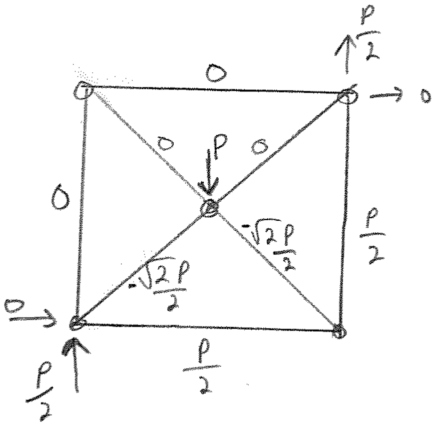
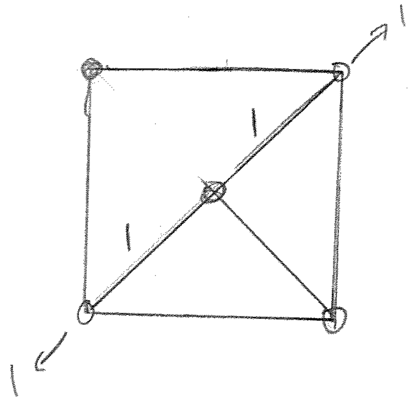
1B Paper 2 - 2016

① (a) $s - m = b + r - Dj = 8 + 4 - 10 = 2$ [2]

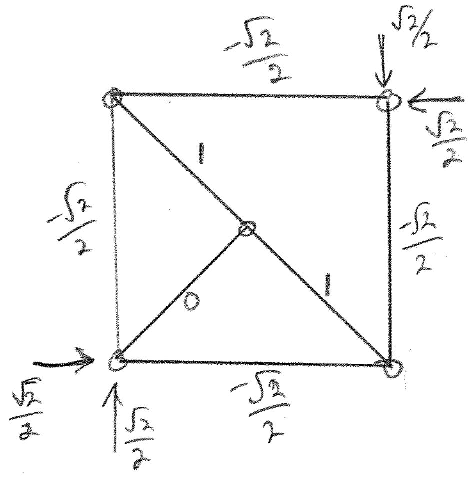
(b) Choose V & VI as redundant



$$t = \begin{bmatrix} 0 \\ 0 \\ P/2 \\ P/2 \\ 0 \\ 0 \\ -\frac{\sqrt{2}}{2}P \\ -\frac{\sqrt{2}}{2}P \end{bmatrix} \qquad \text{lengths} = L \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ \sqrt{2} \\ \sqrt{2} \\ \sqrt{2} \\ \sqrt{2} \end{bmatrix}$$



$$s_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$



$$s_2 = \begin{bmatrix} -\sqrt{2}/2 \\ -\sqrt{2}/2 \\ -\sqrt{2}/2 \\ -\sqrt{2}/2 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$t = \underline{t}_0 + x_1 \underline{s}_1 + x_2 \underline{s}_2$$

$$\underline{e} = \underline{F}\,\underline{t} = \frac{PL}{AE}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ -1 \\ -1 \end{bmatrix} + \frac{x_1 L}{AE}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sqrt{2} \\ 0 \\ \sqrt{2} \end{bmatrix} + \frac{x_2 L}{AE}\begin{bmatrix} -\sqrt{2} \\ -\sqrt{2} \\ -\sqrt{2} \\ -\sqrt{2} \\ \sqrt{2} \\ 0 \\ \sqrt{2} \\ 0 \end{bmatrix}$$

① (b) cont.

$$\underline{s}_1 \cdot \underline{e} = (-1)\frac{PL}{AE} + 2\sqrt{2}\,\frac{x_1 L}{AE} + 0 = 0$$

$$-P + 2\sqrt{2}\,x_1 = 0 \quad \rightarrow \quad \underline{\underline{x_1 = \frac{\sqrt{2}}{4}P}}$$

$$\underline{s}_2 \cdot \underline{e} = \left[2\left(-\frac{\sqrt{2}}{2}\right) - 1\right]\frac{PL}{AE} + \left[4 + 2\sqrt{2}\right]\frac{x_2 L}{AE} = 0$$

$$(-\sqrt{2} - 1)P + (4 + 2\sqrt{2})x_2 = 0 \rightarrow \underline{\underline{x_2 = \frac{1+\sqrt{2}}{4+2\sqrt{2}}P = \frac{\sqrt{2}}{4}P}}$$

