

Q1

- (a) (i)  $\alpha$  decay is the release of a He-4 nucleus, usually accompanied by  $\gamma$ -rays (electromagnetic radiation). The mass number falls by 4, the atomic number by 2.
- (ii)  $\beta$  decay is the emission of a negatively charged particle equivalent to an electron by the transformation of a neutron to a proton, accompanied by  $\gamma$ -rays and a neutrino. The mass number is unchanged; the atomic number increases by 1.
- (iii)  $\gamma$  decay is the emission of electromagnetic radiation. If only a  $\gamma$ -ray is emitted, there is no change in mass number nor atomic number. [15%]

- (b) Exposure is minimised by minimising the time of exposure, maximising the distance from the source, and the use of shielding where appropriate.

$\alpha$ 's are highly charged and heavy and are absorbed by the outer layers of the skin (dead tissue), so they are only dangerous when ingested or inhaled. To work with  $\alpha$ 's no shielding is needed, but work should be carried out in glove boxes to ensure the material cannot enter the body.

$\beta$ 's do require some shielding and, again, precautions must be taken to ensure that they do not enter the body. It must be remembered that when  $\beta$ 's are attenuated by shielding they give rise to secondary radiation (bremsstrahlung, a form of electromagnetic radiation) that may require secondary shielding.

$\gamma$ 's are much more difficult to attenuate and can cause serious damage to living tissue, even if the source does not enter the body. Significant shielding using materials such as lead can be required. [25%]

- (c) For the reaction  ${}^9\text{Be} + {}^4\text{He} \rightarrow {}^{12}\text{C} + \text{n}$

$$\Delta u = 9.01219 + 4.00260 - 12.00000 - 1.00867 = 6.12 \times 10^{-3} \text{ u}$$

$$\therefore \Delta u = 6.12 \times 10^{-3} \times 931.5016 = 5.701 \text{ MeV}$$

Assuming the neutron receives all this energy plus the energy of the  $\alpha$ , the maximum neutron energy is

$$5.70 + 5.49 = 11.19 \text{ MeV} \quad [15\%]$$

- (d) The number of  ${}^{241}\text{Am}$  atoms present in 10 g is

$$N = \frac{m}{M} L = \frac{0.01}{0.241} \times 6.022 \times 10^{23} = 2.499 \times 10^{22}$$

The  ${}^{241}\text{Am}$  decay constant is

$$\lambda = \frac{\ln(2)}{T_{1/2}} = \frac{\ln(2)}{432 \times 365 \times 24 \times 3600} = 5.088 \times 10^{-11} \text{ s}^{-1}$$

Thus, the  ${}^{241}\text{Am}$  activity is

$$A = \lambda N = 5.088 \times 10^{-11} \times 2.499 \times 10^{22} = 1.271 \times 10^{12} \text{ Bq}$$

So, the neutron source rate is

$$S = 0.25A = 0.25 \times 1.271 \times 10^{12} = 3.179 \times 10^{11} \text{ s}^{-1}$$

Thus, assuming no significant absorption in air, the flux at 1 m is

$$\phi = \frac{S}{4\pi r^2} = \frac{3.179 \times 10^{11}}{4\pi \times 1^2} = 2.529 \times 10^{10} \text{ m}^{-2}\text{s}^{-1} \quad [25\%]$$

- (e) From the result in (c) the Am-Be source produces fast neutrons. To make an effective shield against fast neutrons you require
- (i) a moderator to slow the neutrons to energies where they are more readily captured – water, graphite, or a polymer, e.g. polythene, are possible choices depending on the application,
  - (ii) a material with a high neutron capture cross-section for thermal neutrons (e.g. cadmium, boron, indium) – the same materials used in control rods/materials,
  - (iii) something to shield against the  $\gamma$ 's arising from the neutron capture reactions – lead would be suitable for this. [20%]

### Assessor's Comments:

*All candidates: 72 attempts, Average raw mark 15.0/20, Maximum 19, Minimum 8.*

A very popular question attempted by 90% of candidates and generally done very well.

Part (a) was answered very well by almost all candidates.

Some answers to part (b) confused the basic principles of dose minimization with the ALARA principles, while others confused “working with” with “disposing of”.

