

SECTION A     *Introductory Business Economics 2016*

*CRIBS*

1     (a)     *Consider the market for wine. Draw a diagram representing conventionally shaped demand and supply curves. Represent and explain changes in the market equilibrium following: an increase in consumer income; the adoption of more efficient bottling equipment among producers; a fall in the price of beer; a poor harvest due to bad weather.*     [5]

The question requires a good understanding of the basic market model. First of all, the concepts of demand and supply need to be represented as negative and positive, respectively, monotonic relationships between prices and quantities. The intersection of these conventional demand (D) and supply (S) curves will identify the initial (unique) market equilibrium. From this point, movements of either the demand or the supply curve following changes in consumer income (an increase shifts the D curve to the right), better technology (the new bottling equipment shifts the S curve to the right), the price of a substitute good (an increase shifts the D curve to the left), and the poor harvest (which shifts the S curve to the left), will move the market equilibrium in a clockwise fashion.

(b)     *With reference to the problem of resource allocation:*

(i)     *Why is perfect competition generally considered to be the most efficient market structure?*     [5]

Perfect competition favours efficiency because it induces an optimal allocation of resources in production and in exchange. From an allocative viewpoint, it is the market structure that generates the greatest consumers' surplus (defined as the total value that consumers attach to all of the units of the good that they consume, minus what they pay to enjoy this consumption). The best students will be able to briefly compare perfect competition with a monopoly model and identify the deadweight loss generated by market power as a measure of the social cost of the units of a good not traded under monopoly.

(ii)     *If perfect competition is the most efficient market structure, why is it so rare in real markets?*     [5]

A perfectly competitive market is an ideal model based on a number of strict conditions (many suppliers each with an insignificant share of market; each supplier is unable to affect the market price; identical output; perfect information; even access to productive resources; no entry and exit barriers in the long run etc...). The co-occurrence of these conditions in real

markets is very rare. This does not mean to say that pure competition is empirically irrelevant: it provides a theoretical benchmark against which we compare and contrast imperfectly competitive markets and it is extremely useful, for example, to identify and measure the extent of market failure or the gains from free trade.

*(c) Define the components of national income. Identify and explain instances when specific injections and withdrawals might not be in equilibrium.* [10]

A good answer will include a definition of aggregate consumption, investment, government expenditure and net trade as the building blocks of the national income identity. The students will then need to identify withdrawals (savings, taxes and imports) and injections (investment, government expenditure and export) in the circular flow of the macroeconomy. At equilibrium injections should be equal to related withdrawals. When this does not occur we will observe budget surpluses or deficits, a positive or negative balance of payment, mismatches between savings and investments. These have important implications for fiscal, monetary, trade and industrial policy. The best students should be able to comment on this.

Examiner's comments: The simple diagrammatic analysis of the wine market posed no particular problem. The best answers provided a parsimonious sketch of all the relevant movements of demand and supply curves, correctly explained. Part b was more challenging: first of all, it required good understanding of the implications of a perfectly competitive market structure beyond a simple definition; secondly, it required a critical appreciation of the theoretical vs. empirical validity of the model. The final part of the question (c) focused on the macroeconomic definition of national income and asked the candidates to reason about possible inequalities between related components. The question was very effective in stretching the students' performances.

2 *(a) Define a firm's fixed and variable costs. By means of a diagram or diagrams represent and explain the relevant (conventionally shaped) total, average and marginal costs curves in the short run.* [5]

A good answer will define fixed costs as costs related to the factors of production that do not change directly with the level of output in the short term (e.g. typically capital equipment, rent and business rates, depreciation of capital assets, insurance charges; marketing and advertising costs...). Instead, variable costs are costs that vary directly with output such as labour costs or the cost of inputs such materials and components. Excellent answers will specify that what is fixed and what is variable may depend on the particular industry of reference. The definition and correct representation of the marginal costs curve is extremely important. An excellent answer will refer to the principle of marginal diminishing returns to

the variable factor of production. The best students could also discuss the fact that the marginal cost curve will intersect the average cost curve at its minimum.

(b) *Consider an oligopolistic market structure:*

(i) *Compare and contrast the Cournot and Bertrand models of oligopoly;* [5]

A good answer will include the main assumptions behind the models (the number of firms producing homogenous or slightly differentiated goods, perfect information, entry barriers, the strategic and simultaneous nature of decisions). The difference in the output- vs. price-setting decision of Cournot vs. Bertrand respectively is of particular importance. The students are expected to discuss how the Cournot equilibrium will lie between competitive and monopoly prices, while the outcome of Bertrand competition will be equivalent to a perfectly competitive outcome.

(ii) *Does competition among oligopolists maximise total industry profit? Explain your answer* [5]

Typically oligopolistic firms maximise their own profits in the absence of co-operation or collusion. If they instead co-ordinate their strategic decisions, they can deliberately aim to optimise total industry profits as if they were one single monopolist, and then split the total profit according to a quota system.

(c) *Illustrate the fundamental principles of Keynesian consumption theory.* [10]

The students should start from components of aggregate demand in a closed economy (no trade) and therefore consider consumption (C), investment (I) and government demand (G). They should define disposable income as total income minus total taxes:  $Y - T$  and specify a simple consumption function in the form  $C = C(Y - T)$ . They should then define the Marginal propensity to consume (MPC) as the change in C due to changes in disposable income. The main conjectures of the theory are: 1) Income is the main determinant of consumption; 2)  $0 < MPC < 1$  and 3) the average propensity to consume (APC) will fall as income rises. The best students will provide accurate diagrammatic representations of the curves. The basic model predicts that the distribution of income will affect total consumption; that economies may suffer from 'underconsumption' as they grow; and that governments can remedy potential underconsumption by expanding demand through fiscal policy. The best students will comment that the Keynesian theory of consumption was found able to explain cross-sectional differences between households, but unable to explain longitudinal variations (the so-called 'consumption puzzle').

Examiner's comments: The main features of the standard costs' structure of a firm were broadly there. The best students were able to provide sharp and detailed answers, including comments on the theoretical rationale behind the shape and interaction of the marginal cost and average cost curve. Most candidates were able to cover the main characteristics of the two oligopoly models. The best students immediately grasped the comparative nature of the question and structured their answers accordingly. The problem of industry-wide profit maximization under oligopoly was generally well understood, but only the better students were able to refer to foundational microeconomic principles with the required accuracy. The quality of the candidates' answers to the macro question (c) varied considerably – as expected – but overall both the key features of Keynesian consumption theory and their policy implications have been absorbed.

**IB Paper 8 – Section B – Civil Crib Dr Brilakis**

3. (a)

(i)  $w = eS_r/G_s = 1.20 \times 1/2.70 = 0.444 = \underline{44.4\%}$

$$\gamma = \left( \frac{G_s + S_r e}{1 + e} \right) \gamma_w = ((2.70 + 1 \times 1.2)/(1 + 1.2)) 9.8 \text{ kNm}^{-3} = \underline{17.4 \text{ kNm}^{-3}}$$

(ii)  $\sigma_v = 17.4 \times 10 = \underline{174 \text{ kNm}^{-2}}$

$$u = 9.8 \times (10 - 1) = 88.2 \text{ kNm}^{-2}$$

$$\sigma'_v = 174 - 88.2 = \underline{85.8 \text{ kNm}^{-2}}$$

(iii)  $\sigma_h = \sigma_v + 2c_u = 174 + 2 \times 50 = \underline{274 \text{ kNm}^{-2}}$

(iv)  $K_p = (1 + \sin\phi')/(1 - \sin\phi') = (1 + \sin 25^\circ)/(1 - \sin 25^\circ) = 2.46$

$$\sigma'_h = K \sigma'_v = 2.46 \times 85.8 = 211.0 \text{ kNm}^{-2}$$

$$\sigma_h = \sigma'_h + u = 211.0 + 88.2 = \underline{299 \text{ kNm}^{-2}}$$

(b)

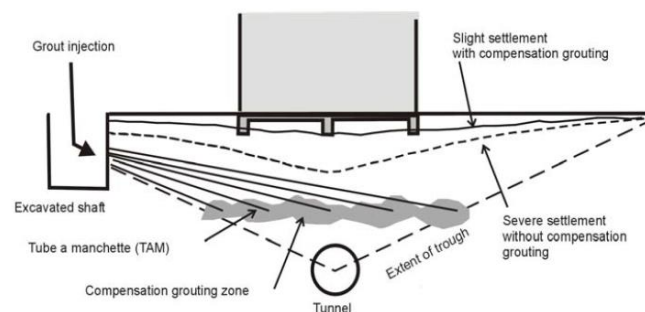
(i) Stability ratio is defined as  $N = (\sigma_v - \sigma_t)/c_u$ , where  $\sigma_v$  is the total vertical stress,  $\sigma_t$  is the support pressure and  $c_u$  is the undrained shear strength of clay.

(ii) Open face mode ( $\sigma_t = 0$ ) can be safely used when  $N$  is about 3.

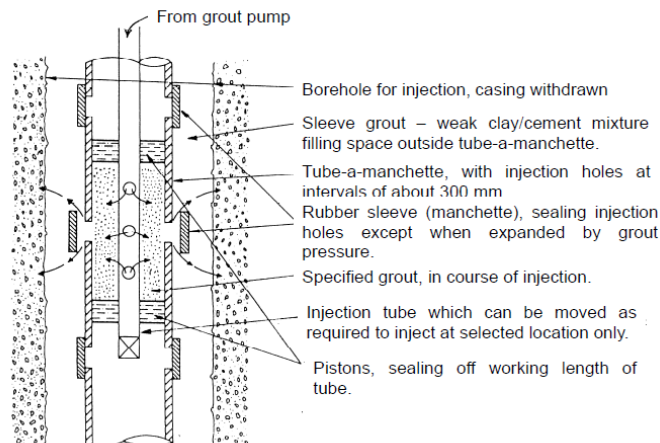
$\sigma_v = 3 \times 100 = 300 \text{ kNm}^{-3}$ . If  $\gamma = 20 \text{ kNm}^{-3}$ , this is equivalent to  $300/20 = \underline{15 \text{ m}}$ .

(iii)

Compensation grouting is a technique to offset subsidence caused during underground excavation and bored tunnelling. The basic principle is that grouts are injected in the zone between the tunnel and overlying buildings to compensate for the ground loss and stress relief induced by underground excavation. A common configuration of the compensation grouting operation is a fan array of grouting holes radiating horizontally from a vertical shaft as shown below. Grout injection is usually undertaken contemporaneously with tunnelling in response to detailed monitoring, so that settlements and distortions are limited to specified amounts.



Grout injection is commonly achieved through the use of a sleeved tube known as a tube a` manchette (TAM). A TAM is a plastic or steel tube with pairs of holes drilled at intervals of 0.3–1.0 m along the length of the tube, each pair being covered by a tight rubber sleeve. These ports act as one-way valves. The TAMs allow re-injection of grout from the same port; appropriate amounts of grout are injected at the right place and time.



