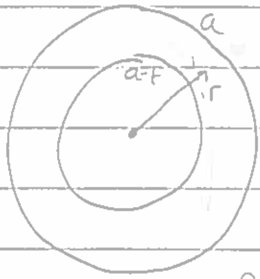
1a)



per unit length

$$\dot{Q} = -2\pi r \lambda \frac{\partial T}{\partial r}$$

$$\dot{Q}\int_{a-t}^{a} \frac{1}{r}\,dr = \int_{T_f}^{T_s} -2\pi\lambda\, dT$$

$$\dot{Q}\ln\left(\frac{a}{a-t}\right) = -2\pi\lambda\,(T_s - T_f).$$

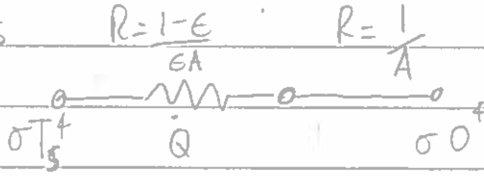
$$\therefore \quad \dot{Q} = \frac{2\pi\lambda(T_s - T_f)}{-\ln\left(\frac{a}{a-t}\right)}$$

$$= \frac{2\pi\lambda\,(T_s - T_f)}{\ln\left(1-\frac{t}{a}\right)}.$$

b) i) Thin wall $\Rightarrow$ $\ln\left(1-\frac{t}{a}\right) \approx -\frac{t}{a}$

$$\dot{Q} = \frac{2\pi\lambda a(T_s - T_f)}{-t} \Rightarrow \frac{\dot{Q}\,t}{2\pi\lambda a} = T_f - T_s.$$

Radiative loss using resistances



$$\Rightarrow \dot{Q} = \frac{\sigma T_s^4 - 0}{\frac{1-\epsilon}{\epsilon A} + \frac{1}{A}} = \epsilon A \sigma T_s^4$$

N.b. - space is black - large enclosure

So $\dot{Q} = \epsilon 2\pi a \sigma T_s^4$

N.B you could also simply note that space is a black body at zero K so radiosity $= A\epsilon\sigma T_s^4$ since there is no irradiance to reflect which also implies $\dot{Q}$ = radiosity.

Two expressions for $\dot{Q}$ can be written as.

$$\frac{\dot{Q}\, t}{2\pi\lambda a} = T_f - T_s .$$

$$\frac{\dot{Q}}{2\pi\epsilon a\sigma T_s^3} = T_s .$$

Adding

$$\dot{Q}\left[\frac{t}{2\pi\lambda a} + \frac{1}{2\pi\epsilon a\sigma T_s^3}\right] = T_f$$

c)i) From (b), can neglect tube wall when.

$$\frac{t}{2\pi\lambda a} \ll \frac{1}{2\pi\epsilon a\sigma T_s^3}$$

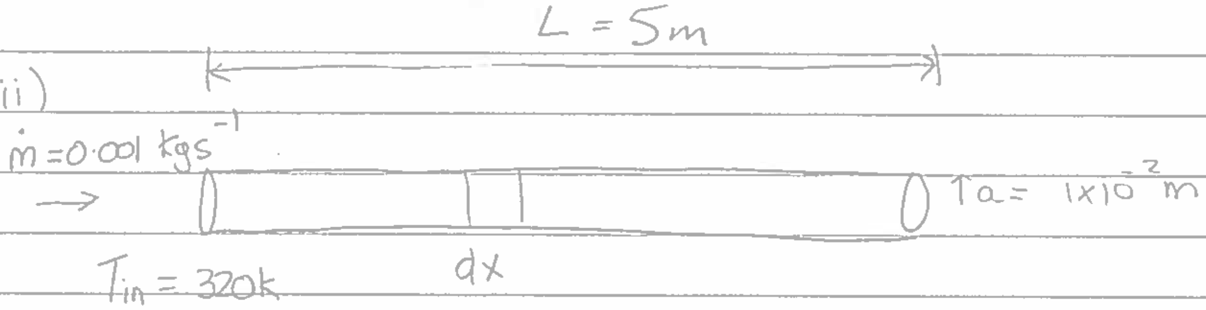
or $\dfrac{t\sigma T_s^3\epsilon}{\lambda} \ll 1$ The worst possible value of T_s would be at the inlet and since $T_s \le T_f$

$$\frac{t\sigma T_f^3\epsilon}{\lambda} \ll 1$$

$$\frac{t\sigma T_f^3\epsilon}{\lambda} = \frac{1\times10^{-3} \times 5.6\times10^{-8} \times 320^3 \times 1}{200} = 9.29\times10^{-6}$$