A number of candidates overlooked the mass of the neutron or the energy of alpha particle in the maximum neutron energy calculation in part (c). There was also a surprisingly high incidence of calculator errors, with the correct expression leading to an incorrect numerical answer.

The calculation in part (d) was generally done well but the accompanying assumptions were not recognised or justified by many candidates.

Many answers to part (d) failed to consider all aspects of the challenge of shielding against fast neutrons.

### Q2

- (a) One-group diffusion theory assumes that all neutrons are at a single constant energy. This is, of course, not the case in practice, even in a fast reactor.

Diffusion theory also assumes that the region being modelled by an equation with given parameter values is homogeneous. In reality the various core components (fuel elements, cladding, coolant, moderator, control rods etc.) have very different neutronic properties, so the use of a single equation to model an entire core represents a gross simplification. [10%]

(b) 
$$\frac{dn}{dt} = -\nabla \cdot \underline{j} + (\eta - 1)\Sigma_a\phi + S$$

Steady-state:  $\therefore \frac{dn}{dt} = 0$

Source-free:  $\therefore S = 0$

Fick's law:  $\underline{j} = -D\nabla\phi$

$$\therefore 0 = -\nabla \cdot (-D\nabla\phi) + (\eta - 1)\Sigma_a\phi$$

If  $D$  is spatially invariant (as implied by the fact that the system is homogeneous):

$$\begin{aligned}\therefore 0 &= D\nabla^2\phi + (\eta - 1)\Sigma_a\phi \\ \therefore \nabla^2\phi + \frac{(\eta - 1)\Sigma_a}{D}\phi &= 0\end{aligned}\quad [15\%]$$

(c)

(i) For cylindrical geometry with rotational symmetry

$$\begin{aligned}\nabla^2 &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2} \\ \therefore \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{\partial z^2} + \frac{(\eta - 1)\Sigma_a}{D}\phi &= 0\end{aligned}$$

Let  $B^2 = \frac{(\eta - 1)\Sigma_a}{D}$  and assume that  $\phi(r, z) = F(r)Z(z)$

$$\begin{aligned}\therefore \frac{Z}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + F\frac{\partial^2 Z}{\partial z^2} + B^2 FZ &= 0 \\ \therefore \frac{1}{Fr}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + \frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + B^2 &= 0\end{aligned}$$

This implies that

$$\frac{1}{Z}\frac{d^2 Z}{dz^2} + \beta^2 = 0$$

and

$$\frac{1}{Fr}\frac{d}{dr}\left(r\frac{dF}{dr}\right) + \alpha^2 = 0$$

with

$$\alpha^2 + \beta^2 = B^2 \quad (2.1)$$

The equation in  $z$  is an SHM equation, so the general solution is

$$Z(z) = A\sin(\beta z) + C\cos(\beta z)$$

By symmetry arguments

$$A = 0$$

If extrapolation distances are neglected, the boundary condition is that  $\phi = 0$  at  $z = \pm \frac{H}{2}$ , so

$$\beta\frac{H}{2} = \frac{\pi}{2} \Rightarrow \beta = \frac{\pi}{H}$$

The equation in  $r$  is a Bessel equation of zero order, so the general solution is

$$F(r) = PJ_0(\alpha r) + QY_0(\alpha r)$$

As  $r \rightarrow 0$ ,  $Y_0(\alpha r) \rightarrow -\infty$ . This would give infinite flux at the centre of the reactor, which is physically impossible, so  $Q = 0$ .

The first zero of  $J_0(x)$  occurs at  $x = 2.405$ , so with a boundary condition that  $\phi = 0$  at  $r = R$

$$\therefore \alpha R = 2.405 \Rightarrow \alpha = \frac{2.405}{R}$$

Thus

$$\phi(r, z) = PCJ_0(\alpha r)\cos(\beta z)$$

If the flux at the centre of the core ( $r = 0, z = 0$ ) is  $\phi_0$  and given that  $J_0(0) = \cos(0) = 1$

$$\therefore PC = \phi_0$$

$$\therefore \phi(r, z) = \phi_0 J_0(\alpha r) \cos(\beta z) \quad [40\%]$$

(ii) From equation (2.1)

$$\alpha^2 + \beta^2 = B^2$$

$$\therefore \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \frac{(\eta-1)\Sigma_a}{D} \quad [5\%]$$