(iv) Excess pore pressure may generate when conduct grout injection in clayey soils. This excess pore pressure dissipates with time, accompanied with soil consolidation. Hence the compensation or heaving effect obtained during construction may reduce with time.

Examiner's comments: This question tested the knowledge of theory on soil stability conditions for tunnelling and general underground work. Most students attempted to answer it, and were able to answer most parts well. The first halves of sub-questions (a) and (b) were answered by most students. The second half of sub-question (a) required them to understand how a Mohr's circle would apply in this case to calculate stresses. While most answered the question, around half were able to answer it correctly. The second half of sub-question (b), part (iii), was only partially answered by most students as they, in their rush, gave very simplistic responses that missed essential information expected in the answer. Only the best candidates included precise diagrammatic details and descriptions. Part (b)(iv) was answered relatively well.

4.

$$(a) (i) K_p = (1 + \sin\phi') / (1 - \sin\phi') = (1 + \sin 30^\circ) / (1 - \sin 30^\circ) = 3$$

$$K_a = (1 - \sin\phi') / (1 + \sin\phi') = (1 - \sin 30^\circ) / (1 + \sin 30^\circ) = 0.33$$

On the soil side

$\sigma_v = 17 \times z$  (kNm<sup>-2</sup>) and  $\sigma'_v = 7 \times z$  (kNm<sup>-2</sup>), where  $z$  is the depth from the original ground surface.

$$\sigma'_h = 0.33 \times 7 \times z = 2.31z \text{ (kNm}^{-2}\text{)}, \sigma_h = 2.31z + 10z = 12.31z \text{ (kNm}^{-2}\text{)}.$$

On the excavation side

From  $0 < z < 10$

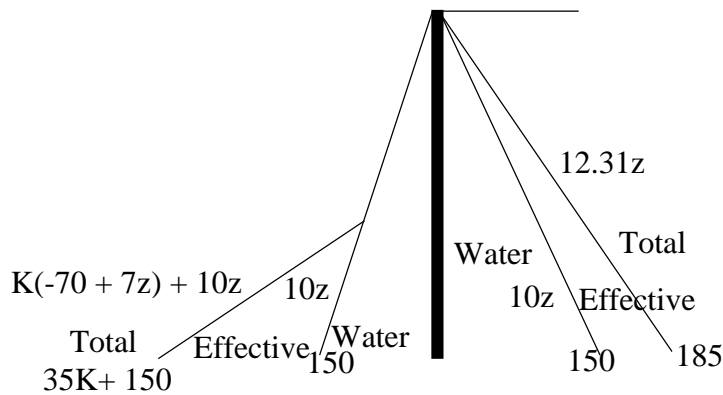
$$\sigma_v = 10 \times z \text{ (kNm}^{-2}\text{)} \text{ and } \sigma'_v = 0 \text{ (kNm}^{-2}\text{)}$$

$$\sigma'_h = 0 \text{ (kNm}^{-2}\text{)} \text{ and } \sigma_h = 10z \text{ (kNm}^{-2}\text{)}$$

From  $10 < z < 15$

$$\sigma_v = 100 + 17 \times (z - 10) = -70 + 17z \text{ (kNm}^{-2}\text{)} \text{ and } \sigma'_v = -70 + 17z - 10z = -70 + 7z \text{ (kNm}^{-2}\text{)}$$

$$\sigma'_h = K(-70 + 7z), \sigma_h = K(-70 + 7z) + 10z$$



(ii) The two sides of the water pressure balance each other and hence only the effective stress part is considered here. By taking a moment around the prop location,  $K$  can be evaluated.

$$0.5 \times 35 \times 15 \times 10 = 0.5 \times K \times 35 \times 5 \times (10 + 5 \times 2/3)$$

$$K = 2.25$$

$$K_p / K = 3 / 2.25 = 1.33$$

(iii) The “effective” force on the soil side =  $0.5 \times 35 \times 15 = 262.5$  kN/m

The “effective” force on the excavation side =  $0.5 \times 2.25 \times 35 \times 5 = 196.9$  kN/m

Prop force  $P = 262.5 - 196.9 = 65.6 \text{ kN/m}$ .

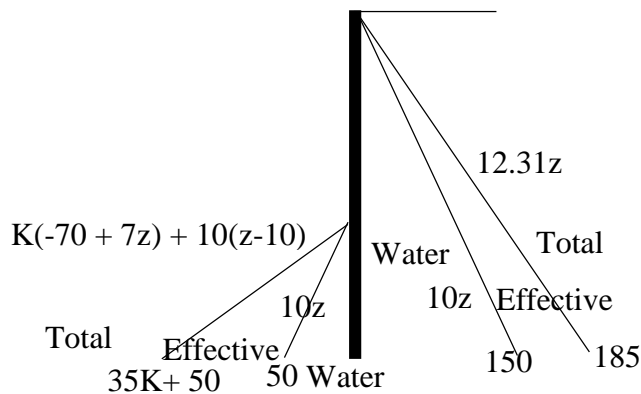
(b) (i)  $z$  is the depth from the original ground surface.

The soil side – same as (a)

The excavation side

$$\sigma_v = 17(z-10) = 17z-170 \text{ (kNm}^{-2}\text{)}, \sigma'_v = 7(z-10) = 7z-70 \text{ (kNm}^{-2}\text{)}$$

$$\sigma'_h = K(7(z-10)) = K(7z-70), \text{ (kNm}^{-2}\text{)} \quad \sigma_h = K(7z-70) + 10(z-10). \text{ (kNm}^{-2}\text{)}$$



Using the total stress, compute the moments around the prop for both side.

The soil side =  $0.5 \times 185 \times 15 \times 10 = 13,875 \text{ (kN/m)·m}$

The excavation side (using  $K = K_p = 3$ ) =  $0.5 \times (35 \times 3 + 50) \times 5 \times (10 + 5 \times 2/3) = 5,154 \text{ kN/m·m}$

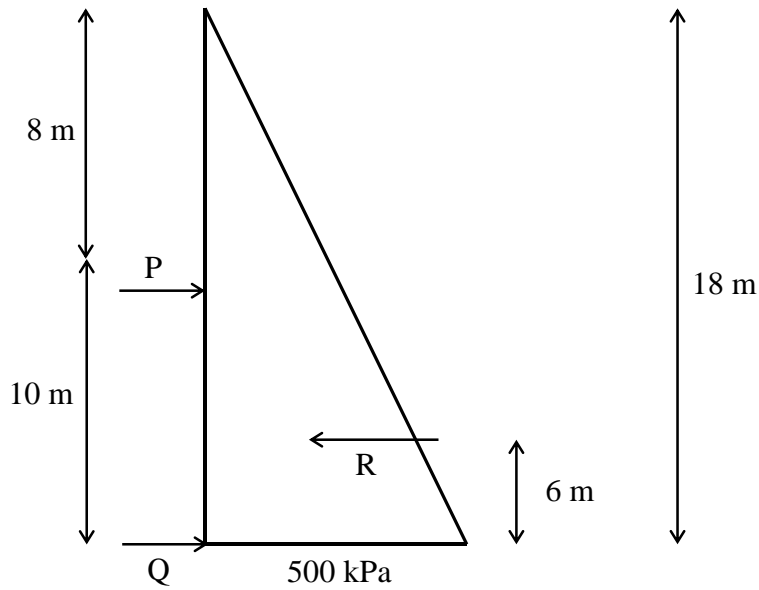
Because the active moment is greater than the resisting moment, the wall is not safe.

(ii) More props are needed and/or the retaining wall needs to be keyed deeper into the rock.

Examiner’s comments: This question tested the student’s understanding of the forces on both sides of a wall meant to protect an excavation and the methods behind calculating it. Two thirds of the students answered it well, and those able to answer sub-question (a) were usually able to answer sub-question (b). Those who did not answer well the first sub-question were almost always unable to continue. The mistakes were partly numerical and partly lack of being able to visualize the forces correctly and understand how to put the equations together. A few students run out of time.



5.



Resultant force on the wall is:

$$R = 500 \times \frac{1}{2} \times 18 \times 10^3 N = 4,500 \text{ KN/m}$$

Taking moments about the base

$$4,500 \times 6 = P \times 10 \Rightarrow P = 2,700 \text{ KN/m}$$

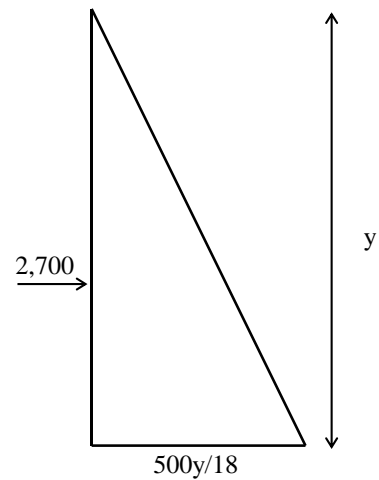
$$\therefore Q = R - P = 1,800 \text{ KN/m}$$

$\therefore$  At depth  $y$  below prop ( $y > 8 \text{ m}$ )

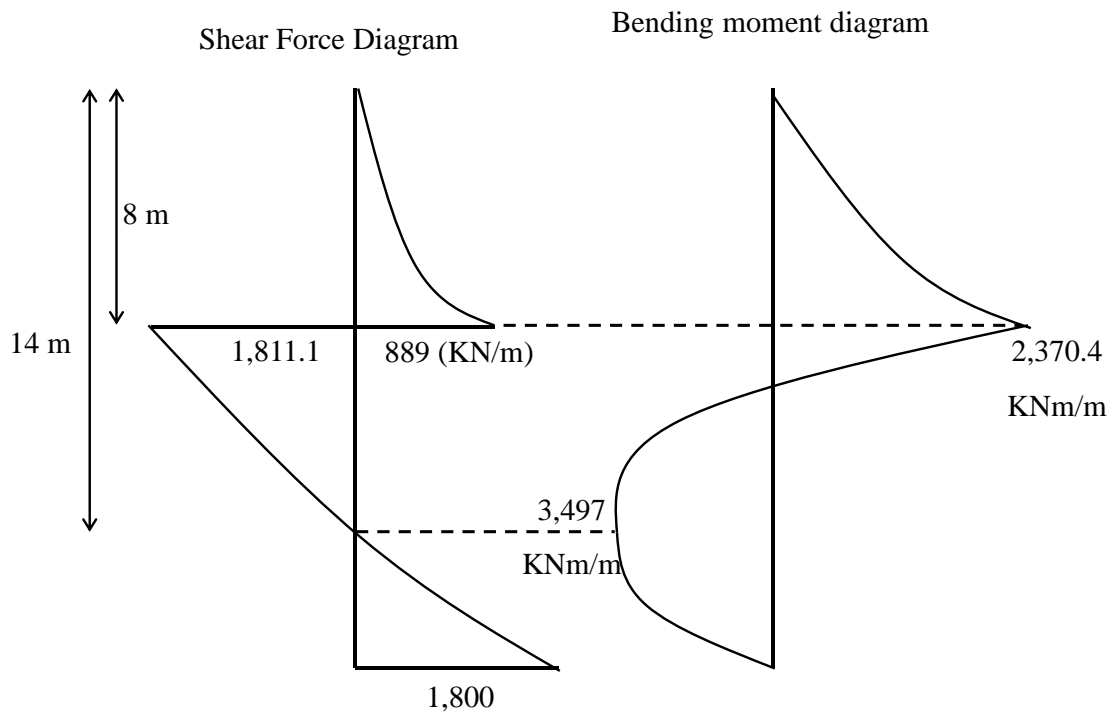
$$\text{Shear force} = \frac{1}{2} \times \frac{500}{18} \times y \times y - 2,700 \text{ KN/m}$$

$$\therefore SF = 0 \text{ when } 13.889y^2 = 2,700 \Rightarrow y = 13.94 \text{ m} \cong 14 \text{ m}$$

$$\text{Bending moment} = \frac{\frac{500}{18}y^2}{2} \times \frac{y}{3} - 2,700 \times (y - 8)$$



