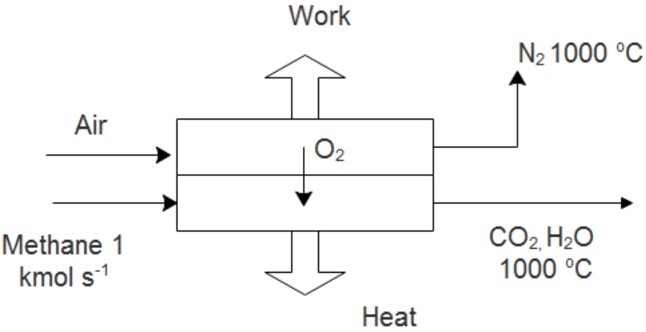
1.

The availability of a stream is defined so that the change in availability is equal to the maximum work that can be obtained from the change in state. Exergy is simply the availability with respect to a defined reference state, the environment. The exergy therefore gives an absolute measure of the work value of a material stream. Most of the irreversibility in a conventional power station comes from the combustion reaction itself, and the transfer of heat from the hot gases to the much colder steam cycle.

[20%]



b) i)

For the fuel cell

$$0 = Q\left(1 - \frac{T_o}{T}\right) + W - T_0 \Delta S_{irrev} + \sum B_{in} - \sum B_{out}$$

And

$$0 = Q + W + \sum H_{in} - \sum H_{out}$$

The overall enthalpy change over the fuel cell is

$$\sum H_{out} - \sum H_{in} =$$

$$1 \times (-344889) + 2 \times (-204068) + 2 \times \frac{0.79}{0.21} \times (30587) - 1 \times (-74600) = -448294$$

The overall enthalpy change in entropy is

$$\begin{aligned}\sum S_{out} - \sum S_{in} = {} & 1 \times \left(282.7 - 8.314 \times \ln\left(\frac{1}{3}\right)\right) + 2 \times \left(243.1 - 8.314 \times \ln\left(\frac{2}{3}\right)\right) + 2 \times \frac{0.79}{0.21} \\ & \times (236.2) - 1 \times (186.4) - 2 \times \frac{0.79}{0.21} \times (191.6 - 8.314 \times \ln(0.79)) - 2 \\ & \times (205.1 - 8.314 \times \ln(0.21)) = 483.0\end{aligned}$$

Therefore

$$\sum B_{out} - \sum B_{in} = -448294 - 298.15 \times 483.0 = -592300$$

[15%]

b) ii) So for the reversible fuel cell

$$-592300 = Q\left(1 - \frac{298.15}{1273.15}\right) + W - T_o\Delta S_{irrev}$$

$$-448294 = Q + W$$

Combining (note that this is just the entropy balance)

$$\Delta H - To\Delta S = Q\left(1 - \frac{298.15}{1273.15}\right) + W - T_o\Delta S_{irrev}$$

$$\Delta S = Q\left(\frac{1}{1273.15}\right) + \Delta S_{irrev}$$

$$483 - 500 = Q\left(\frac{1}{1273.15}\right)$$

$$Q = -17 \times 1273.15 = -21644\, kW$$

$$W = -448294 + 21644 = -426650\, kW$$

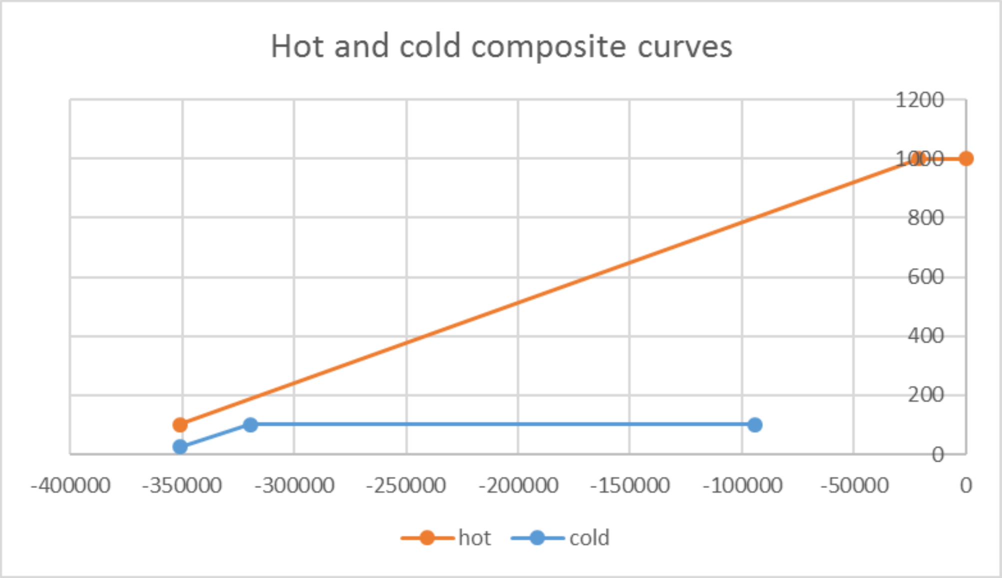
[15%]