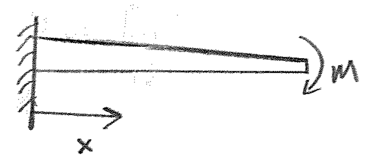
$$\underline{t} = \underline{t}_0 + x_1 \underline{s}_1 + x_2 \underline{s}_2 = \frac{P}{4}\begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ \sqrt{2} \\ \sqrt{2} \\ -\sqrt{2} \\ -\sqrt{2} \end{bmatrix}$$

[19]

(c) $\delta_{B,H} = e_{IV} = \frac{P}{4}\left(\frac{2L}{AE}\right) = \underline{\underline{\frac{1}{2}\frac{PL}{AE}}}$

[4]

② (a)



$$b^* = b\left(1 - \frac{x}{2L}\right)$$

$$I = \frac{(b^*)^4}{12} = b^4\left(1-\frac{x}{2L}\right)^4 = \frac{b^4}{192L^4}(2L-x)^4$$

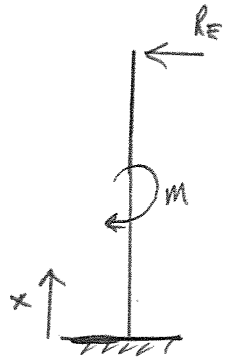
$$\left.\begin{array}{l} m(x) = M \\ \kappa(x) = \dfrac{M}{EI(x)} \end{array}\right\} \qquad M\theta = \int_0^L m(x)\,\kappa(x)\,dx$$

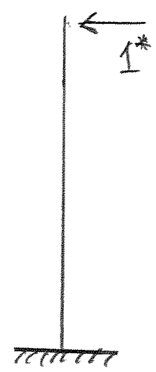
$$\cancel{M}\theta = \int_0^L \cancel{M}\left[\frac{M}{E}\,\frac{192L^4}{b^4}(2L-x)^{-4}\right]dx$$

$$\theta = \frac{M}{E}\,\frac{192L^4}{b^4}\left[\frac{(2L-x)^{-3}}{3}\right]_0^L$$

$$= \frac{M}{E}\,\frac{192L^4}{3b^4}\left(\frac{7}{8L^3}\right) \rightarrow \underline{\underline{\theta = \frac{56L}{b^4}\,\frac{M}{E}}} \qquad [8]$$

(b) (i)





$x = 0 \rightarrow L$: $m(x) = -(2L-x)R_E + M$

$x = L \rightarrow 2L$; $m(x) = -(2L-x)R_E$

$$m^*(x) = -(2L-x)(1)$$

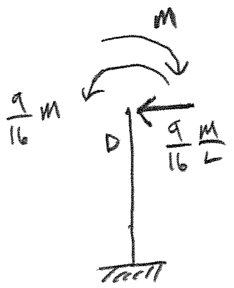
$$\Sigma F^*\delta = 1^*(0) = \int_0^{2L}\left[\frac{-(2L-x)R_E}{EI}\right](-(2L-x))\,dx + \int_0^L \frac{M}{EI}(-(2L-x))\,dx$$

$$0 = \left[\frac{-R_E}{EI}\,\frac{(2L-x)^3}{3}\right]_0^{2L} + \frac{M}{EI}\left[\frac{(2L-x)^2}{2}\right]_0^L$$

$$0 = \frac{-R_E}{\cancel{EI}}\left(-\frac{8L^{\cancel{3}}}{3}\right) + \frac{M}{\cancel{EI}}\left(\frac{\cancel{L^2}}{2} - \frac{4\cancel{L^2}}{2}\right) \longrightarrow \frac{8}{3}R_E L = \frac{3}{2}M$$

$$\underline{\underline{R_E = \frac{9}{16}\,\frac{M}{L}}} \qquad [12]$$

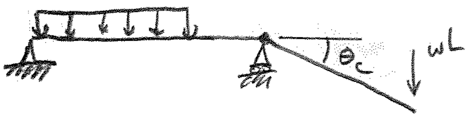
② (b)(ii) Find rotation @ Point D:



Use data book: $\theta_D = \left(\frac{-7}{16}M\right)\frac{L}{EI} + \left(\frac{9}{16}\frac{M}{L}\right)\frac{L^2}{2EI}$

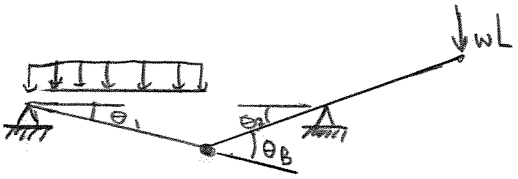
$$= \left(\frac{-7}{16} + \frac{9}{32}\right)\frac{ML}{EI} = -\frac{5}{32}\frac{ML}{EI}$$ [5]

3 (a) (i)



$$M_p \phi_c = (2L - \alpha L)\, \phi_c\, wL$$

$$M_p = (2-\alpha)\, wL^2 \qquad (1)$$



$$\theta_B = \theta_1 + \theta_2$$

$$\theta_1 L = \theta_2 (\alpha L - L)$$

$$\theta_1 = \theta_2 (\alpha - 1)$$

$$\theta_B = \theta_2 (\alpha - 1) + \theta_2 = \alpha \theta_2$$

$$M_p (\alpha \theta_2) = wL \frac{\theta_2 (\alpha L - L)}{2} - wL\, \theta_2 (2L - \alpha L)$$

$$M_p \alpha = \frac{wL^2}{2}(\alpha - 1) - \frac{wL^2}{2}(4 - 2\alpha) = \frac{wL^2}{2}(3\alpha - 5) \qquad (2)$$

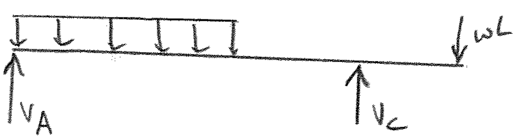
Combine (1) & (2): $(2-\alpha)\cancel{wL^2}\,\alpha = \frac{\cancel{wL^2}}{2}(3\alpha - 5)$

$$4\alpha - 2\alpha^2 - 3\alpha + 5 = 0$$

$$2\alpha^2 - \alpha - 5 = 0 \rightarrow \underline{\underline{\alpha = 1.85}}$$

[9]

(ii) Equilibrium:

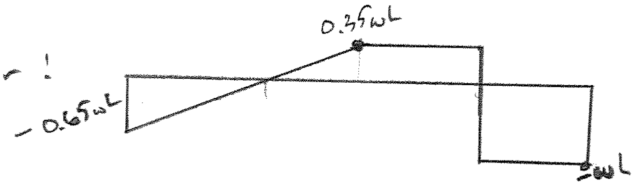


$$\Sigma M_A: \; V_C (1.85 \cancel{L}) = wL (2\cancel{L}) + wL \left(\frac{\cancel{L}}{2}\right)$$

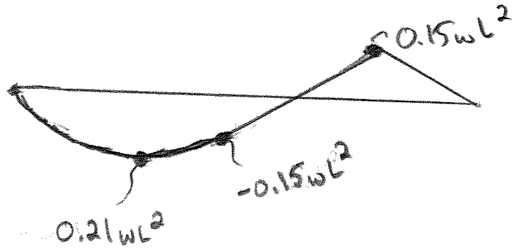
$$V_C = \frac{2.5}{1.85} wL = 1.35 wL$$

$$V_A = 0.65\, wL$$

Shear:

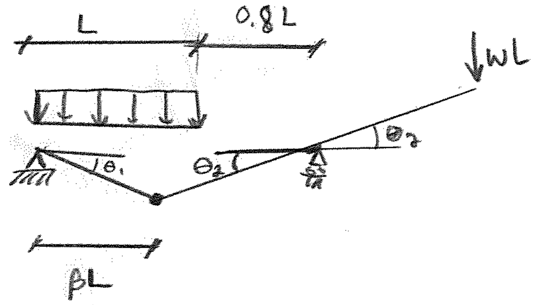


Moment:



[4]

③ (b)(i)



$$\theta_1 \beta L = \theta_2 (1.8-\beta) L$$

$$\theta_1 = \theta_2 \frac{(1.8-\beta)}{\beta}$$

$$M_p(\theta_1+\theta_2) = w(\beta L)\left(\theta_1 \frac{\beta L}{2}\right) + (L-\beta L)w\left(\theta_2\left(0.8L + \frac{L-\beta L}{2}\right)\right) - wL(0.2L\theta_2)$$

$$= w\frac{(\beta L)^2}{2}\,\theta_2\frac{(1.8-\beta)}{\beta} + (L-\beta L)w\,\theta_2\left(1.3L - \frac{\beta L}{2}\right) - 0.2wL^2\theta_2$$

$$M_p\left(\frac{1.8\theta_2}{\beta} - \cancel{\theta_2} + \cancel{\theta_2}\right) = w\frac{\beta L^2}{2}\theta_2(1.8-\beta) + wL^2\theta_2(1-\beta)\left(1.3-\frac{\beta}{2}\right) - 0.2wL^2\theta_2$$

$$M_p = \frac{\beta}{1.8}\left[wL^2\left(\frac{\beta}{2}(1.8-\beta) + (1-\beta)\left(1.3-\frac{\beta}{2}\right) - 0.2\right)\right]$$

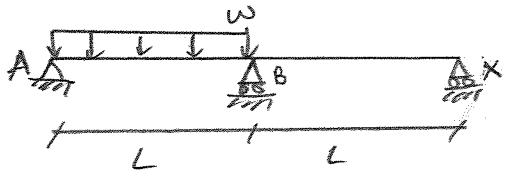
$$= wL^2\frac{\beta}{1.8}\left(-0.9\beta + 1.1\right) \rightarrow \underline{\underline{M_p = wL^2\beta(-0.5\beta + 0.61)}}$$

[9]

(ii) Set $\frac{dM_p}{d\beta} = 0 \rightarrow$ solve for β

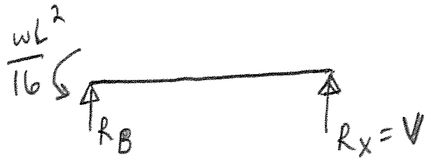
Plug β into equation for M_p from (b)(i). [3]

④ (a) Use symmetry:



Databook: $M_B = \frac{wL^2}{16}$

Cut at B:

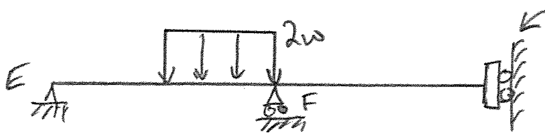


$\Sigma M_B = 0 \rightarrow VL = \frac{-wL^2}{16}$

$$\underline{\underline{V = \frac{-wL}{16}}}$$

$(\underline{\underline{M_{center} = 0}})$

(b) Use symmetry:

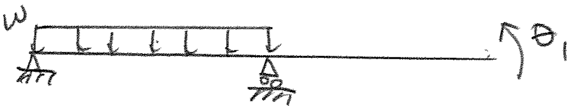


$M \neq 0$

$V = 0$

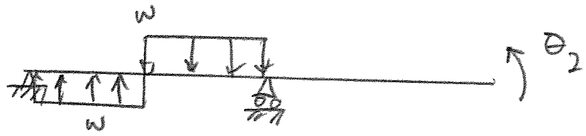
rotation $= 0$

|||



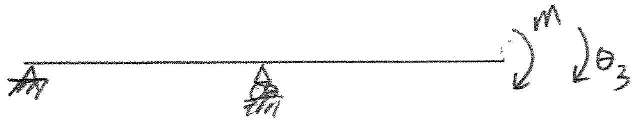
$\theta_1 \rightarrow \theta_1 = \theta_{F_1} = \frac{wL^3}{24EI}$