$\therefore$ ok to neglect tube wall

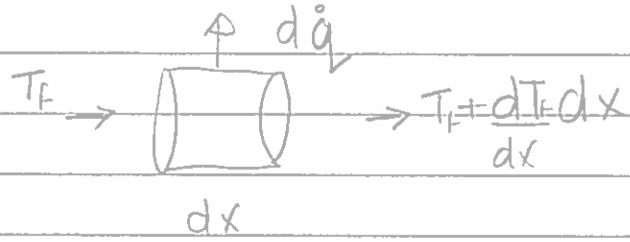
c(ii)



$c_p = 1000 \, J \, kg^{-1} \, K^{-1}$

q = total heat flow

Consider a small section dx.



$$d\dot{q} = -\dot{m} c \frac{dT_f}{dx} dx$$

local rate of heat loss per unit length

$$\dot{Q} \, dx = -\dot{m} c \frac{dT_f}{dx} dx$$

When radiation is largest resistance $T_s = T_f$ and

$$Q = 2\pi \epsilon a \sigma T_f^4 = 2\pi a \sigma T_f^4$$

$$\Rightarrow \quad 2\pi a \sigma T_f^4 = -\dot{m} c \frac{dT_f}{dx}$$

$$\Rightarrow \quad \int_0^L \frac{2\pi a \sigma}{\dot{m} c} dx = \int_{320}^{T} -\frac{1}{T_f^4} dT_f$$

$$\frac{2\pi a \sigma L}{\dot{m} c} = \left[\frac{1}{3T_f^3} \right]_{320}^{T} = \frac{1}{3} \left[\frac{1}{T^3} - \frac{1}{320^3} \right]$$

$$T^{-3} = (320^{-3}) + \frac{6\pi a \sigma L}{\dot{m} c}$$

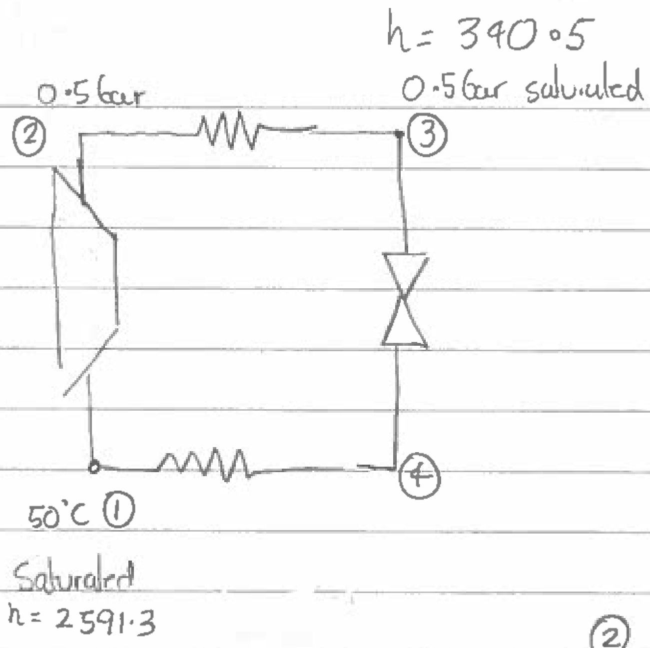
$$= (320)^{-3} + \frac{6 \times \pi \times 0.01 \times 5.67 \times 10^{-8} \times 5}{0.001 \times 1000}$$