c)i)

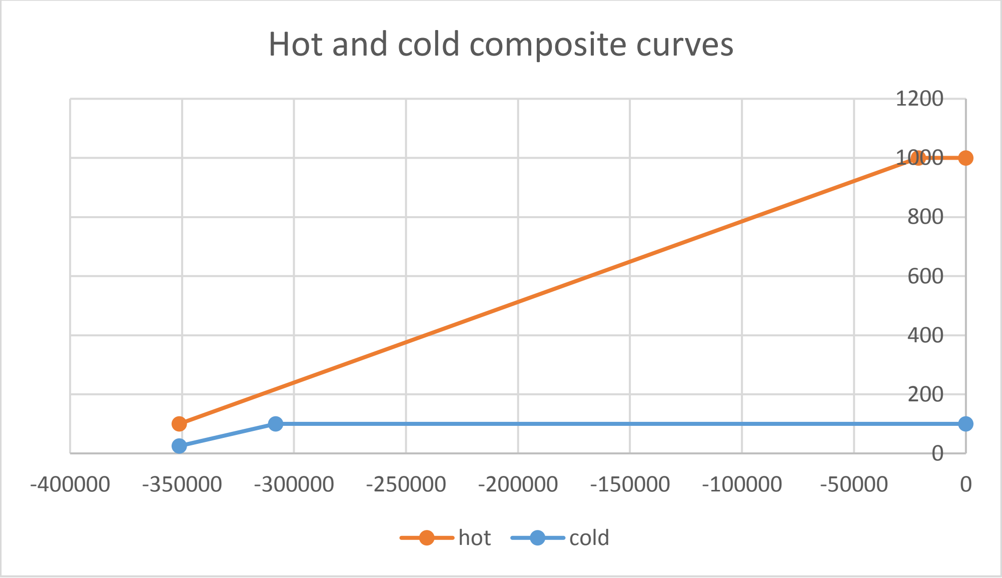
The exit streams can be combined as they cover the same temperature interval, so the heat loads are

Temperature (celcius)		
Start	end	Interval heat load (kW)
	1000	
1000	1000	-21644
1000	100	-852734-(-522894)= -329840

The hot composite is shown below. The cold composite is made up of two sections, first heating from 25 to 100 with an enthalpy change of $(419.2 - 104.8) \times \dot{m}_w$ and the second vapourisation with an enthalpy change of $(2256.4) \times \dot{m}_w$. The cold composite curve therefore can be stretched, below it is shown with $\dot{m} = 100$. As all of the heat from the fuel cell is available at temperatures above that for the water boiling, there is no pinch, hence all the heat can be utilised.



Thus the actual composite curves are shown below.



Since all the heat can be utilised

$$\dot{m}_w = -\frac{329840}{2570.8} = 136.7 \text{ kg s}^{-1}$$

[25%]

c)ii) The Exergy outputs are the work and the steam

For the steam the exergy output is $\Delta H - T_0 \Delta S$ where $\Delta$is the difference between the environment state (saturated liquid water at 298 K) and steam (100 °C and 1 atm) i.e.

$$\Delta H - T_0 \Delta S = (2675.6 - 104.8) - 298.15 \times (1.307 - 0.367) = 2290.53 \; kJ \; kg^{-1}$$

Multiplying by the mass flow gives

$$E_{steam} = 313117 \; kW$$

The work is $E_w = 426650$ kW

So total exergy output is $E_{out} = 739767\ kW$

The exergy input is the availability of the CH4 relative to the environment state. This calculation is greatly simplified by being given the exergies of pure CO_2 and O_2.