(d)

$$V = \pi R^2 H$$

From (c)(ii)

$$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = B^2$$

$$\therefore R^2 = \frac{(2.405)^2}{B^2 - (\pi/H)^2}$$

$$\therefore V = \frac{\pi(2.405)^2 H}{B^2 - (\pi/H)^2}$$

$V$  is minimized when  $\frac{dV}{dH} = 0$

$$\frac{dV}{dH} = \pi(2.405)^2 \left[ \frac{B^2 - \left(\frac{\pi}{H}\right)^2 + H \left[-\frac{2\pi^2}{H^3}\right]}{\left[B^2 - (\pi/H)^2\right]^2} \right] = \pi(2.405)^2 \left[ \frac{B^2 - 3\left(\frac{\pi}{H}\right)^2}{\left[B^2 - (\pi/H)^2\right]^2} \right]$$

Thus,  $\frac{dV}{dH} = 0$  when  $B^2 - 3\left(\frac{\pi}{H}\right)^2 = 0 \Rightarrow H = \frac{\pi\sqrt{3}}{B}$

$$\therefore R^2 = \frac{(2.405)^2}{B^2 - (\pi/H)^2} = \frac{(2.405)^2}{B^2 - (B/\sqrt{3})^2} = \frac{(2.405)^2}{\frac{2}{3}B^2} \Rightarrow R = \sqrt{\frac{3}{2}} \frac{2.405}{B}$$

$$\frac{H}{R} = \frac{\pi\sqrt{3}}{B} \sqrt{\frac{2}{3}} \frac{B}{2.405} = \frac{\pi\sqrt{2}}{2.405} = 1.847 \quad [30\%]$$

### Assessor's Comments:

*All candidates: 78 attempts, Average raw mark 15.0/20, Maximum 20, Minimum 6.*

Another very popular question attempted by all but two candidates and generally done very well.

A surprising number of candidates apparently did not know that “one-group” meant “constant neutron energy” in answering part (a).

Parts (b) and (c) were generally done rather well. The most common reasons for losing marks were the use of incorrect boundary conditions in solving the equations and/or poor justifications of steps. There was some lack of rigour (or confusion) about partial vs ordinary differentials in the solution of partial differential equation governing the flux distribution in (c)(i).

Part (d) was done less well in comparison, despite its similarity to an Examples Paper question.

Q3

- (a) Xenon-135, which is produced directly in small quantities as a fission product and by the decay of the more commonly produced iodine-135, has a very high capture cross-section for thermal neutrons and therefore captures these extensively. This affects the neutron balance for a reactor and, if not compensated for by the withdrawal of control rods, for example, can take the reactor subcritical. To maintain steady-state operation the unavoidable poisoning due to the equilibrium xenon-135 population must be allowed for in the design of the reactivity control system.

When a reactor is shut down xenon-135 will continue to be produced by the decay of iodine-135, but will no longer be eliminated by neutron capture. The xenon-135 population will therefore rise initially. Xenon-135 is radioactive and decays to caesium-135 with a half-life of about 9 hours, so the post-shutdown xenon-135 population does eventually decrease, but there may be of several hours during which there is insufficient excess reactivity available to overcome the effect of the xenon poisoning and restart the reactor. [20%]

(b)

(i) In steady state 
$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I = 0 \Rightarrow I_0 = \frac{\gamma_i \Sigma_f \phi}{\lambda_i}$$

and 
$$\frac{dX}{dt} = \lambda_i I - \lambda_x X - \phi \sigma X = 0 \Rightarrow X_0 = \frac{\lambda_i I_0}{\lambda_x + \phi \sigma} = \frac{\gamma_i \Sigma_f \phi}{\lambda_x + \phi \sigma}$$

For a high power reactor, the flux  $\phi$  is high, so  $\phi \sigma \gg \lambda_x$  and hence  $X_0 \rightarrow \frac{\gamma_i \Sigma_f}{\sigma}$

The reactivity loss (poisoning) 
$$\rho_{Xe} = -\frac{\sigma X}{\nu \Sigma_f}$$

$$\therefore \rho_0 = -\frac{\sigma X_0}{\nu \Sigma_f} = -\frac{\sigma}{\nu \Sigma_f} \frac{\gamma_i \Sigma_f}{\sigma} = -\frac{\gamma_i}{\nu} \quad [20\%]$$