(a)



(b)

From Data Sheet:

$$M = 0.15 \times f_{cu} \times bd^2 \Rightarrow 3,497 \times 10^6 = 0.15 \times 40 \times 1000 \times d^2 \Rightarrow$$

$$d = 763.4 \text{ mm} \cong 765 \text{ mm}$$

This is the effective depth.

$\therefore$  Actual wall thickness will be greater to allow for cover etc.

To find steel required at prop position

Guess  $x = 0.5$  and from Data Sheet

$$M = 0.87 \times f_y \times A_s \times d \times \left(1 - \frac{x}{2}\right)$$

$$2370.4 \times 10^6 = 0.87 \times 460 \times A_s \times 765 \times \frac{3}{4} \Rightarrow A_s = 10,323.4 \text{ mm}^2/\text{m}$$

$$\cong 10,324 \text{ mm}^2/\text{m}$$

$$\text{But } x = 2.175 \frac{f_y}{f_{cu}} \times \frac{A_s}{bd} \text{ (Data Sheet)}$$

$$\text{So } x = 0.34 \Rightarrow \text{Calculate new } A_s = 9,328.4 \text{ mm}^2/\text{m}$$

(could iterate further, but stop here)

$$\therefore A_s = 9,329 \text{ mm}^2/\text{m}$$

$$40 \text{ mm bars} = 1256.6 \text{ mm}^2, \text{ hence need } 7.4/\text{m}$$

Say 40 mm bars at 125 mm centres => 8 bars

At Max moment position

$$M = 3,497 \times 10^6 \text{ Nmm, choose } x = 0.5$$

$$\therefore A_s = 15,230 \text{ mm}^2/\text{m}$$

(no need to iterate since  $d$  chosen to give balanced section here)

Answer: 40 mm bars @80mm centres or 50 mm bars @ 125 mm centres

(c)

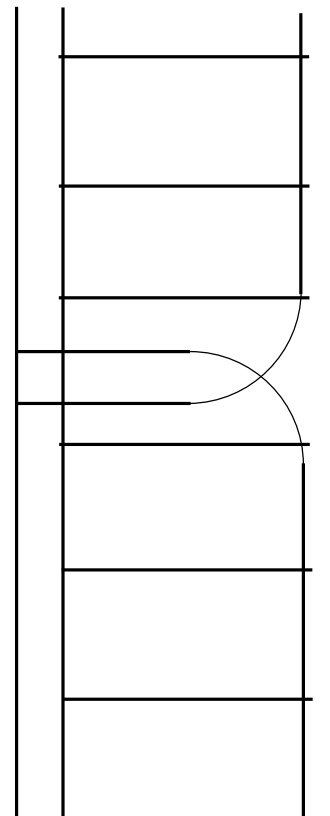
No real benefits of double reinforcing the wall. It would need to be applied over significant length. However, the trench could be narrower so that less material would need to be encountered; so there could be benefit depending on relative costs of excavation and steel.

(d)

The total effective area (curving bars) should be 9,329 mm<sup>2</sup>/m.

Shear reinforcement or link bars are not designed here but would be needed.

Examiner's comments: This question is related to the design of a reinforced concrete wall. Half of the candidates were able to calculate the reaction forces, shear and bending moments, as well as to determine whether single reinforcement was enough for hogging and sagging moments correctly. Most of the candidates attempted to propose solutions for reinforcement at the prop level. About half of the candidates manage to answer the entire question. In general, the best candidates generated correct diagrams for the location of reinforcement bars given shear and bending moments. The appropriate number, dimensions, and spacing of bars was partly well answered.



## Engineering Tripos Part 1B

### Paper 8, Selected Topics, Section C

#### Crib 2015/6 (Michael Sutcliffe)

6 (a) Suggested points to cover:

- loading on various elements from wind shear, and turbulence
- critical frequencies of vibration relative to load spectrum (1P and 3P)
- life cycle fatigue analysis for critical locations on structure (including joints)
- start-up
- gearbox and blade noise
- location relative to habitation
- spectrum of noise and relative nuisance value
- need to be away from habitation, with the distance dependent on the turbine size.

See notes for details.

(b) See figure for rainflow analysis below.

Once the individual cycles of loading have been calculated, a spectrum of loading can be characterised by binning the cases into different mean stress and stress amplitude, or by fitting the data to mathematically distribution functions such as the exponential or Raleigh distributions. Goodman's rule can be used to include the effect of mean stress and Miner's rule used to sum up the different cycle contributions to failure.

(c)

Assume self-similar **planform** and **cross section** both of which scale with blade length  $L$ . Assume uniform pressure loading  $p$  with the aerofoil acting with the same drag and lift coefficients and same relative velocity for a change in  $L$  (which implies different rotational velocities).

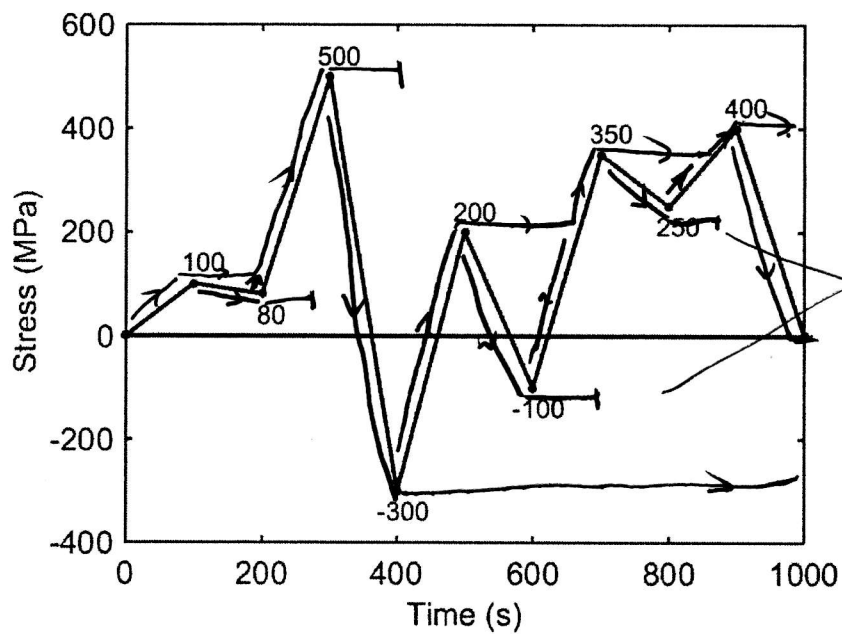
Parameter	Growth exponent $n$ ( $L^n$ )
(i)	
Second moment of area $I$	4 [assume all dimensions of cross section $\sim L$ ]
Self-weight root bending moment $M_{SW} \approx \int mgr dr$	4 [ $m$ = mass per unit length $\sim L^2$ ]
Self weight stress $\sigma_{\max,sw} = \frac{M_{sw} b_0}{I_{NN} 2}$	1
(ii)	
Aerodynamic loading intensity $F_N = \rho c$	1 [ $c$ = chord length $\sim L$ ]
Total aerodynamic load $\int F_N dr$	2
Aerodynamic root bending moment $M_N \approx \int F_N r dr$	3
Aerodynamic stress $\sigma_{\max,aero} \approx \frac{M_N}{I_{TT}} d_o$	0

[Good points were raised in (a), with full marks given when most of the points listed above were discussed. The rainflow analysis was generally done well. The scaling analysis tended to be done by deriving equations in full, rather than inspecting the governing equations to identify which parameters scaled with  $L$  and which parameters did not vary.]

EGT1

ENGINEERING TRIPOS PART IB

Friday 3 June 2016, Paper 8, Question 6.



Extra copy of Fig. 3 for Question 6.

Cycles:

0 → 500	MPa	100 → 80	MPa
80 → 100	"	500 → -300	"
-300 → 400	"	200 → -100	"
-100 → 200	"	350 → 250	"
250 → 350	"	400 → 0	"

Q7

(a) (i) Since  $\lambda = \frac{\omega R}{v}$  and  $\omega$  and  $R$  are both fixed,  
 $\lambda \propto 1/v$ . So if  $\lambda = 8$  at  $v = 6 \text{ ms}^{-1}$  then  $\lambda = 4$   
 at  $v = 12 \text{ ms}^{-1}$ . Since  $C_p \propto \lambda$  and  $C_p = 0.4$  when  
 $\lambda = 8$  then  $C_p = 0.2$  when  $\lambda = 4$ .

$$P = \frac{1}{2} C_p \rho A v^3$$

At rated wind speed:  $P = 1.2 \text{ MW}$ ,  $C_p = 0.2$ ,  $v = 12 \text{ ms}^{-1}$

$$\Rightarrow 1.2 \times 10^6 = \frac{1}{2} \times 0.2 \times 1.23 \times A \times 12^3$$

$$\Rightarrow A = 5646 \text{ m}^2 \Rightarrow \underline{d = 84.8 \text{ m}}$$

$$(ii) \omega_s = \frac{2\pi f}{p} = \frac{2\pi \times 50}{8} = \underline{39.3 \text{ rad s}^{-1}}$$

$$\lambda = 8 = \frac{\omega R}{v} \quad \text{when } v = 6 \text{ ms}^{-1} \quad \left( R = \frac{d}{2} = 42.4 \text{ m} \right)$$

$$\omega_e = 1.13 \text{ rad s}^{-1}$$

$$\therefore \text{ratio} = \frac{39.3}{1.13} = \underline{35}$$

Q7 b) Generator speed is fixed at synchronous speed of  $39.3 \text{ rad/s}$

(i) At rated wind speed  $P = -1.2 \text{ MW} = T_g \omega_s$

$$\Rightarrow T_g = -30.5 \text{ kNm}$$

Using simplified torque equation:-

$$T = \frac{3V^2}{\omega_s k_2'} \Rightarrow -30.5 \times 10^3 = \frac{3 \times \left(\frac{6.6 \times 10^3}{\sqrt{3}}\right)^2}{39.3 \times 1} s$$

$$\Rightarrow s = -0.0275$$

$$I = \frac{V_{ph}}{R_1 + k_2'/s + j(X_1 + X_2')} = \frac{6.6 \times 10^3 / \sqrt{3}}{1 + 1/(-0.0275) + j4}$$
$$= \underline{107 \text{ A}}$$