+

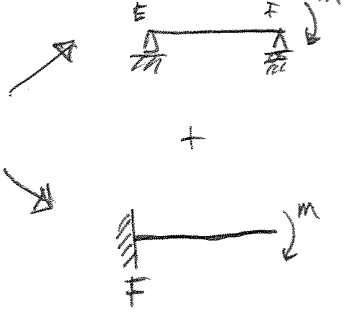


$\theta_2 \rightarrow \theta_2 = \theta_{F_2} = \frac{w\left(\frac{L}{2}\right)^3}{24EI} = \frac{wL^3}{24EI}\left(\frac{1}{8}\right)$

+



θ_3

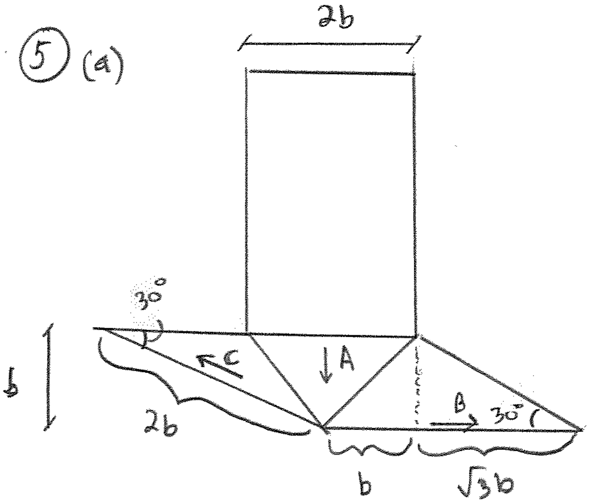


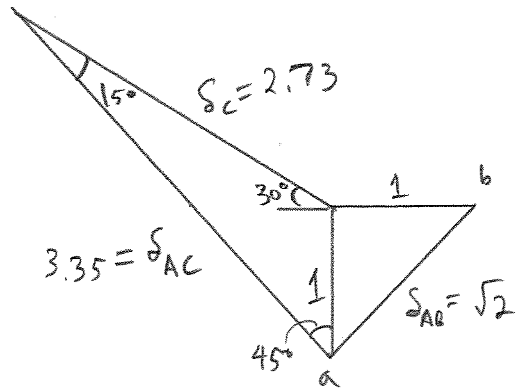
$\theta_3 = \frac{ML}{3EI} + \frac{ML}{EI}$

$= \frac{4}{3}\frac{ML}{EI}$

$\therefore \theta_1 + \theta_2 = \theta_3 \rightarrow \frac{9}{8}\frac{wL^3}{24EI} = \frac{4}{3}\frac{ML}{EI} \rightarrow \underline{\underline{M = \frac{9}{256}wL^2, \; V_{center} = 0}}$

(5) (a)





$$W\,\delta_A = k\Big[\big((1+\sqrt{3})b\big)\,\delta_B + (\sqrt{2}\,b)\,\delta_{AB} + (\sqrt{2}\,b)\,\delta_{AC} + (2b)\,\delta_{AC}\Big]$$

$$\frac{1}{\sin 15^\circ} = \frac{\delta_c}{\sin 45^\circ} = \frac{\delta_{AC}}{\sin 120^\circ}$$

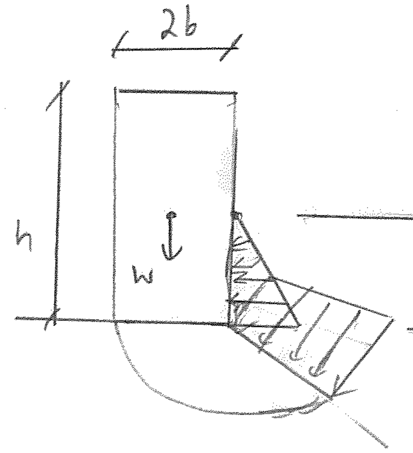
$$\delta_c = \frac{\sin 45^\circ}{\sin 15^\circ} = 2.73$$