$$T = 228 K$$

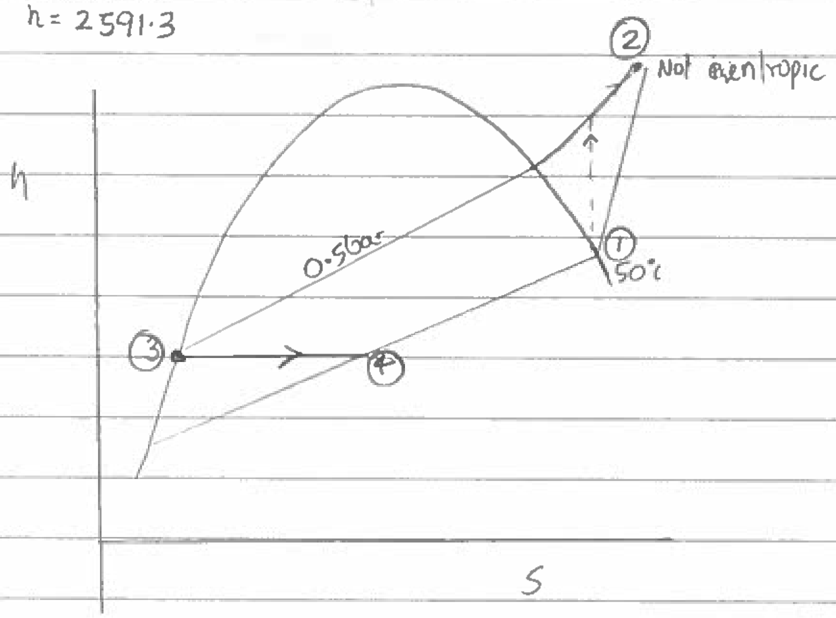
iii) Total rate of heat loss = $\dot{m} c \Delta T$ = $\underline{\underline{92 W}}$

Note - This crib uses the tables. The answers can be found more efficiently ~~by making use of the~~ h-s chart in places.

2)



a)



b i) From saturated tables at ① $T^{sat} = 50°C \Rightarrow p^{sat} = \underline{\underline{12.35 \text{ kPa}}}$

∴ evaporator pressure is $\underline{\underline{12.35}}$ kPa

lowest temperature at which heat is rejected is the condensor outlet ∴ $P_{sat} = 0.5$ bar $\Rightarrow T_{sat} = \underline{\underline{81.3\,°C}}$

b ii) (Note easier to use chart)