(ii) At  $v = 6 \text{ m/s}$ ,  $\lambda = 8$  and  $C_p = 0.4$  so

$$P = \frac{1}{2} \times 0.4 \times \rho \times 5646 \times 6^3 = 300 \text{ kW}$$

$$\Rightarrow T_g = -300 \times 10^3 / 39.3 = \underline{-7.63 \text{ kNm}}$$

$$-7.63 \times 10^3 = \frac{3 \times \left(\frac{6.6 \times 10^3}{\sqrt{3}}\right)^2}{39.3 \times 1} s \Rightarrow s = -0.00688$$

$$I = \frac{6.6 \times 10^3 / \sqrt{3}}{((1 - 1/0.00688) + j4)} = \underline{26.4 \text{ A}}$$

7 (c)

(i) 40GW installed per year, at  $0.012 \text{ kg/kW} = 480 \text{ tonnes Nd/year}$ .

Total annual production is 20,020 tonnes/year. Wind turbines consume approximately 2.4% of world production.

(ii) Materials are classified as 'critical' if access to them could be limited (e.g. by monopoly of supply), and they are essential for national security or important economically.

China produces 97% of Nd globally, so the USA and EU are dependent on China for supplies of this element for wind turbine magnets. Without Chinese supplies wind turbines would consume almost all of the capacity of the rest of the world, so the consumption of Nd is critical. Note that this material would also be used in many other applications too, compounding the problem.

(iii) Total production capacity of CFRP is 104,600 tonnes/yr, so current global production is 45% of this = 46,500 tonnes/yr. Wind turbine consumption of 6,700 tonnes CFRP/year is therefore 14% of the total.

CFRP consumes a much higher percentage of world supply than Nd. But the strategic concerns for the USA and EU are much lower, since Japan, the USA and the EU control 70% of global supply. Moreover there is a capacity for growth and indeed capacity for this material can be increased since it does not rely on a rare raw material.

[Most candidates succeeded with (a) parts (i) and (ii), although with the usual mistakes of mixing up poles and pole-pairs caused errors in the generator speed and hence gearbox ratio. Plenty of good answer to part (b), the main problem being candidates ignoring the torque-slip equation simplification and subsequently ending up in a tangle of algebra when estimating the slip. In (c) general comments were good but often not adequately supported by using the numbers given in the question.]

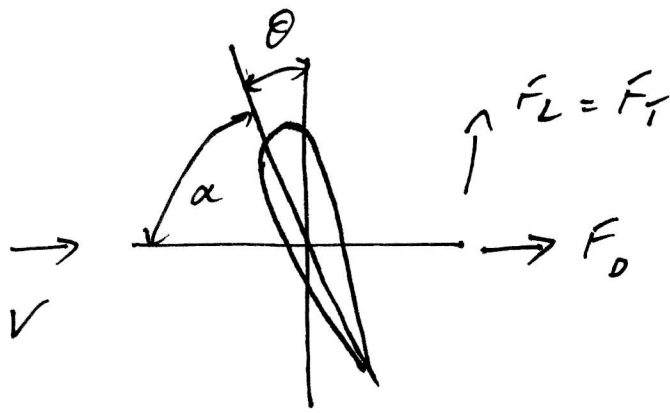
Q8 (a) Lift - initially flow is smooth over the aerofoil and lift is approximately proportional to the angle of attack with small  $C_D$ . Then separation occurs with a sharp drop in  $C_L$  and a corresponding increase in  $C_D$ .

[This question was answered well by many who remembered that the lift and drag force on an aerofoil are defined to be perpendicular and parallel, respectively, to the incident wind direction. Since the turbine is initially stationary only the lift force produces torque and the angle of attack is simply  $\alpha = 90^\circ - \theta$ . Unfortunately a significant number of candidates who attempted the question defined their lift and drag forces incorrectly to be aligned to the chord line.]



Q8

(b)



$$\phi = \alpha + \theta$$

where  $\tan \phi = \frac{V}{wT} = \infty$

$$\therefore \phi = 90^\circ$$

$$\alpha = 90^\circ - \theta$$

To get initial torque need  $F_L = \frac{1}{2} \rho V^2 c C_L$

$$= \frac{1}{2} \times 1.2 \times 8^2 \times c \times C_L$$

$$= 38.4 \times c \times C_L$$

	$r$ (m)	2	6	10	14	18
M	mass (kg)	520	440	360	200	100
	$\theta$ (deg)	20	13	5	2	0.5
	$\alpha$ (deg)	70	77	85	88	89.5
	$C_L$	0.46	0.35	0.19	0.12	0.08
	$c$ (m)	1.6	1.5	1.3	1	0.7
	$F_T$ (N/m)	27	20	9.5	4.6	2.15
	Force (N)	104	80	38	18.4	8.6
	Torque ( $F \times r$ ) (Nm)	208	480	380	258	155

Total torque = 1480 Nm / blade  
= 4440 Nm for 3 blades.

Q 8 (c)

$$J = \sum r^2 M$$

$$= 4 \times 520 + 36 \times 460 + 100 \times 360 + 196 \times 200 + 324 \times 100$$

$$= 123520 \text{ kg m}^2 \text{ per blade}$$

$$T = J \dot{\omega} \Rightarrow \dot{\omega} = \frac{1480}{123520} = 0.012 \text{ rad/s}^2$$
$$= 0.69 \text{ deg/s}^2$$

(d) Assume that this initial acceleration holds for the acceleration up to 30 rpm

$$\omega = \dot{\omega} t$$

$$\Rightarrow t = \frac{30 \times 2\pi}{60} \times \frac{1}{0.012} = 260 \text{ s}$$

(e) Now we would need to modify the aerodynamic calcs for lift and drag to take into account the changing relative motion associated with the rotation. And the acceleration will vary with time. Both lift and drag forces will have components giving rise to acceleration moments

9. (a)

$$\frac{T_{03}}{T_{02}} = \left[ \frac{(p_{03}/p_{02})^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} + 1 \right] \quad \therefore T_{03} = 288 \times \left[ \frac{(24)^{\frac{\gamma-1}{\gamma}} - 1}{0.9} + 1 \right] = 761.4 \text{ K}$$

$$n_{stage} \geq \frac{\Delta h_{0,total}}{U^2 \left( \Delta h_0 / U^2 \right)_{max}} = \frac{c_p (T_{03} - T_{02})}{U^2 \times 0.45} = \frac{1005 \times (761.4 - 288)}{300^2 \times 0.45} = 11.75$$

\(\therefore\) The minimum number of stages is 12.

[5]

(b)

$$\frac{\dot{m} \sqrt{c_p T_{02}}}{A_x p_{02}} = \frac{\gamma}{\sqrt{\gamma-1}} M \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{-(\gamma+1)/2(\gamma-1)} = 1.056 \quad \text{for } M = 0.58$$

$$A_x = \frac{\dot{m} \sqrt{c_p T_{02}}}{Q(0.58) p_{02}} = \frac{60 \times \sqrt{1005 \times 288}}{1.056 \times 98000} = 0.3119 \text{ m}^2$$

$$\text{The mean radius, } \bar{r} = \frac{A_x}{2\pi h} = \frac{0.3119}{2\pi \times 0.2} = \underline{0.248 \text{ m}}$$

[5]

(c) Work balance for the shaft:

$$c_p (T_{03} - T_{02}) = c_p (T_{04} - T_{05})$$

$$\therefore T_{05} = T_{04} - (T_{03} - T_{02}) = 1500 - (761.4 - 288) = \underline{1026.6 \text{ K}}$$

$$\text{For the turbine, } \frac{p_{05}}{p_{04}} = \left( 1 - \frac{T_{05}}{T_{04}} \right)^{\frac{\gamma}{\gamma-1}} \Rightarrow p_{05} = 24 \times 0.98 \times \left( 1 - \frac{1026.6}{1500} \right)^{\frac{7}{2}} = \underline{5.189 \text{ bar}}$$

For the exhaust jet, given the nozzle is isentropic:

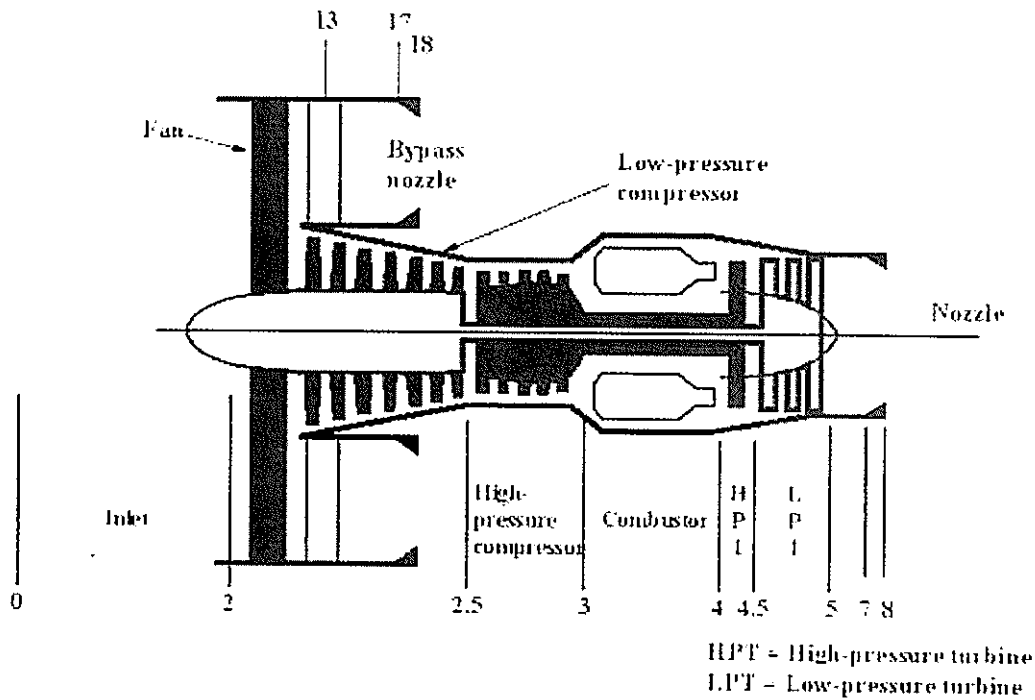
$$0.5V_9^2 = c_p (T_{05} - T_9) = c_p T_{05} \left( 1 - (p_9/p_{05})^{(\gamma-1)/\gamma} \right)$$

$$V_9 = \sqrt{2c_p T_{05} \left( 1 - (p_9/p_{05})^{(\gamma-1)/\gamma} \right)} = \sqrt{2 \times 1005 \times 1026.6 \times \left( 1 - (0.98/5.189)^{2/7} \right)} = \underline{884.2 \text{ m/s}}$$

[7]

(d) The turbojet has a very high jet velocity, which leads to low propulsive efficiency (and high jet noise). A turbofan extracts kinetic energy from core and uses this to drive a fan, which pushes a large mass flow of air through the bypass duct. The jet velocity of the core jet

and the bypass jet are similar and much lower than for a turbojet, leading to high propulsive efficiency and low noise. The increased mass flow means the required thrust is still achieved.