For the combustion of methane (using pure O_2 to pure CO_2 and Pure liquid water).

$$\Delta B = \Delta H - T_0 \Delta S =$$

$$(2 \times -285830 - 393510 + 74600) - 298.15 \times (2 \times -69 - 213.8 + 186.4)$$

$$= -841256$$

So the Exergy of the methane is

$$841256 + 19400 - 3900 =$$

$$856756\ kJ\ s^{-1}$$

The efficiency is then

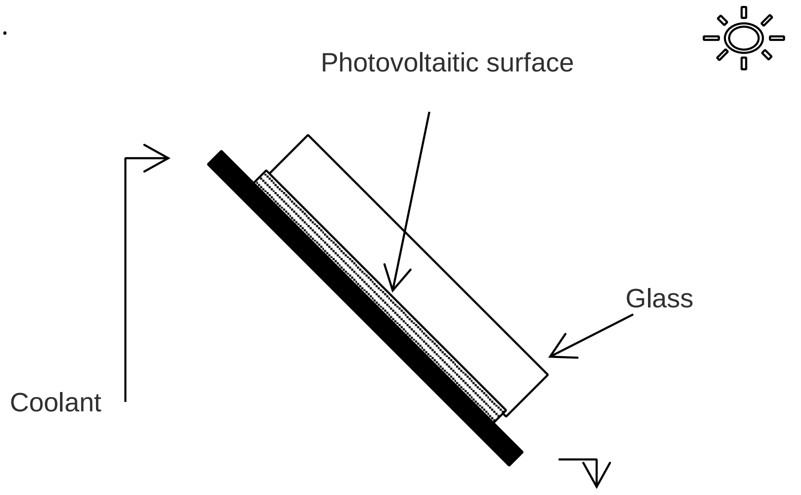
$$\frac{739767}{856756} = 0.86$$

[25%]

Examiners Comments:

Students correctly described the difference between exergy and availability. The heat and availability changes were usually correctly calculated. Many students took the enthalpy changes as the heat output, and forgot that this is the sum of the work and heat. A small proportion of students plotted the heat integration curves correctly, but most were able to make some headway and so gained marks on this part of the question. Many students noticed the system wasn't pinched so all the heat could be utilised. Very few students were able to make progress son the final part. Some made a good attempt at calculating the exergy of each stream from first principles, but failed to make use of the standard exergies provided in the question and so ran out of time.

2.



a)

In a semi-conductor, there is an equilibrium population of free electron and hole pairs. In a P type there is an excess of holes, and for n electrons. When brought together this produces a p-n junction. At the junction holes and electrons recombine to produce a depletion zone, across which there is now a potential difference. A photon hitting this depletion zone can create an electron hole pair, which then gets pumped out of the depletion zone by the potential gradient. However, to excite an electron from the valence band to the conduction band, requires the photon to have an energy greater than the band gap for the particular semi conductor. Any excess energy will simply be dissipated as heat.

[20%]

b)

The electrical power output is given by

$$\int \eta \times f d\lambda$$

Where P is the incident power, f is the distribution of the photons, and η is the efficiency at which each wavelength can be converted.

The band gap of the semi-conductor is 1.24 eV, photons with an energy less than this are unused, whilst those with more can only use 1.24eV. The conversion between photon energy E in electron volts and wavelength is $E = \frac{hc}{\lambda}/e$ where h is planck's constant , c is the speed of light and e is the charge on an electron (i.e. J per electron volts). This band gap energy corresponds to a wavelength of

$$\lambda = \frac{hc}{1.24e} = 1.001 \times 10^{-6} m = 1000\,nm$$

And for wavelengths below those

$$\eta = \frac{1.24}{\frac{hc}{\lambda e}}$$

The power, remembering that 10% of the incoming power is reflected, is then

$$0.9\int_{-\infty}^{1000} \eta \times f d\lambda$$

Taking the average efficiency for each of the intervals the integral can be approximated numerically.

