_{i}}+B \Rightarrow B=X_{0}-\frac{\lambda_{i} I_{0}}{\lambda_{x}-\lambda_{i}} \\
\therefore \quad X=X_{0} \exp \left(-\lambda_{x} t\right)+\frac{\lambda_{i} I_{0}}{\lambda_{x}-\lambda_{i}}\left[\exp \left(-\lambda_{i} t\right)-\exp \left(-\lambda_{x} t\right)\right]
\end{gathered}
$$

Using results found in part (b):

$$
I_{0}=\frac{\gamma_{i} \Sigma_{f} \phi_{\max }}{\lambda_{i}}
$$

and

$$
\begin{gather*}
X_{0}=\frac{\left(\gamma_{x}+\gamma_{i}\right) \Sigma_{f} \phi_{\max }}{\lambda_{x}+\sigma \phi_{\max }} \\
\therefore \quad X=\frac{\left(\gamma_{x}+\gamma_{i}\right) \Sigma_{f} \phi_{\max }}{\lambda_{x}+\sigma \phi_{\max }} \exp \left(-\lambda_{x} t\right)+\frac{\gamma_{i} \Sigma_{f} \phi_{\max }}{\lambda_{x}-\lambda_{i}}\left[\exp \left(-\lambda_{i} t\right)-\exp \left(-\lambda_{x} t\right)\right]
\end{gather*}
$$

(e) The Xe - 135 population will rise immediately after the shutdown, go through a maximum and then fall (eventually to zero). As prior to the shutdown the reactor was operating at the highest possible flux level (determined by the amount of xenon poisoning that can be overcome), it will not be possible to restart the reactor until the $\mathrm{Xe}-135$ population falls again to $X_{0}$, its level when shutdown occurred.
Hence, using the results in part (d), this will occur at time $t$ when

$$
\begin{gathered}
\frac{\left(\gamma_{x}+\gamma_{i}\right) \Sigma_{f} \phi_{\max }}{\lambda_{x}+\sigma \phi_{\max }}=\frac{\left(\gamma_{x}+\gamma_{i}\right) \Sigma_{f} \phi_{\max }}{\lambda_{x}+\sigma \phi_{\max }} \exp \left(-\lambda_{x} t\right)+\frac{\gamma_{i} \Sigma_{f} \phi_{\max }}{\lambda_{x}-\lambda_{i}}\left[\exp \left(-\lambda_{i} t\right)-\exp \left(-\lambda_{x} t\right)\right] \\
\therefore \frac{\gamma_{x}+\gamma_{i}}{\lambda_{x}+\sigma \phi_{\max }}\left[1-\exp \left(-\lambda_{x} t\right)\right]=\frac{\gamma_{i}}{\lambda_{x}-\lambda_{i}}\left[\exp \left(-\lambda_{i} t\right)-\exp \left(-\lambda_{x} t\right)\right]
\end{gathered}
$$

This time is in practice of the order of several hours, so spurious trips can result in significant unnecessary loss of generation (and therefore revenue).

## Assessor's Comments:

All candidates: 78 attempts, Average mark 14.8/20, Maximum 20, Minimum 4.
A popular question attempted by $93 \%$ of candidates, many of whom made good attempts.
Most answers to parts (a) and (b) were very good.
Several candidates had difficulties with signs in part (c). Many candidates did not know the correct units for neutron flux. Some candidates calculated the maximum flux to be negative without comment.
A surprising number of candidates did not recognize that the neutron flux would be zero when the reactor was shut down and therefore tried to solve the wrong differential equations in their attempt at part (d).
In part (e), the majority of candidates erroneously thought that the reactor could be started when the $\mathrm{Xe}-135$ population stops increasing. That is not the case in the scenario here.