$S_1 = 8.075$

$S_2^{isen} = 8.075 \quad P = 0.5 \text{ bar} \quad$ - superheated

From tables

T	s	h
175	8.0531	2828.9
200	8.1592	2877.8

interpolating $\quad T = 180.16°C \quad h_2^{isen} = 2839.0$

However isentropic efficiency is 0.9

$$\Rightarrow h_2 - h_1 = \frac{1}{0.9}(h_2^{isen} - h_1)$$

$$h_2 = h_1 + \frac{1}{0.9}\left(h_2^{isen} - h_1\right)$$

$h_1 = 2591.3$

$\therefore h_2 = \underline{\underline{2866.5 \text{ kJ/kg}}}$

iii) Throttle valve is isenthalpic $\Rightarrow h_4 = h_3 = 340.5$

④ is in two phase region. At $50°C$ $\quad h_f = 209.3$
$h_g = 2591.3$

$$\Rightarrow 340.5 = 2591.3\,x + (1-x)\,209.3$$

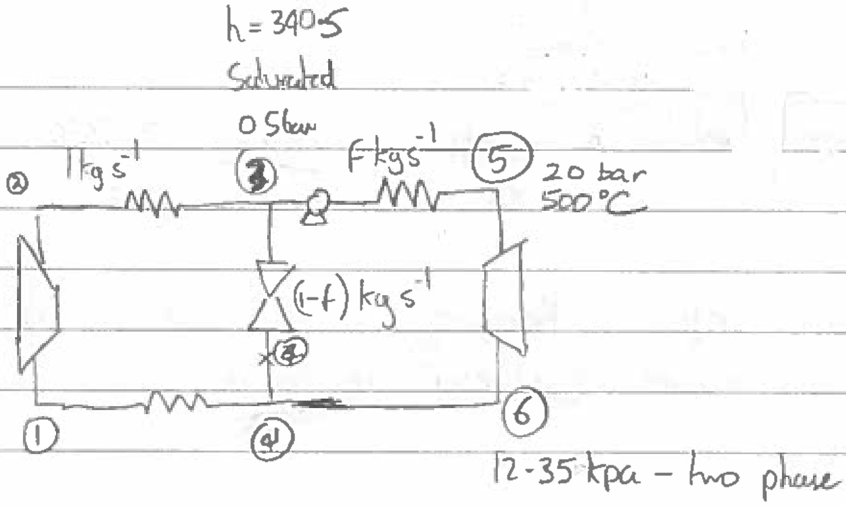
$$x = \frac{340.5 - 209.3}{2591.3 - 209.3} = \underline{\underline{0.055}}$$

iv)

$Q_{in} = h_1 - h_4 = 2591.3 - 340.5 = 2250.8 \text{ kJ/kg}$

$W_{compressor} = h_2 - h_1 = 2866.5 - 2591.3 = 275.2$

$$\therefore COP_R = \frac{2250.8}{275.2} = \underline{\underline{8.18}}$$



i) Conditions at points 1, 2, 3, 4 unchanged

Need turbine work $= f(h_5 - h_6)$.

$h_5 = 3468.2 \qquad s_5 = 7.4337$

$s_6 = s_5$

At 12.35 kPa $s_f = 0.704 \quad s_g = 8.075$

$$\Rightarrow \quad x = \frac{7.434 - 0.704}{8.075 - 0.704} = \underline{\underline{0.913}}$$

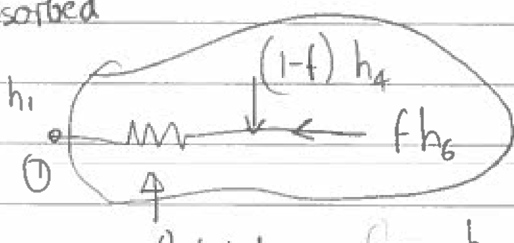
$$\Rightarrow \quad h_6 = 209.3 \times (1 - 0.913) + 0.913 \times 2591.3$$
$$= 2384.1$$

$$\therefore \quad W_T = f(3468.2 - 2384.1) = f\,1084.1 \text{ kJ/kg}$$

Match compressor work

$$275.2 = f\,1084.1 \quad \Rightarrow \quad f = \underline{\underline{0.254}}$$

ii) Heat absorbed



$$Q_{absorbed} = h_1 - fh_6 - (1-f)h_4 = 2591.3 - 0.254 \times 2384.1$$
$$- 0.746 \times 340.5$$

$$Q_{absorbed} = 1731.7 \text{ kJ/kg}$$

$$Q_{input} = f(h_5 - h_3) = 794.4$$

Suitable metric : $\dfrac{\text{heat absorbed by evaporator}}{\text{heat supplied to generate work}}$

This is <u>2.18</u>

<u>Much</u> lower than COP, which does not take account of efficiency with which work for the cycle is generated.

a)

(i) $b = h - T_o s$, where T_o is environmental temp.

(ii) $\dot{m}_o h_o - \dot{m}_i h_i = \dot{Q} - \dot{W}_x$

or per kg

$$h_o - h_i = q - w_x \quad \text{(A)}$$

And

$$s_o - s_i = \int \frac{dq}{T} + \Delta s_{irrev} \quad \text{(B)}$$

multiply (B) by T_o and subtract

$$\underbrace{(h_o - T_o s_o)}_{b_o} - \underbrace{(h_i - T_o s_i)}_{b_i} = \int \left(1 - \frac{T_o}{T}\right) dq - T_o \Delta s_{irrev} - w_x$$

$$\Delta b = \int \left(1 - \frac{T_o}{T}\right) dq - T_o \Delta s_{irrev} - w_x$$