Interval		Distribution	Mean photon use efficiency
280	500	1.03	0.38961
500	750	1.29	0.62437
750	1000	0.77	0.87412

$$0.9\int_{-\infty}^{1000} \eta \times f d\lambda = 0.9 * (220 \times 1.03 * 0.38961 + 0.250 * 1.29 * 0.62437 + 0.250 * 0.77 * 0.87412)$$
$$= 422\ W$$

Note this is effectively the limiting power for a solar panel and real solar panels would not be this efficient.

The heat load is the $Q = 900 - (200 + 45) - 422 = \ 233\ W$

[30%]

c) The saving can be broken down as follows

Saving in CO2 for every kWh of electricity displaced	0.4 kg of CO2
Saving in CO2 by displacing gas	Each kWh of electricity produces 233/422 kwh of heat. Generating 40 MJ of heat produces 44/16 kg of CO_2, so 1 kwh of heat generates $\frac{44}{16}/(\frac{40}{3.6})$ kg of CO_2. The CO2 generated is then $\frac{44}{16} \times \left(\frac{3.6}{40}\right) \times \frac{233}{422} \times \frac{1}{0.9}$=0.151
Panel foot print	Total kWh generated over lifetime = 8*422/1000*365*10 So the CO2 produced for every kwh of electricity delivered is 2000/(8*422/1000*365*10)= 0.162 kg
Total saving	0.31149+0.4 - .162 = 0.389 kg CO2/kWh

[20%]

d) There are number of points to discuss here

- The panel efficiency is very high, probably unrealistically slow.
- Clearly, the simplified solar distribution could be improved.
- The calculation relies on both the heat and electricity to be in demand, there will be times in the year when either electricity or heat are not required. This is countered somewhat by using a hot water to tank to store energy or the grid as an electricity buffer.

- It is very likely that the heat would displace natural gas.
- Displacing the average electricity mix can lead to large errors. Electricity cannot be readily stored, so when it is generated matters. Solar panels generate most of their electricity on a warm summer day, when in the uk demands are not peaking, and there may already be a large amount of low carbon electricity on the grid. In contrast, in the winter they will generate nothing, when the CO2 footprint of the electricity is highest. You could consider using the instantaneous average and integrating over the year, but even this does solve all the problems. The real question is what plants had to be throttled back when the panel generated its electricity, i.e. what was the marginal footprint of the load following plant on the grid. All this means the estimates of saving for the electricity produced is likely to be overestimated.

[30%]

Examiners Comments

Students seemed to have a good understanding of the principles of solar PV systems. The calculation of heat and work proved to be more difficult, though a significant proportion of the students were able to work through to the correct answer. The biggest difficulty was in working out the phone use efficiency at each wavelength. Many students simply assumed that all the photons with energy above the band gap were converted fully to work. Attempts at the lifecycle calculation resulted in a wide variety of incorrect numerical answers, but the general direction of the calculation was correct. The use of kwh as a unit seemed to confuse some students. The last descriptive part of the question was generally well answered.

Question 3

The identity for gCO_2/km is similar to the one described in the lectures.

a)

$$\frac{\mathbf{CO_2}}{\mathbf{Fuel}}$$

describes the carbon intensity of the fuel, with various fuels (i.e. diesel, petrol, natural gas) having different intensity values. Estimated values: ≈70 CO_2 per MJ (gasoline), ≈75 CO_2 per MJ (diesel)

$$\frac{\mathbf{fuel}}{\mathbf{Tracive\ work\ (i.e. useful\ energy)}}$$

describes the efficiency of the conversion device (engine and drive system) and how effectively fuel (final energy) is converted in to driving force at the car wheels. Estimated values: ≈20% efficiency for petrol engines, therefore ≈5.

$$\frac{\boldsymbol{Tractive\ work\ (i.e. useful\ energy)}}{\mathbf{km}}$$

describes the efficiency of the passive system (vehicle body) and how effectively it converts the driving force (useful energy) into kilometres travelled by the car (service). ≈0.3 MJ/pkm, ≈1.2pkm/vkm, therefore ≈0.36 MJ/vkm

[15%]

b)

Cars can be powered by either petrol or diesel engines.

Petrol engines
are constrained in size by the need to ensure that the fuel does not ignite spontaneously before the flame from the spark can sweep the cylinder. Spontaneous ignition causes knocking and will damage the engine. This size is determined by the induction time for the fuel air mix, and the speed of the flame front. In addition, petrol engines have an additional drawback, which is the need to fix the air to fuel to make sure the fuel can burn. Air is drawn into the engine by the down stroke, fixing the volume of air in the cylinder. Thus, to change the power output, when the amount of fuel changes, the only way to change the amount of air is to throttle the intake. Thus an engine is always throttled (causing irreversibility) when the engine is not running at full load.