The factors limiting the upper value of the bypass ratio include the nacelle weight and drag, since the engine diameter increases as BPR rises. The fan and LP turbine weight also increase as the fan diameter increases and more power is required from the fan but at a lower rotational speed .

This question worked well. Part (a) on calculating the number of compressor stages was answered perfectly by almost all the candidates. However, part (b) on applying continuity to a compressible flow was found to be very difficult. Most candidates wanted to use  $\dot{m} = \rho AV$  , but few determined the correct area or the correct (static) density. Only a couple of candidates saw the easy way to do this using the non-dimensional mass flow function. Part (c) involved calculating conditions at turbine exit and the jet velocity. Some candidates forgot to balance the compressor and turbine work or applied the turbine isentropic efficiency incorrectly. Otherwise this part was well answered. For part (d) most candidates explained well why modern passenger aircraft use high bypass ratio engines, but drew poor quality sketches of the layout of a turbofan. Most knew a couple of factors that limit the upper value of bypass ratio, but surprisingly there was more mention of installation effects than increasing engine weight.

10. (a)

$$\frac{dp}{dz} = -\rho g = -\frac{pg}{RT} \quad , \quad \int_{p_T}^p \frac{dp}{p} = -\frac{g}{RT_T} \int_{z_T}^z dz \Rightarrow \ln\left(\frac{p}{p_T}\right) = -\frac{g}{RT_T}(z - z_T)$$

$$\therefore p = p_T \exp\left\{-\frac{g}{RT_T}(z - z_T)\right\}$$

[5]

(b)

$$SFC = \frac{\dot{m}_f}{F_N} \Rightarrow \frac{dm}{dt} = -SFC \cdot F_N = -SFC \cdot D = -SFC \frac{mg}{L/D}$$

$$\int_{m_1}^m \frac{dm}{m} = -\frac{SFC \cdot g}{L/D} \int_0^s \frac{ds}{V}$$

$$\therefore \ln\left(\frac{m}{m_1}\right) = -\frac{SFC \cdot g}{V L/D} s \Rightarrow m = m_1 \exp\left(-\frac{g SFC}{V L/D} s\right)$$

[5]

(c)

$$C_L = \frac{L}{0.5 \rho A V^2} = \frac{mg}{0.5 \frac{p}{RT_T} A V^2} \Rightarrow p = \frac{mgRT_T}{0.5 A V^2 C_L}$$

$$\text{Using equations above, } p_T \exp\left\{-\frac{g}{RT_T}(z - z_T)\right\} = \frac{m_1 g RT_T}{0.5 A V^2 C_L} \exp\left(-\frac{g SFC}{V L/D} s\right)$$

$$\therefore z - z_T = \frac{RT_T \cdot SFC}{V L/D} s - \frac{RT_T}{g} \ln\left(\frac{m_1 g RT_T}{0.5 A p_T V^2 C_L}\right)$$

This is a linear relationship between  $z$  and  $s$  since all other parameters are constants.

[6]

(d)

$$C_L = \frac{m_1 g}{0.5 \frac{p_T}{RT_T} A V^2} = \frac{m_1 g}{0.5 p_T A \gamma M^2}$$

$$M = \sqrt{\frac{m_1 g}{0.5 p_T A \gamma C_L}} = \sqrt{\frac{150000 \times 9.81}{0.5 \times 0.226 \times 10^5 \times 270 \times 1.4 \times 0.5}} = \underline{0.830}$$

$$V = M \sqrt{\gamma RT_T} = 0.83 \sqrt{1.4 \times 287 \times 216.65} = 244.9 \text{ m/s}$$

Mass of fuel burned,  $m_{fuel} = m_1 - m$

$$= m_1 \left( 1 - \exp \left( - \frac{g \text{ SFC}}{V L/D} s \right) \right) = 150 \left\{ 1 - \exp \left( - \frac{9.81 \times 0.016}{244.9 \times 20} \times 6000 \right) \right\} = \underline{26.24 \text{ tonne}}$$

From above,  $z - z_T = \frac{RT_T \cdot \text{SFC}}{V L/D} s$  when cruise starts at  $z_T$  since  $\frac{m_1 g RT_T}{0.5 A p_T V^2 C_L} = 1$

Therefore, the final altitude,

$$z = \frac{RT_T \cdot \text{SFC}}{V L/D} s + z_T = \frac{287 \times 216.65 \times 0.016 \times 6000}{244.9 \times 20} + 11000 = \underline{12220 \text{ m}}$$

[9]

This was the least standard question and as a result the least popular. However, it was by far the best answered and despite tough marking had an average of 70%! Parts (a) and (b) were proofs of the pressure variation with altitude above the tropopause and of a form of the Breguet range equation. Both of these were completed perfectly in almost all cases. Part (c) involved combining the results of parts (a) and (b) to find a relationship between cruise distance and altitude. Almost all candidates saw how to do this, using the requirement of constant lift coefficient, but many were a bit careless with the algebra involved. Part (d) was a calculation of fuel burn and altitude change for the cruise of a specific aircraft. This was solved more easily than the assessor expected (partly because several candidates spotted they could find the final altitude without using their result in part (c)).

11. (a)

$$\frac{p_{02}}{p_a} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow p_{02} = 23.8 \times \left(1 + \frac{0.4}{2} 0.85^2\right)^{\frac{7}{2}} = \underline{38.17 \text{ kPa}}$$

$$T_{02} = T_a \left(1 + \frac{\gamma-1}{2} M^2\right) = 219 \times \left(1 + \frac{0.4}{2} 0.85^2\right) = \underline{250.6 \text{ K}}$$

$$V = M \sqrt{\gamma R T} = 0.85 \times \sqrt{1.4 \times 287 \times 219} = \underline{252.1 \text{ m/s}}$$

(b)

$$(i) \quad \eta_p = \frac{2V}{V_j + V} = 0.84 \quad \therefore V_j = V \left(\frac{2}{\eta_p} - 1\right) = 252.14 \left(\frac{2}{0.84} - 1\right) = 348.2 \text{ m/s}$$

$$\dot{m} = \frac{F_N}{V_j - V} = \frac{180 \times 10^3 / 4}{348.2 - 252.1} = \underline{468.5 \text{ kg/s}}$$

$$(ii) \quad F_G = \dot{m} V_j = 468.5 \times 348.2 = \underline{163.1 \text{ kN}}$$

$$(iii) \quad \eta_o = \eta_p \eta_{th} = \frac{F_N V}{\dot{m}_{fuel} LCV} = \frac{1}{sfc} \times \frac{V}{LCV}$$

$$\Rightarrow sfc = \frac{1}{\eta_p \eta_{th}} \times \frac{V}{LCV} = \frac{1}{0.84 \times 0.48} \times \frac{252.14}{43 \times 10^6} = \underline{14.54 \text{ g kN}^{-1} \text{ s}^{-1}}$$

(c)

$$(i) \quad \left(\frac{\dot{m} \sqrt{c_p T_{02}}}{A_N p_{02}}\right)_{cruise} = \left(\frac{\dot{m} \sqrt{c_p T_{02}}}{A_N p_{02}}\right)_{test}$$

$$\Rightarrow \dot{m}_{test} = \frac{\left(\dot{m} \sqrt{T_{02} / p_{02}}\right)_{cruise}}{\left(\sqrt{T_{02} / p_{02}}\right)_{test}} = \frac{468.5 \times \sqrt{250.6 / 38.17}}{\sqrt{290 / 102}} = \underline{1163.8 \text{ kg/s}}$$

$$(ii) \quad \left(\frac{F_G + p_a A_N}{A_N p_{02}}\right)_{cruise} = \left(\frac{F_G + p_a A_N}{A_N p_{02}}\right)_{test}$$

$$F_{G, test} = A_N p_{02} \left(\frac{F_G + p_a A_N}{A_N p_{02}}\right)_{cruise} - p_a A_N = A_N p_{02} \left( \left(\frac{F_G + p_a A_N}{A_N p_{02}}\right)_{cruise} - 1 \right)$$

[4]

[3]

[2]

[3]

[2]

$$\therefore F_{G, test} = 3.8 \times 102 \left( \frac{163.1 + 23.8 \times 3.8}{3.8 \times 38.17} - 1 \right) = \underline{289.9 \text{ kN}}$$

[3]

$$(iii) \quad \left( \frac{\dot{m}_f LCV}{A_N p_{02} \sqrt{c_p T_{02}}} \right)_{cruise} = \left( \frac{\dot{m}_f LCV}{A_N p_{02} \sqrt{c_p T_{02}}} \right)_{test}$$

$$\dot{m}_f_{test} = \left( p_{02} \sqrt{T_{02}} \right)_{test} \left( \frac{sfc \times F_N}{p_{02} \sqrt{T_{02}}} \right)_{cruise} = 102 \times \sqrt{290} \times \left( \frac{0.01454 \times 180/4}{38.17 \sqrt{250.6}} \right) = 1.881 \text{ kg/s}$$

$$\therefore sfc_{test} = \frac{\dot{m}_f}{F_N} = \frac{1.881}{289.9} = \underline{6.49 \text{ g kN}^{-1} \text{ s}^{-1}}$$

[4]

(d) At top-of-climb the inlet stagnation conditions are the same as start of climb. However, the thrust required is greater (to enable a climb rate of typically 300 ft/min). Therefore all the aerodynamic non-dimensional groups are higher at top-of-climb (e.g.  $T_{04}/T_{02}$  around 6.1 at top-of climb, but 5.6 at start-of climb).

The top-of-climb condition is important in the design of a jet engine because it is the most aerodynamically demanding condition (the engine is operating hardest non-dimensionally). The size of the engine is fixed by the top-of-climb condition. The fuel consumption of the engine is fixed by the cruise condition.