$$\delta_{AC} = \frac{\sin 120^\circ}{\sin 15^\circ} = 3.35$$

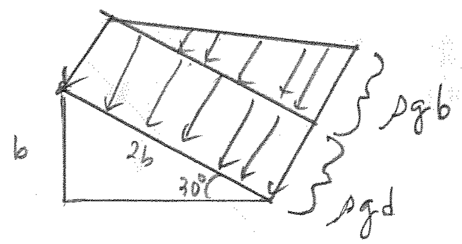
$$(2\cancel{b}h)(2\rho_w g)\,1 = k\Big[(1+\sqrt{3})\cancel{b}\,(1) + \sqrt{2}\,\cancel{b}\,(\sqrt{2}) + \sqrt{2}\,\cancel{b}\,(3.35) + 2\cancel{b}\,(2.73)\Big]$$

$$h = \frac{k}{4\rho_w g}(14.9) \rightarrow h = 3.73\frac{k}{\rho_w g} = (2+\sqrt{3})\frac{k}{\rho_w g}$$

⑤ (b)



ρ = density of water



W.D = E.D

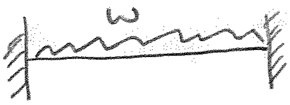
$$(2bh)(2\rho g)(b\cancel{\theta}) + \frac{\rho g d^2}{2}\left(\frac{d}{3}\cancel{\theta}\right) - (\rho g d)(2b)(b\cancel{\theta}) - (\rho g b)\left(\frac{2b}{2}\right)\left(\frac{4}{3}b\cancel{\theta}\right)$$

$$= k(2b)\left(\frac{5\pi}{6}\right)(2b\cancel{\theta})$$

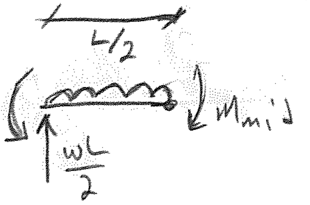
$$\rho g\left(4b^2\overset{2d}{\cancel{h}} + \frac{d^3}{6} - 2b^2 d - \frac{4}{3}b^3\right) = k\left(\frac{10}{3}\pi b^2\right)$$

$$\underline{\underline{k = \frac{3}{10\pi b^2 \rho g}\left[6db^2 + \frac{d^3}{6} - \frac{4}{3}b^3\right]}}$$

⑥ (a) Data book case:



$M_{ends} = \frac{wL^2}{12}$



$\frac{wL^2}{12} = M$

$\rightarrow M_{mid} = \frac{wL^2}{12} + w\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) - \frac{wL}{2}\left(\frac{L}{2}\right)$

$M_{mid} = -\frac{wL^2}{24}$

$w = 364 \ N/m$

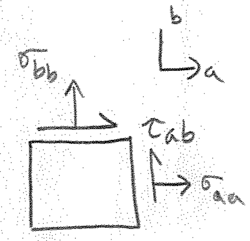
$I = \pi r^3 t = (1\times10^{-6})\pi \ m^4$

$\sigma_\ell = \frac{My}{I} = \frac{\frac{wL^2}{24} r}{I} = 189 \ MPa$ (σ_ℓ from pressure $= 0$, pipe open @ ends)

$\sigma_{hoop} = \frac{pr}{t} = 100p$

Tresa: $(100p + 189) = 275 \ MPa \rightarrow p = 0.86 \ MPa$

(b)(i) $\sigma_{aa} = \frac{\overset{=210\ GPa}{E}}{(1-\nu^2)}\left(14 + \nu(20)\right) = 4.62 \ MPa$