The only way to provide the torque reserve in a traditional petrol engines vehicle is to gear the drive train so that when cruising the engine operates at low torque. This mean that when power is required the engine can rapidly increase torque by simply adding fuel (and reducing the throttling). Maintaining the torque reserve means that engines are therefore operating far below their optimum load.

Diesel engines
do not suffer from knocking this as fuel is consumed throughout the cylinder homogenously as it evaporates from small droplets. Thus, diesel engines can have much larger cylinders and compression ratio. Car diesel engines operate at higher efficiencies than petrol engines.

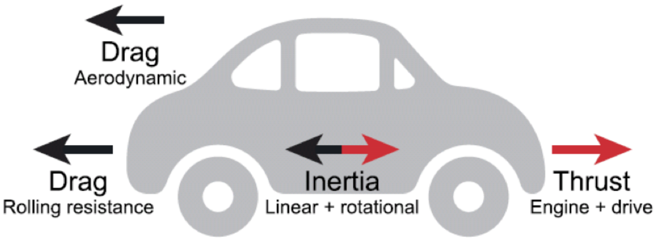
Turbocharging

A downsized engine whose operating point under normal cruising conditions is close to the optimum for fuel consumption, will operate very efficiently, but will not be able to respond to sudden demands for power. The power of both petrol and diesel engines is largely determined by how much oxygen is present in the cylinder before combustion is initiated. Thus, to provide extra power, more air must be fed to the cylinders. Turbo chargers are devices which compress the air going into the cylinder on the intake stroke, so that rather than being sucked in at sub atmospheric pressure, the air is at a higher pressure. For a fixed volume cylinder this means more oxygen in the cylinder, more fuel can be injected and more power extracted. A turbocharged small engine could therefore have similar characteristics to a larger conventional engine, but with the added advantage that when the turbocharging is inactive the engine is "small" and better matched to cruising. The only disadvantage is the delay in activating at the compression device, which can lead to turbo lag. The torque reserve is therefore provided by the engine being able to suddenly become larger.

[25%]

c)

The final term relates to the car passive system, and how the tractive force applied at the wheels is dissipated (to low grade heat) in exchange for the service (distance travelled). Three factors (see below) affect the efficiency of the passive system: aerodynamic drag (how easily the cars pushes through the air); mechanical drag (the rolling resistance between the wheels and road); inertia (changes in speed). The mass of the car has a significant effect on the passive system efficiency.



$$F = F_M + F_A + F_I = \mu m g + \frac{1}{2}\rho \nu^2 C_D A_f + m\frac{d\nu}{dt}$$

Mechanical drag
(rolling resistance)
friction coefficient
mass of vehicle

Aerodynamic drag
(air resistance)
drag coefficient
frontal area of vehicle

[30%]

d)

The new identity should include the upstream conversion of primary energy sources to electricity.

$$\frac{CO_2}{km} = \frac{CO_2}{energy\ source} \times \frac{energy\ source}{Electricity} \times \frac{electricity}{tractive\ force} \times \frac{tractive\ force}{km}$$

Students should identify that the conversion device efficiency $\frac{\text{electricity}}{\text{tractive force}}$ is more efficient than the previous $\frac{\text{fuel}}{\text{tractive force}}$ and how the inefficient combustion process has been shifted upstream to the $\frac{\text{energy source}}{\text{electricity}}$ term. Renewable energy sources can be used to decarbonise the electricity production. Electric vehicles can employ regenerative braking to recover some of the tractive force, thus improving the efficiency of the passive system. Electric car batteries are energy intensive to manufacture, therefore the embodied emissions of the electric vehicle may increase. A whole lifecycle energy use study would be needed to calculate the trade off between use phase and embodied energy use.

[30%]

Examiners Comments

This question proved to be very popular and most of the answers were to a high standard. The students correctly described how the equation in part (a) was decomposed. For the most part, students seemed to understand the limitations of internal combustion engines in part (b), with many students sketching the operating maps for the engines. Answers to Parts (c) and (d) were generally good.