[4]

**Dr C. A. Hall**

This was the most popular question, which was slightly surprising as in many ways it was the most involved and required a lot of effort to get all the answers. Part (a) was a standard test of determining stagnation conditions at engine inlet. Over 90% of candidates got this completely right. Part (b) was unusual in terms of the information provided and required candidates to be clear on the mathematical definitions of thrust, engine efficiencies and specific fuel consumption. Generally it was well answered. Part (c) involved relating the results in part (b) to a static engine test at the same non-dimensional operating condition. A surprising number of candidates didn't realise that for a static test the inlet stagnation conditions would equal the ambient conditions. Those that were careful with the use of the non-dimensional groups given generally found the correct answers. Part (d) was not well answered. Few candidates stated clearly that at top-of-climb all the engine non-dimensionals are higher than at cruise and most didn't explain why this was the case or why this is important to the engine design. Most candidates recited standard extracts from the notes about cruise requiring high efficiency and top of climb being aerodynamically demanding.



**PAPER 8 SECTION E PART IB 2016**

Q.12

(a) UV exposure is the current lithography standard used in industry to achieve nanometer scale gate lengths in MOSFETs – it provides high throughput and whole wafer exposure using a step-and-repeat process. For example, the spectrum from a typical mercury UV lamp source shows the strongest emission at a wavelength of 365 nm. The practical limit in UV lithography is diffraction at the edges of the mask used.

On the other hand, E-beam lithography provides smaller feature sizes but is an expensive process. Exposure is by serial means using an e-beam focused to a small spot of typical diameter of 100 nm. The desired pattern is traced using a raster or vector scan, hence, low throughput. For example, for a 30 keV source, the wavelength is  $h/(2mE)^{1/2} = 7 \times 10^{-12}$  m. The practical limit in e-beam lithography is not the diffraction limit to the spot because the wavelength is so small but the scattering of secondary electrons, which expose the resist.

(b) Diffraction effects limit the imaging characteristics of the optical system. The resolution is roughly  $0.5(\lambda/NA)$  where NA is the numerical aperture of the projection optics and  $\lambda$  the wavelength. For a 200 nm source,  $100 \sim 0.5 \times 200/NA$  and  $NA = 1$ .

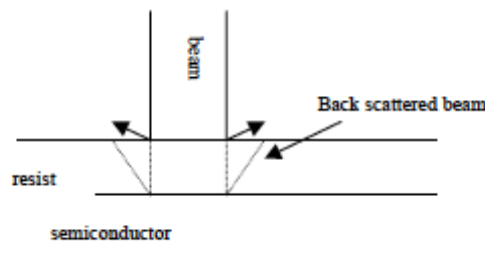
(c)

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 40 \times 10^3}}$$

$$\lambda = \sim 6 \times 10^{-12} \text{ m}$$

However back scattering will limit resolution.

e.g.



(d) Volume of 1 mole of Si = Si mol wt/Si density = 28.09/2.33 = 12.06 cm<sup>3</sup>/mol.

Volume of 1 mole of SiO<sub>2</sub> = 60.08/2.27 = 26.46 cm<sup>3</sup>/mol

→ thickness of Si area/thickness of SiO<sub>2</sub> area = 12.06/26.46 = 0.45

Hence if the required oxide thickness  $t$  is reduced by  $0.45t$ , then the overall total thickness will be increased by  $(1-0.45)t = 0.55t$ .

For 40nm of oxide →  $0.45 \times 40\text{nm} = 18\text{nm}$  of Si consumed

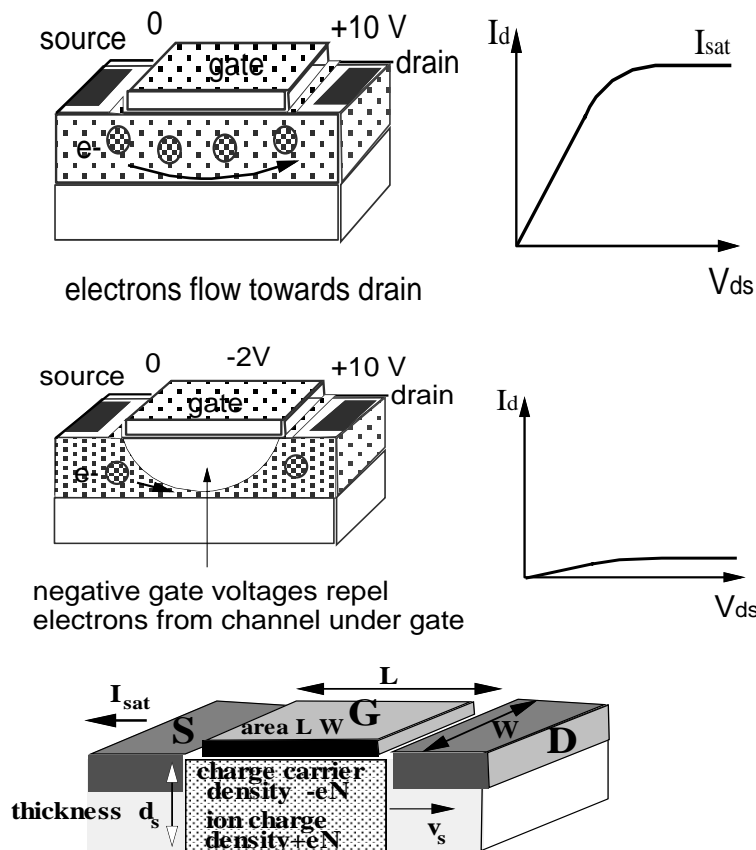
New Si thickness =  $(475-0.018)$  microns = 474.982 microns

New total thickness =  $475+(0.55 \times 0.040) = 475.022$  microns

(e) New materials augmenting (i.e. layered on top of the SiO<sub>2</sub>) include high dielectric constant (i.e. high-k) insulators such as hafnium oxide, zirconium oxide, tantalum oxide, etc.

*Examiner's comments: This was the least popular question. It related to semiconductor manufacturing and the quintessential lithography steps and its possible evolution for scaling of transistor sizes. This question may have not been popular, as some parts were not covered in past exams. However similar questions were covered in the examples sheet.*

Q.13 (a)



(b)

For a rod of radius  $r$ , and length  $L$ , then

$$\text{charge enclosed} = N.e.\pi.r^2.L$$

$$\text{total electric flux through its surface} = 2.\pi.r.L.\epsilon.E$$

$$\text{so } 2.\pi.r.\epsilon.E = N.e.\pi.r^2.L \quad \text{OR} \quad 2.\epsilon.E = N.e.r$$

To get gate turn-off voltage,

$$\text{integrate } E = N.e.r/(2.\epsilon) \text{ over } dr \text{ to give } V = -N.e.r^2/(4.\epsilon)$$

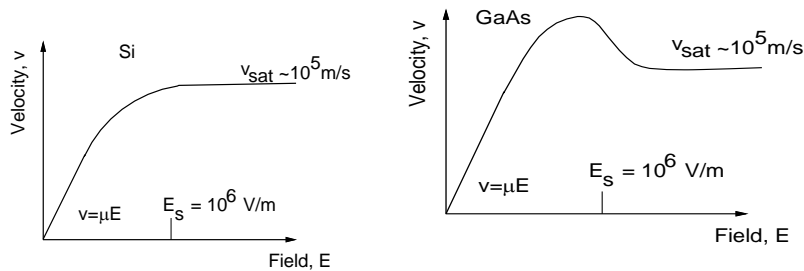
(c)

$$\text{Substitute in quantities, } V = 10^{25} \times 1.6 \cdot 10^{-19} \times (10^{-8})^2 / (4 \times 12 \times 8.8 \cdot 10^{-12}) = 0.379 \text{ V.}$$

The value is small, because of scaled down dimensions.

*Examiner's comments: This question was reasonably well answered, although the students found the last two parts of the question quite challenging. Here they were asked the operating principles of depletion mode field effect transistors and to come up with an approximate expression for the gate voltage for turn-off in a silicon nanowire depletion mode transistor.*

Q.14 (a)



Velocity remains constant (and independent of E) after a certain critical E due to energy loss by scattering – at high velocities, time between collision is so short that carriers don't reach higher velocity.

(b)  $v = \mu \cdot E = \mu \cdot V/L$ , transit time  $\tau = L/v$ . so  $\tau = L^2/\mu \cdot V$  or  $L^2 = t \cdot \mu \cdot V$   
 substituting,  $L^2 = 10^{-5} \times 18 \times 10^{-4} \times 0.2 = 36 \times 10^{-10}$ , so gate length  $L = 6 \times 10^{-5}$  m.

(c) electric field  $E = 0.2 / 6 \times 10^{-5} = 3.33 \times 10^3$  V/m, carrier velocity  $v = E \cdot \mu = 6$  m/s.

(d)  $L = 6 \times 10^{-5}$  m, so  $W = 300 \times 10^{-5}$  m. ( $W/L = 50$ )

channel height  $h = (I \cdot \tau) / (W \cdot L \cdot N \cdot e) = (10^{-3} \times 10^{-5}) / (300 \times 6 \times 10^{-10} \times 10^{26} \times 1.6 \times 10^{-19}) = 3.47 \times 10^{-9}$  m.

number of layers =  $3.47 / 0.336 = 10.33 \rightarrow 11$ .

*This was a very popular question and was also the best answered with quite a number of students obtaining very high marks. It was a multi-part question on carrier transport in semiconductors. It also involved calculations on channel geometrical properties such as channel length and channel layers. Most of the equations required were either derived or on the data sheet.*

Arokia Nathan

# Solutions Paper 8 - Section F

(2016)

## SECTION F: Information Engineering

Answer not more than two questions from this section.