Change in power potential

power potential change owing to heat transfer

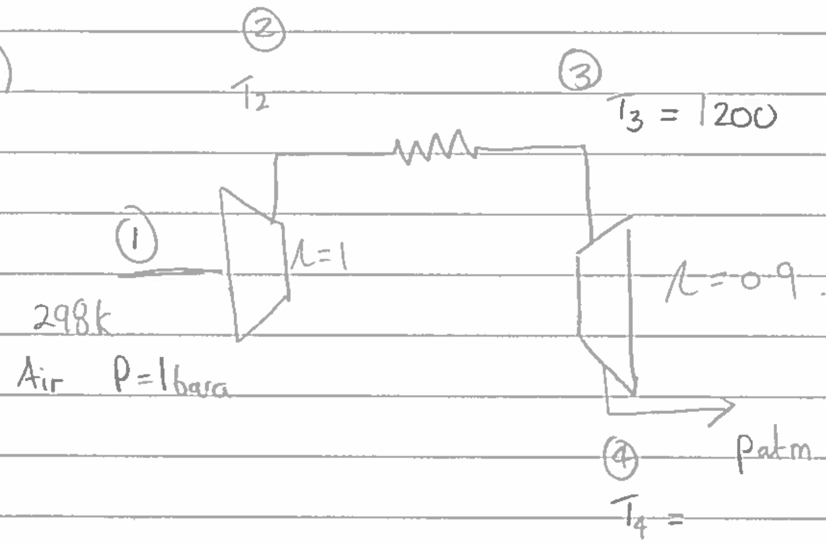
lost work

work output.

Q3/2

$R = 287 \quad C_p = 1005 \quad \gamma = 1.4$

b)



$r = \frac{P_2}{P_1} = \frac{P_3}{P_4} = 10$

$T_2 = r^{\frac{\gamma-1}{\gamma}} T_1$
$= 575.3\,K$

$T_3 - T_4 = \eta (T_3 - T_4^{\text{isen}})$
$= \eta T_3 \left(1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right)$
$T_4 = T_3 - \eta T_3 \left(1 - \left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right)$
$= T_3\left(1 - \eta + \eta\left(\frac{1}{r}\right)^{\frac{\gamma-1}{\gamma}}\right) = T_3\, 0.566$

i) lost work in compressor $= 0$

lost work in turbine $= T_0 \Delta s_{irrev}$

(adiabatic $\Rightarrow \Delta S = \Delta S_{irrev}$)

$= T_0 \Delta s$

$= T_0 \left[C_p \ln\left(\frac{T_4}{T_3}\right) - R \ln\left(\frac{P_4}{P_3}\right) \right]$

$= 298 \left[1005 \ln 0.566 - 287 \ln \left(\tfrac{1}{10}\right) \right]$

$= 26.5$ kJ / kg.

ii) Availability rise between ② → ③

$\Delta G = \underbrace{C_p (T_3 - T_2)}_{\Delta h} - T_0 \left(\underbrace{C_p \ln \frac{T_3}{T_2}}_{\Delta s} \right) = 407.6$ kJ/kg

iii) First law efficiency $\eta = \dfrac{W_T - W_c}{Q_{in}}$

$$W_T = C_p(T_3 - T_4) = 523 \text{ kJ/kg}$$
$$W_c = C_p(T_2 - T_1) = 279 \text{ kJ/kg}$$
$$Q = C_p(T_3 - T_2) = 628 \text{ kJ/kg}$$

$$\eta_{1st} = 38.9\%$$

Second law efficiency is ratio of work output to availability input, i.e.

$$\eta_{second} = \frac{W_T - W_c}{\Delta b_{input}} = 0.60 = 60\%$$

c) System now used to generate cold air. T_3 altered to 308K and heat removed rather than added.

i) Analysis from previous parts can be reused $\frac{T_4}{T_3}$ is same as before so $T_4 = 0.566 \times 308 = 174$ K