- (ii) At shutdown  $\phi = 0$ ,  $I = I_0$  and  $X = X_0$

$$\therefore \frac{dI}{dt} = -\lambda_i I \Rightarrow I = I_0 \exp(-\lambda_i t) \quad \text{by inspection}$$

and 
$$\frac{dX}{dt} + \lambda_x X = \lambda_i I = \lambda_i I_0 \exp(-\lambda_i t)$$

By inspection, the particular integral  $X_{PI} = A \exp(-\lambda_i t)$

$$\therefore -\lambda_i A \exp(-\lambda_i t) + \lambda_x A \exp(-\lambda_i t) = \lambda_i I_0 \exp(-\lambda_i t)$$

$$\therefore A = \frac{\lambda_i I_0}{\lambda_x - \lambda_i}$$

Also by inspection, the complementary function  $X_{CF} = B \exp(-\lambda_x t)$

Therefore the general solution is

$$X_{GS} = \frac{\lambda_i I_0}{\lambda_x - \lambda_i} \exp(-\lambda_i t) + B \exp(-\lambda_x t)$$

With the initial condition that  $X = X_0$  at  $t = 0$

$$X_0 = \frac{\lambda_i I_0}{\lambda_x - \lambda_i} + B \Rightarrow B = X_0 - \frac{\lambda_i I_0}{\lambda_x - \lambda_i}$$

$$\therefore X_{\text{GS}} = \frac{\lambda_i I_0}{\lambda_x - \lambda_i} \exp(-\lambda_i t) + \left[ X_0 - \frac{\lambda_i I_0}{\lambda_x - \lambda_i} \right] \exp(-\lambda_x t)$$

Substituting for  $I_0$  and  $X_0$  using the expressions found in part (b)

$$\therefore X = \frac{\gamma_i \Sigma_f \phi}{\lambda_x - \lambda_i} \exp(-\lambda_i t) + \left[ \frac{\gamma_i \Sigma_f \phi}{\lambda_x + \phi \sigma} - \frac{\gamma_i \Sigma_f \phi}{\lambda_x - \lambda_i} \right] \exp(-\lambda_x t)$$

If  $|\lambda_x + \phi \sigma| \gg |\lambda_i - \lambda_x|$ , then  $\left| \frac{\gamma_i \Sigma_f \phi}{\lambda_x - \lambda_i} \right| \gg \left| \frac{\gamma_i \Sigma_f \phi}{\lambda_x + \phi \sigma} \right|$

$$\therefore X \approx \frac{\gamma_i \Sigma_f \phi}{\lambda_x - \lambda_i} [\exp(-\lambda_i t) - \exp(-\lambda_x t)] \quad [40\%]$$

(iii)  $X$  is maximized when  $\frac{dX}{dt} = 0$

$$\therefore \lambda_i \exp(-\lambda_i t) = \lambda_x \exp(-\lambda_x t)$$

$$\therefore \ln(\lambda_i) - \lambda_i t = \ln(\lambda_x) - \lambda_x t$$

$$\therefore t = \frac{\ln(\lambda_i / \lambda_x)}{\lambda_i - \lambda_x}$$

$$\lambda_i = \frac{\ln(2)}{T_{1/2i}} = \frac{\ln(2)}{6.7} = 0.10345 \text{ hr}^{-1}$$

$$\lambda_x = \frac{\ln(2)}{T_{1/2x}} = \frac{\ln(2)}{9.2} = 0.07534 \text{ hr}^{-1}$$

$$\therefore t = \frac{\ln(0.10345/0.07534)}{0.10345 - 0.07534} = 11.28 \text{ hours}$$

If we assume that it takes at least the same amount of time again for the xenon-135 concentration to return to its level when shutdown occurred, then the implication is that it could take the best part of a day before the reactor can be restarted – exactly how long depends on the amount of excess reactivity available (e.g. through control rod movement) when shutdown occurred. [20%]

### Assessor's Comments:

*All candidates: 76 attempts, Average raw mark 14.3/20, Maximum 19, Minimum 3.*

Another very popular question attempted by all but four candidates and generally done very well. Many of the descriptions of xenon poisoning and its operational implications in part (a) were rather superficial. Some candidates evidently confused xenon poisoning with the kinetics of delayed neutrons.