- 1 (a) (i) In the bilinear interpolation for point  $(p, q)$  we now derive the constants  $\alpha, \beta, \gamma, \delta$ .

$$x_{pq} = \alpha x_{ac} + \beta x_{bc} + \gamma x_{ad} + \delta x_{bd}$$

If we have a 1D case, where we want to find the value  $x_p$  at a point  $p$  between points  $a$  and  $b$ , which have values  $x_a, x_b$ , we apply the standard linear interpolation formula so that

$$x_p = x_a + \frac{(p-a)}{(b-a)}(x_b - x_a) = \frac{(b-p)x_a + (p-a)x_b}{b-a}$$

So, taking the values in Fig 1, let us first do linear interpolation in the vertical direction to give  $x_1$  and  $x_2$  (which are shown in figure)

$$x_1 = \frac{(b-p)x_{ac} + (p-a)x_{bc}}{b-a}$$

$$x_2 = \frac{(b-p)x_{ad} + (p-a)x_{bd}}{b-a}$$

Now we interpolate horizontally between  $x_1$  and  $x_2$ :

$$x_{pq} = \frac{(d-q)x_1 + (q-c)x_2}{d-c}$$

Substituting for  $x_1$  and  $x_2$  gives

$$\begin{aligned} x_{pq} &= \frac{(d-q)\frac{(b-p)x_{ac} + (p-a)x_{bc}}{b-a} + (q-c)\frac{(b-p)x_{ad} + (p-a)x_{bd}}{b-a}}{d-c} \\ &= \frac{(d-q)[(b-p)x_{ac} + (p-a)x_{bc}] + (q-c)[(b-p)x_{ad} + (p-a)x_{bd}]}{(b-a)(d-c)} \end{aligned}$$

Which therefore tells us that

$$\alpha = \frac{(d-q)(b-p)}{(b-a)(d-c)}$$

$$\beta = \frac{(d-q)(p-a)}{(b-a)(d-c)}$$

$$\gamma = \frac{(q-c)(b-p)}{(b-a)(d-c)}$$

$$\delta = \frac{(q-c)(p-a)}{(b-a)(d-c)}$$

[8]

(ii) *cubic interpolation* in 1D to a point  $p$  uses 4 consecutive samples (rather than 2 as in bilinear interpolation) located at  $\{a, b, c, d\}$ , with  $b \leq p \leq c$ . If  $u = (p-b)/(c-b)$ , then  $u$  goes from 0 to 1, and we fit a cubic equation in  $u$  such that

$$x(u) = \alpha u^3 + \beta u^2 + \gamma u + \delta$$

and solve for  $\alpha, \beta, \gamma, \delta$  with constraints  $x(0) = x_b, x(1) = x_c, dx/du = (x_c - x_a)/2$  at  $u = 0, dx/du = (x_d - x_b)/2$  at  $u = 1$ .

*Bi-cubic interpolation* in 2D at a point  $x_{pq}$  is obtained by applying cubic equation in both the vertical and horizontal directions using the  $4 \times 4$  neighbourhood of pixels surrounding  $(p, q)$ .

The technique of bicubic interpolation produces less blurring of edges and other distortion artifacts than bilinear interpolation, but is more computationally demanding (about 4 times the computational cost) – in practice, bilinear interpolation is more commonly used. Less blurring of edges by bicubic interpolation occurs because higher frequencies in the signal are attenuated less by cubic than by linear interpolation.

[5]

(b) (i) Using the expression for the Gaussian function we can write  $g_{rc}(x, y)$  as

$$\begin{aligned} g_{rc}(x, y) &= \frac{1}{\sqrt{2\pi b}} e^{-\frac{x^2}{2b^2}} \frac{1}{\sqrt{2\pi b}} e^{-\frac{y^2}{2b^2}} \\ &= \frac{1}{2\pi b^2} e^{-\frac{x^2+y^2}{2b^2}} \end{aligned}$$

giving, in polar coordinates  $((r, \theta))$ ,

$$g_{rc}(r, \theta) = \frac{1}{2\pi b^2} e^{-\frac{r^2}{2b^2}}$$

with  $r^2 = x^2 + y^2$ . Note that this is independent of the  $\theta$ , so the filter is circularly symmetric and therefore *isotropic*.

[6]

(ii) It is not hard to show that the frequency response of our isotropic Gaussian filter is also isotropic. If  $b$  increases, the ‘width’ (any definition of width

here is acceptable) of our isotropic Gaussian filter increases and the filter has a greater smoothing/blurring effect on the image. Correspondingly, the width of the frequency response (also an isotropic Gaussian) decreases: cutting off more of the frequency plane is equivalent to increased spatial smoothing.

[6]

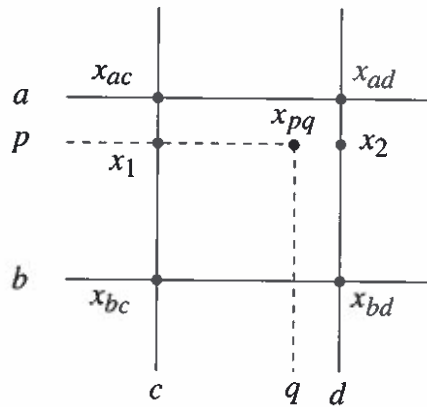


Fig. 1

**END OF PAPER**

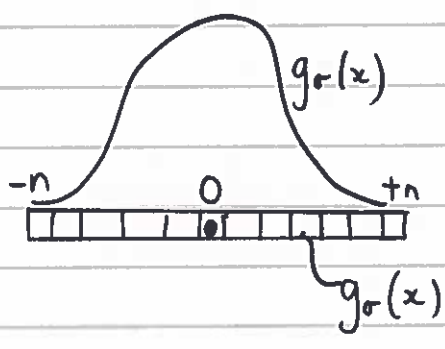
2016 Paper 8 - section F

Q16(a)

(i) Low pass filter is  $g_{\sigma}(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$   
 (2D Gaussian)  
 $= g_{\sigma}(x) g_{\sigma}(y)$

(ii)  $S(x, y) = \sum_{-n}^n \sum_{-n}^n I(x-u, y-v) g_{\sigma}(u) g_{\sigma}(v)$

where Gaussian kernels are sampled  $N = 2n+1$  with  $\frac{1}{1000}$  peak value



(ii)  $\nabla S = \nabla (g_{\sigma}(x, y) * I(x, y)) = (S_x, S_y)$  where  $S_x = \frac{\partial S}{\partial x}$

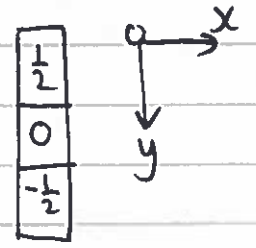
By Taylor series expansion:

$\frac{\partial S}{\partial x} \approx \frac{S(x+1, y) - S(x-1, y)}{2}$

1D convolution kernels

$\frac{1}{2}$	0	$-\frac{1}{2}$
---------------	---	----------------

$\frac{\partial S}{\partial y} \approx \frac{S(x, y+1) - S(x, y-1)}{2}$



Q (iii).

Generate scale-space representation  $S(x, y, \sigma_i)$  sampled logarithmically  $s$  image, per each octave.

$$S(x, y, \sigma_i) = I(x, y) * g_{\sigma_i}(x, y)$$

$$\sigma_i = 2^{\frac{i}{s}} \sigma_0 \quad \text{and} \quad \sigma_{i+1} = \sigma_i 2^{\frac{1}{s}}$$

— implement all 2D gaussian blurs as  $2 \times 1D$  gaussian blurs for efficiency

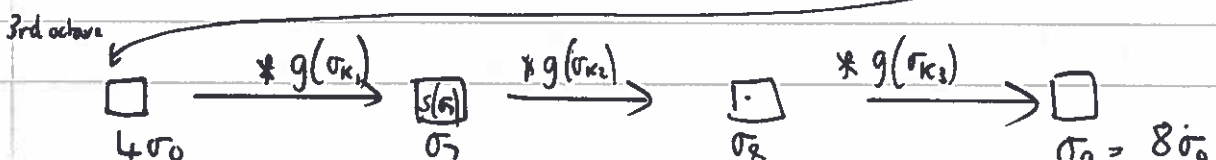
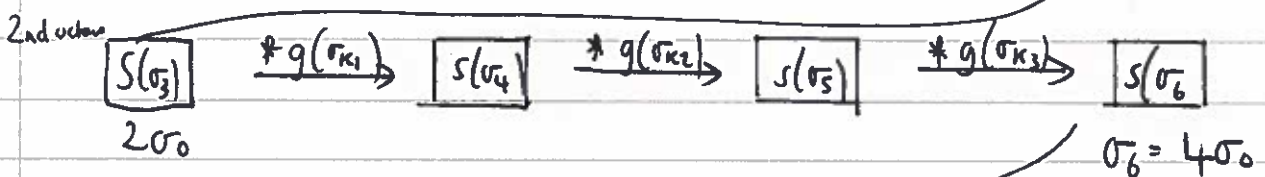
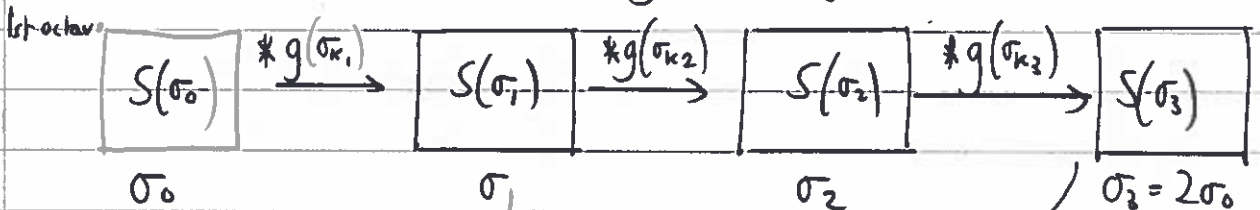
— apply incremental blurs  $g(\sigma_k)$  with small kernels to reduce computation

$$g(\sigma_{i+1}) = g(\sigma_i) * g(\sigma_k)$$

$$\sigma_k = \sigma_i \sqrt{2^{\frac{2}{s}} - 1}$$

— After scale is doubled, after  $s$  blurs,  $\sigma_i = 2\sigma_0$ , subsample image to  $\frac{1}{2}$  size to generate a new octave.

— Apply same incremental blurs  $g(\sigma_k)$  to generate images in next octave.



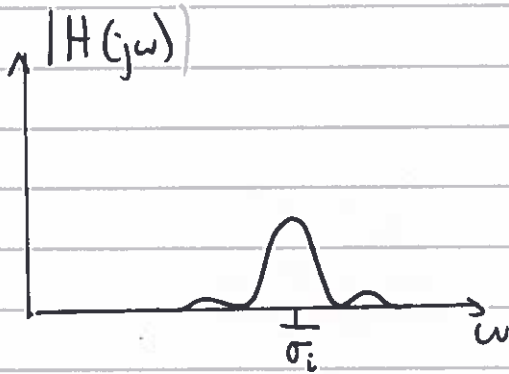


16(b)

(i) Band-pass filtering with the Laplacian of a Gaussian

$$\nabla^2 g(x, y, \sigma_i) * I(x, y) = \nabla^2 g * I = \nabla^2 S(\sigma_i) \approx \underline{S(\sigma_{i+1}) - S(\sigma_i)}$$

This is a band-pass filter since difference of gaussian in space + frequency



Look for max/min of  $\nabla^2 S(\sigma_i)$

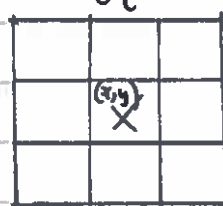
(ii) Efficiently compute by  $S(\sigma_{i+1}) - S(\sigma_i)$  ie. subtract neighbouring images in same octave

Generate LOG Pyramid and search for max/min

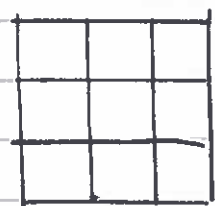
Check 26 neighbours and location  $(x, y)$  and scale,  $\sigma_i$  are given by  $\frac{\text{local min}}{\text{max}}$



$\nabla^2 S(\sigma_{i-1})$



$\nabla^2 S(\sigma_i) = S(\sigma_{i+1}) - S(\sigma_i)$



$\nabla^2 S(\sigma_{i+1})$

(b)(iii)

Dominant orientation:

- Sample  $l \times l$  pixels from  $S(x, y, \sigma_i)$  at intercept coords  $(x_i, y_i)$
  - Compute gradients  $\nabla S(x, y, \sigma_i)$ : magnitude and orientation
  - produce a histogram of gradient magnitudes (binned) against orientation at  $10^\circ$  resolution for bins.
  - Smooth histogram to find dominant/max peak
-

2016 section F (Paper 8)

Q17

(a) SIFT (scale invariant feature transform)

(i)

- sample 16x16 pixels centred at  $(x_i, y_i)$  found orientated at  $\theta_i$  (dominant orientation) from  $S(x, y, \sigma_i)$

(ie. 16x16 pixels at appropriate scale, position and orientation)

- Compute gradients  $\nabla S$

- Smooth/Weight gradients by  $g(\sigma_i \times 1.5)$  to emphasise gradients close to blob-centre. (alternatively  $\frac{1}{2}$  window size)

- Produce 4x4 (=16) cells

- In each cell, compute a HOG at 45° orientations (ie. 4 bins) Add / sum gradient magnitudes to each bin (by interpolation)

- concatenate HOGs to produce 16x8 = 128D vector

- normalize to unit length

- truncate any element  $t \leq 0.2$

- renormalize 128D vector to unit norm.