The lost work in the turbine only depends on $\frac{p_4}{p_3}$, T_4/T_3 so is the same ie 26.6 kJ/kg

Compressor input work unchanged $= 279$ kJ/kg
Turbine work $= C_p(T_3 - T_4) = 135$ kJ/kg

$\therefore$ Net work input $= 144$ kJ/kg

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Question number 3 Sheet number 4

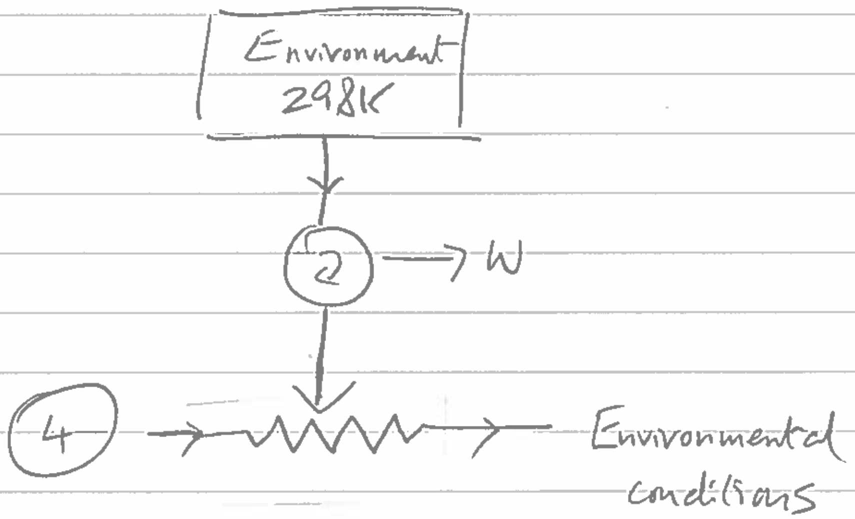
c) ii) Power potential of exhaust gas is difference in availability from environmental value, ie

$$(h_4 - T_0 s_4) - (h_{atm} - T_0 s_{atm})$$

$$= c_p (T_4 - T_0) - T_0 c_p \ln\left(\frac{T_4}{T_0}\right)$$

$$= 36.5 \text{ kJ/kg}$$

Positive because heat transfer from environment to exhaust gas can be used to generate work:

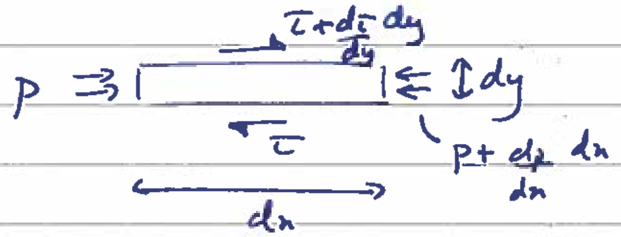


Question number 4 Sheet number 1

a) (i) ① Continuity for a streamtube implies u independent of x. (Equally, can argue that flow is 'fully-developed'.)

② Parallel streamlines imply p independent of y, via y-momentum for streamtube (or streamline-normal eqn, noting that Newton's law of viscosity implies no viscous forces $\perp$ streamlines in this flow)

③ Combine results of ① and ② with x-momentum for streamtube :



and Newton's law of viscosity : $\tau = \mu \dfrac{du}{dy}$.

(ii) Integrate 2x : $u = A + By + \dfrac{y^2}{2\mu}\dfrac{dp}{dx}$;

A and B from B.C.s, to get $u = -\dfrac{h^2 - y^2}{2\mu}\dfrac{dp}{dx}$

(iii) $$Q = \int_{-h}^{h} u\,dy = -\frac{1}{2\mu}\frac{dp}{dx}\left[h^2 y - \frac{y^3}{3}\right]_{-h}^{h}$$

$$= \frac{2}{3}\frac{h^3}{\mu}\left[-\frac{dp}{dx}\right].$$

b) Apply result of a)(iii) locally : $Q = \frac{2}{3}\frac{h^3}{\mu}\left(-\frac{dp}{dx}\right)$

where now h and dp/dx depend on x

Q is still constant $\Rightarrow$ $-\frac{dp}{dx} = \frac{3\mu Q}{2} \frac{1}{\left[h_A + \frac{h_B - h_A}{L}x\right]^3}$