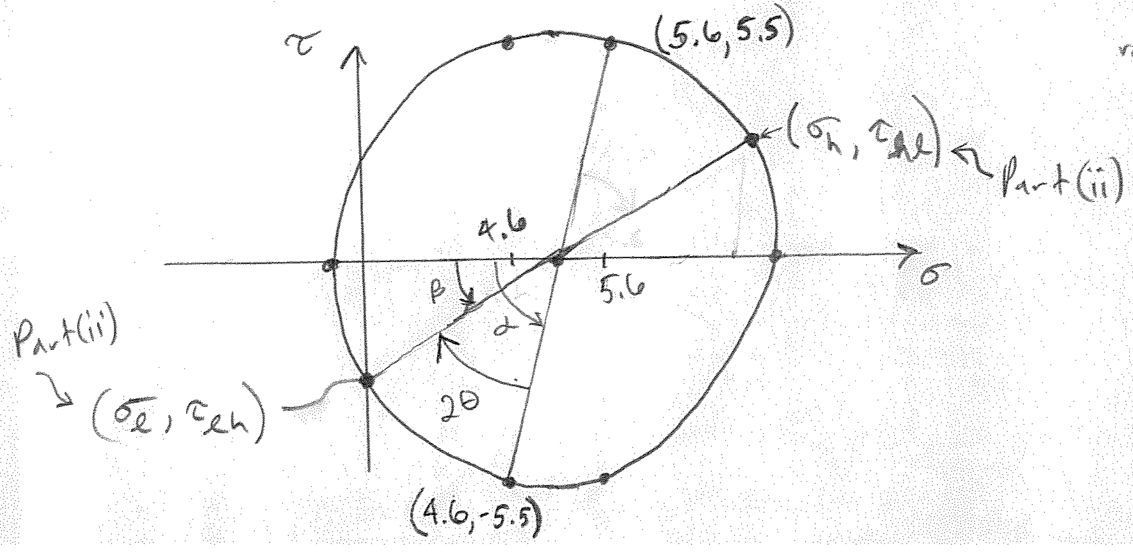


$\sigma_{bb} = \frac{E}{(1-\nu^2)}\left(20 + \nu(14)\right) = 5.58 \ MPa$

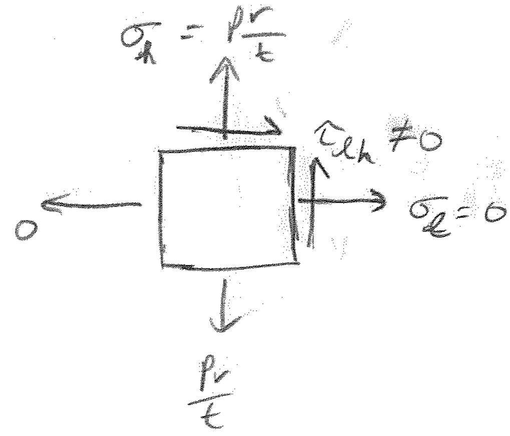
$\tau_{ab} = \overset{=81\ GPa}{G}\gamma_{xy} = G(68) = 5.51 \ MPa$

$\sigma_{center} = 5.1 \ MPa$

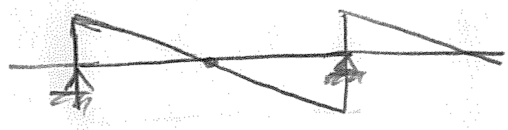
radius $= 5.53$



⑥ (b)(ii)



Shear diagram:



(shear is not necessarily zero)

$$\text{From Mohr's circle: } \overbrace{\sigma_h}^{} = \frac{pr}{t} \Rightarrow \overbrace{(5.1\text{ MPa})}^{\sigma_h}(2) = 100\,p$$

$$\underline{\underline{p = 102\text{ kPa}}}$$

(iii) Find 2θ on Mohr's circle (previous page)

$$\tau_{h\ell} = \sqrt{5.53^2 - 5.1^2} = 2.14\text{ MPa}$$

$$\tan\alpha = \frac{5.53}{5.1 - 4.62} \rightarrow \alpha = 85^\circ \qquad \tan\beta = \frac{2.14}{5.1} \rightarrow \beta = 23^\circ$$

$$\underline{\underline{\theta = \tfrac{1}{2}(\alpha - \beta) = 31^\circ}}$$

(counterclockwise from σ_a to σ_ℓ)

$$\tau = \frac{S A_c \bar{y}}{(\pi r^3 t)(2t)}$$

$$S = \frac{2\pi r^3 t^2 \tau}{\pi r t\left(\frac{2r}{\pi}\right)} = \pi r t \overset{= 2.14\text{ MPa}}{\tau} = 672\text{ N}$$

$$\text{Distance from midspan} = \frac{672\text{ N}}{364\text{ N/m}} = 1.85\text{ m} \rightarrow \underline{\underline{8.15\text{ m from support}}}$$