Answers to parts (b)(i) and (b)(ii) often lacked careful justification of steps in deriving results. A number of answers to (b)(ii) failed to recognise that the flux would be zero after shutdown.

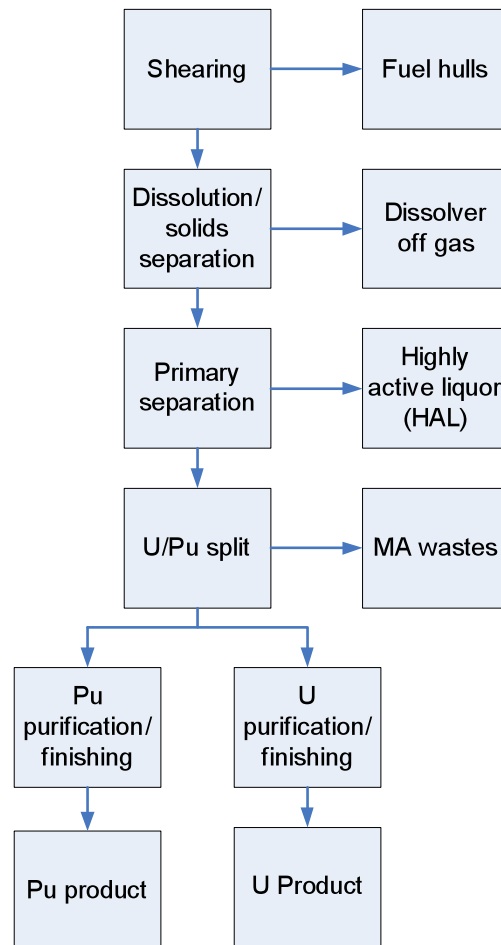
Some attempts to solve the differential equation governing the xenon population showed confusion between the roles of particular integrals and complementary functions.

Some answers in (b)(ii) went straight to the high flux solution for xenon variation without finding the more general solution first, as required.

The time of maximum post-shutdown xenon was correctly found by most candidates. Unsuccessful attempts generally failed because the candidate did not understand the difference between a half-life and a decay constant. Discussions of the practical significance of the result often failed to recognise that the reactor restart time would depend on level of excess reactivity available.

Q4

(a) The basic steps in reprocessing are:



1. Removal of cladding either by shearing (oxide fuel) or use of cutting dies (Magnox).
2. Dissolution of fuel in hot concentrated nitric acid; the resulting liquor is clarified (by centrifuge in the case of oxide fuel).
3. Primary separation by solvent extraction using tri-butyl phosphate dissolved in odourless kerosene (TBP/OK). The fission products remain in the aqueous stream whilst the uranium and plutonium transfer to the organic (TBP/OK) stream. Plutonium/uranium separation by reduction of the valency of plutonium, which renders it insoluble in TBP/OK. The reduction is effected either by the use of ferrous sulphamate or hydrazine/UIV. The former is a better reducing agent but adding further metal ions complicates the waste treatment processes. The plutonium is backwashed into dilute nitric acid leaving the uranium in the organic phase.

4. Uranium purification by backwashing the uranium into the aqueous phase by using very dilute nitric acid. The uranium is purified by adjusting the pH, back-extracting into TBP/OK and then back again into dilute nitric acid. It is then concentrated and oxidised to uranium dioxide.
5. The plutonium is re-oxidised, backwashed into dilute nitric acid, then the cycle is repeated leaving the plutonium in an aqueous solution. It is precipitated as an oxalate, then calcined to produce plutonium dioxide. [30%]

- (b) + Spent nuclear fuel still contains about 96% of the original uranium, of which the  $^{235}\text{U}$  content is around 0.85%, depending on initial enrichment and burn-up. Recovering and reusing the uranium enables about 25% more energy to be produced from the uranium – better resource utilisation.

+ The separation of the highly active fission products and actinides reduces the volume of high-level waste to be disposed of.

+ Plutonium bred from the uranium can be recovered for use in MOX and fast reactor fuels.