Hence $p_A - p_B = \frac{3\mu Q}{4}\left[-\frac{L}{h_B - h_A}\left(h_A + \frac{h_B - h_A}{L}x\right)^{-2}\right]_A^B$

$$= \frac{3\mu Q L}{4(h_B - h_A)}\left[\frac{1}{h_A^2} - \frac{1}{h_B^2}\right]$$

$$= \frac{3\mu Q L (h_A + h_B)}{4 h_A^2 h_B^2}$$

c) Eliminate dp/dx between the expressions for u and Q in (a), to obtain

$$u = \frac{3Q}{4h^3}(h^2 - y^2) \sim O(Q/h)$$

$$\frac{\partial u}{\partial x} = \frac{3Q}{4}\left[-\frac{1}{h^2} + \frac{3y^2}{h^4}\right]\frac{dh}{dx} \sim O\left(\frac{Q}{h^2}\frac{dh}{dx}\right) \qquad \frac{\partial^2 u}{\partial y^2} \sim O\left(\frac{Q}{h^3}\right)$$

So for $\rho u \frac{\partial u}{\partial x} \ll \mu \frac{\partial^2 u}{\partial y^2}$, $\frac{\rho Q}{\mu}\frac{dh}{dx} \ll 1$

N.B. It's tempting to try sidestepping the integration of part (b) by applying SFME to the region AB. In fact, nothing's gained, because the (varying) wall shear stress must still be integrated.

a) (i) Total pressure in reservoir, relative to atmospheric and with surface as height datum, is zero.

Total pressure at top of fountain ($V=0$) is $\rho g H$.

Hence pump must provide $\underline{\underline{\rho g H}}$ increase

(ii) At nozzle exit, total pressure $= \frac{1}{2}\rho V^2$

Hence $\frac{1}{2}\rho V^2 = \rho g H$; $V = \sqrt{2gH}$

Flow rate $Q = AV = \underline{\underline{A\sqrt{2gH}}}$.

b) (i) $\underset{ML^{-1}T^{-2}}{\Delta p_T} = fn\left[\underset{T^{-1}}{N}, \underset{L}{D}, \underset{L^3T^{-1}}{Q}, \underset{ML^{-3}}{\rho}, \underset{ML^{-1}T^{-1}}{\mu}\right]$

Eliminate M : $\underset{L^2T^{-2}}{\dfrac{\Delta p_T}{\rho}} = fn\left[N, D, Q, \underset{L^2T^{-1}}{\dfrac{\mu}{\rho}}\right]$

Eliminate L and T using D and N : $\dfrac{\Delta p_T}{\rho(ND)^2} = fn\left[\dfrac{Q}{ND^3}, \dfrac{\mu}{\rho ND^2}\right]$

Other possible groups from products of these.

(ii) If influence of viscosity is negligible, then

$fn\left[\dfrac{Q}{ND^3}, \dfrac{\mu}{\rho ND^2}\right] = f\left(\dfrac{Q}{ND^3}\right)$ only, because the LHS cannot be affected by changes in $\dfrac{\mu}{\rho ND^2}$.

N.B. Different forms from part (i) must first be manipulated before this argument can be made.

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Question number 5 Sheet number 2

c) Scenario: Δp_T fixed (fountain height); D fixed (same pump).

(i) Reducing Q (to reduce power consumption) at same speed $\Rightarrow Q/ND^3$ changes $\Rightarrow \frac{\Delta p_T}{\rho (ND)^2} = f\left(Q/ND^3 \right)$ also changes. This would imply a change in Δp_T, so N cannot be held constant.