– The principal problem with reprocessing is one of cost. When the decision to build THORP and its equivalent in France was made, it was predicted that the price of uranium would rise to at least \$100 per pound by 1990 (approx. \$500 per pound at 2019 prices) whereas the current price of uranium is only around \$30 per pound. Reprocessing can therefore not be justified on cost grounds alone.

Magnox spent fuel is reprocessed because of the problems that result from contact with water: the cladding corrodes, and the metallic fuel reacts with water to produce uranium hydride (which can ignite spontaneously in air). Contact with water occurs when the spent fuel is cooled under water after being removed from the reactor. Contact with water would also be likely in a repository. [20%]

(c)

- (i) Thermal power is given by:  $P_{\text{th}} = \frac{P_e}{\eta_{\text{th}}} = \frac{1600}{0.33} = 4848 \text{ MW(th)}$

Annual thermal output is given by

$$4848 \text{ MW(th)} \times 0.9 \times 365 \text{ days/year} = 1.593 \times 10^6 \text{ MWd}$$

Annual fuel requirement (this is  $P$  as needed in calculations below) is therefore given by

$$\frac{1.593 \times 10^6 \text{ MWd}}{45 \times 10^3 \text{ MWdte}^{-1}} = 35.40 \text{ te}$$

Mass balance across the enrichment plant requires

$$F = P + W \quad \Rightarrow \quad W = F - P$$

$$x_f F = x_p P + x_w W$$

$$\therefore x_f F = x_p P + x_w (F - P)$$

$$\therefore F(x_f - x_w) = P(x_p - x_w) \quad \Rightarrow \quad F = P \left( \frac{x_p - x_w}{x_f - x_w} \right)$$

Using the values given in the question and calculated above



$$\therefore F = 35.40 \left( \frac{0.04 - 0.003}{0.007 - 0.003} \right) = 327.45 \text{ te} \quad [20\%]$$

(ii) The recovered U, after accounting for reprocessing losses, is

$$0.985 \times 33 = 32.505 \text{ te at } 0.85\% \text{ } ^{235}\text{U}$$

Rearranging the equation in (i), the amount of recovered fuel after enrichment to 4.3% is given by

$$P_r = F_r \left( \frac{x_{fr} - x_w}{x_{pr} - x_w} \right)$$

$$\therefore P_r = 32.505 \left( \frac{0.0085 - 0.003}{0.043 - 0.003} \right) = 4.469 \text{ te}$$

The new amount of fresh product needed is therefore

$$P' = P - P_r = 35.4 - 4.469 = 30.93 \text{ te}$$

Thus, the new amount of UOC needed is given by

$$\frac{P}{F} = \frac{P'}{F'} \Rightarrow F' = F \times \frac{P'}{P} = 327.45 \times \frac{30.93}{35.40} = 286.10 \text{ te}$$

So the saving in UOC is

$$\Delta F = F - F' = 327.45 - 286.10 = 41.35 \text{ te} \quad [15\%]$$

(iii) The cost of reprocessing =  $1200 \times 35,400 = \$42.48 \times 10^6$

The cost of spent fuel disposal =  $400 \times 35,400 = \$14.16 \times 10^6$

$\therefore$  Difference =  $\$28.32 \times 10^6$

To make reprocessing economic, the saving in the cost of UOC must be more than this difference. Therefore the cost of UOC per unit mass must be more than

$$\frac{28.32 \times 10^6}{41.35} = 684.9 \times 10^3 \text{ \$ per te} = 684.9 \text{ \$ per kg} = 1506.7 \text{ \$ per pound}$$

Such a high price is very unlikely any time in the near future. This figure is significantly higher than the original price used as a basis for justifying THORP, although it should be noted that the calculation ignores any credit for the re-use of the plutonium in MOX fuel. [15%]

### Assessor's Comments:

*All candidates: 14 attempts, Average raw mark 11.8/20, Maximum 16, Minimum 3.*

Comfortably the least popular question, attempted by only 14 (17.5%) of candidates, and the least well done.

Several answers to part (a) included unnecessary discussion of the ways of treating the principal waste streams arising from the Purex process, rather than simply identifying them.

Almost no one could recall in any detail why Magnox spent fuel needed to be reprocessed.

Calculations in part (c) were generally on the right lines but often showed insufficient attention to detail. Some answers included unnecessary separative work unit calculations.

A few very desultory attempts dragged down the overall average for Part IIA candidates.