(ii) $Q/ND^3 = 0.05 \Rightarrow \eta = 0.60$, cf. 0.75

$\Rightarrow$ reduced by $\underline{0.8}$ factor

$\frac{\Delta p_T}{\rho (ND)^2}$ increases from 4 to 5.5

but Δp_T is fixed by fountain height $\Rightarrow \frac{N'}{N} = \sqrt{\frac{4}{5.5}} = \underline{0.853}$

$\frac{Q'}{N'} = \frac{1}{2}\frac{Q}{N}$ $\qquad \frac{Q'}{Q} = \frac{1}{2}\frac{N'}{N} = \underline{0.426}$

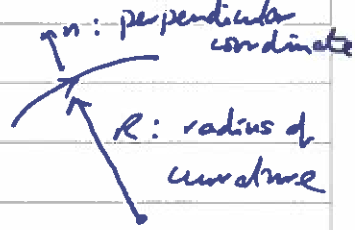
(iii) Electrical power $= \frac{Q \Delta p_T}{\eta}$, and for specific case in (ii), reduction in Q comfortably outweighs reduction in η, so consumption is now $100 \times \frac{0.426}{0.8} = \underline{53\%}$ of previous

Due to smooth and gentle η variation near optimum: hence reduction in costs won't be directly proportional to reduction in Q, but won't be too far off (unless the modification is really extreme).

a) $\frac{\partial p}{\partial n} = \frac{\rho V^2}{R}$

p : pressure
ρ : density
V : velocity

n, R define streamline geometry, via



Physical interpretation : the equation is a statement of Newton II, with $\partial p/\partial n$ the force (per unit volume) on fluid particles with (centripetal) acceleration V^2/R.

b) (i) Consider the flow on the channel bed. From the streamline-normal equation, the pressure must increase with r. The fluid depth must therefore also become greater in order to provide this increase (via the hydrostatic contribution).

(ii) Horizontal streamlines, no curvature in the vertical direction, so bed-level pressure is $\rho g h$. Also $n = r$, $R = r$

Streamline-normal eqn gives $\rho g \frac{dh}{dr} = \rho \frac{V^2}{r}$

$$\Rightarrow \quad V^2 = gr.\frac{2r}{H} \; ; \quad \underline{\underline{V = \sqrt{\frac{2g}{H}}\, r}}$$

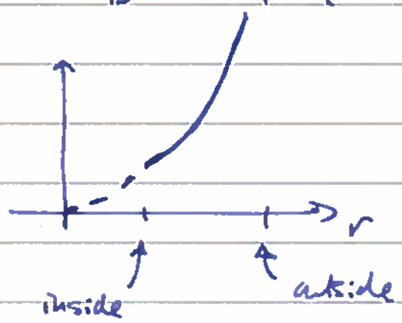
(iii) Pressure and height contributions give $\rho g h$, independent of vertical pos'n.

Hence total pressure $= \rho g h + \frac{1}{2}\rho V^2$

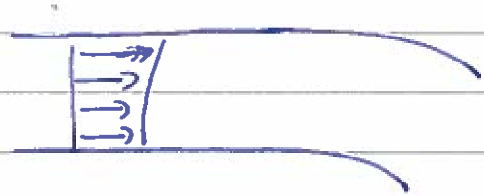
$$= \rho g\left[\frac{r^2}{H} + \frac{r^2}{H}\right] = \underline{\underline{2\rho g \frac{r^2}{H}}}$$

iv) Along streamlines, total pressure is constant (Bernoulli).

Total pressure vs r in bend :



This doesn't translate directly into profile across channel upstream (because continuity isn't satisfied if we assume streamline separation across channel remains constant). However, we can say that total pressure increases across channel. Height of flow is now constant, so the total-pressure increase must be associated with a velocity increase.



c) In the bed boundary layers, streamlines still have very little curvature out of horizontal planes, so pressure variation vertically is close to hydrostatic, and $\partial p/\partial n$ ($\partial p/\partial r$) is thus almost unchanged. However, the velocity retardation implies that the centripetal acceleration of fluid particles is lower (if radius of curvature unchanged).

There is thus an 'excess' force inwards, which will cause radial acceleration and hence velocity. This inwards flow component will lead to radial sediment transport from the outside to the inside of the bend.