(a)(ii). Invariant to lighting by using gradients / normalization. Encodes 2D shape / edge contours with a little invariance to exact alignment.

Q17(a) (iii)

Each interest point produces a SIFT descriptor  $\underline{d}_q$   
(128D vector, unit length)

Find correspondences by searching database for a match that minimised euclidean distance between descriptors.  
ie. Find nearest neighbour in database.

Accept as a match if nearest neighbour  $\underline{d}_1$  and 2nd neighbour  $\underline{d}_2$  satisfy:

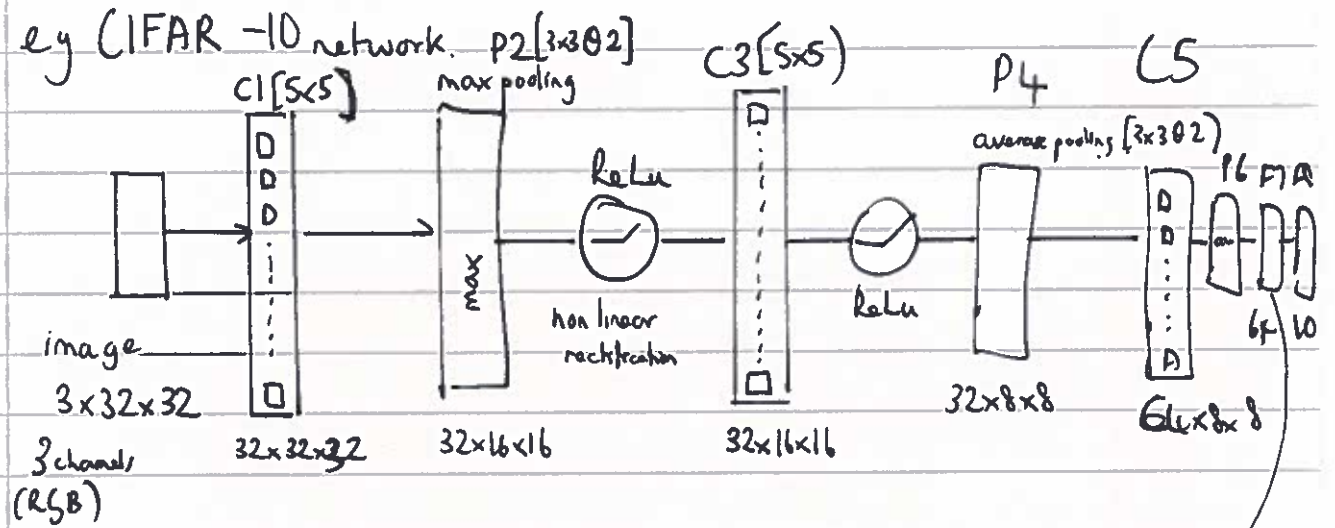
$$\frac{\cos^{-1}(\underline{d}_1 \cdot \underline{d}_q)}{\cos^{-1}(\underline{d}_2 \cdot \underline{d}_q)} < \text{threshold} = 0.6 - 0.7$$

To find nearest neighbours we structure database of descriptors using K-d tree structure (ie. efficient binary search)

(a)(iv) Each match points to a target image.  
Accept the target which has most votes  
(need at least 4 for verification)

# Q17(b)

## (i) CNN architecture



5x5 convolution kernels  
32 channel, 4 5x5 kernels  
2400 parameters

25,600 params.

Fully-connected layers

C1: 32 convolutions  
C3: 32 convolutions  
C5: 64 convolutions

F7 produces = 1024 vector.

- Convolutional layers, eg. 5x5 kernels
- Pooling — average, P4, P6 } sub-sample image to 1/4 size.  
— max, P2
- Rectified Linear Units — non-linearity

• CIFAR-10 network has about 140,000 parameters (filter coefficients)

Anatomy: C1 finds low-level features; P2 gives some flexibility to exact location; C3 looks for parts; P4 smooths response before sub-sampling, to reduce size; C5 finds structures (DPM); P6 sub-samples to a vector of 1024; F7 — sub-category classifier; F8 — find classifier to 1-10.

Q17b)(ii)

Supervised learning — show images with known labels (training set)  
 — compute LOSS f — error between network prediction and actual label  
 (eg. log-loss cross entropy)

Minimize LOSS function by optimizing network using:

Backprop algorithm (~~gradient descent~~) and stochastic gradient descent (SGD) or ADAM.

Learning rate and momentum parameter  $10^{-7}$  and 0.9

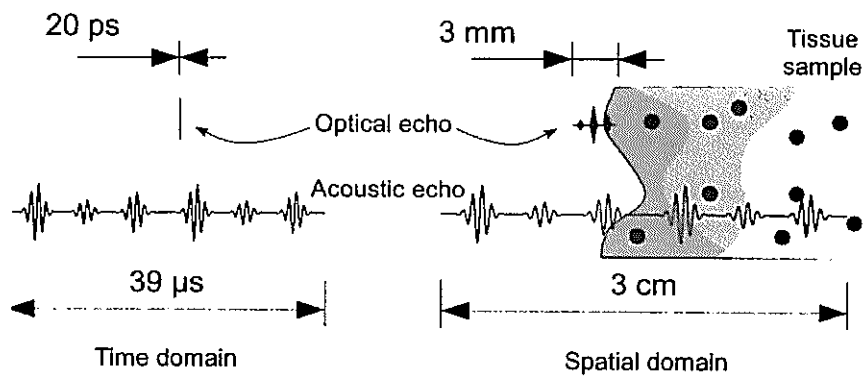
1000s of labelled images needed.

(iii) The output of layer 7 is a 1024D vector (ie. fixed length) that can be used for classification, clustering etc.

(iv) The opp of layer 8 is a soft-max classifier — takes a real-valued vector and gives a number (probability between 0 to 1). The max is the classification.

## SECTION G: Bioengineering

18 (a) A pulse-echo technique is one which sends a pulse of sound (US) or light (optical) into the tissue, which is then reflected and the echo is used to image the tissue. The pulse is broadband with low-coherence, i.e. it has a fairly high bandwidth relative to the centre frequency, which means there are only a fairly small number of oscillations in each pulse. The figure below illustrates this and also shows how the speed differs between the two techniques.



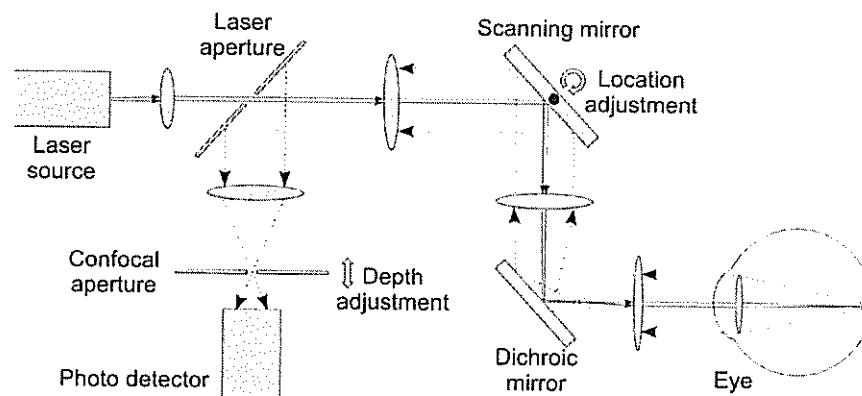
The main differences between US imaging and optical imaging (apart from the speed mentioned above) are:

- Light is strongly scattered by small objects in tissue, whereas US is weakly scattered by such objects.
- Light is reflected by changes in the refractive index of tissue, whereas ultrasound is reflected by changes in the acoustic impedance of tissue (though both signals have some dependency on material density).
- Generally, there is better optical than acoustical contrast between different tissue types, hence different properties are clearer in an optical imaging system.
- The scattering coefficient is nearly 1000 times less for acoustic than optical signals. This makes the acoustic echo much weaker, but it also minimises problems caused by multiple scattering (when an echo itself scatters off other tissues).

(b) Since ultrasound scattering is so weak, attenuation is mostly due to absorption, and is much less than for optical imaging. Typical ophthalmic ultrasound systems at 10MHz or higher can image 3 cm into tissue, which equates to at least 1 cm into the retina. Imaging depth is increased if the ultrasound frequency is reduced. Ultrasound is attenuated very little by water, but dramatically by bone: hence it can 'see' through water but not past bone.

[3]

(c) (i) *Scanning laser ophthalmoscopy (SLO)* is a tomographic technique which can image with more precision than a Fundus Camera. It doesn't take one picture immediately, but scans a laser ('raster scan') over the retina to produce an image. Confocal optics are used to reduce the depth range for a given focus – this allows tomographic imaging, i.e. sectioning, which means we can produce 3D image data. As the SL Ophthalmoscope requires scanning mirrors to gradually sweep the laser spot over the eye to form the whole image, it is slower than the Fundus Camera which uses conventional widefield imaging. A sketch of the SL Ophthalmoscope is given below.



The SL Ophthalmoscope therefore overcomes some of the restrictions of the Fundus Camera (FC) in the following ways:

- As it uses a very small spot of focused laser light to illuminate a particular bit of the fundus, this improves the radial (x) resolution slightly, but greatly improves the illumination efficiency. An FC only sees about one sixteenth of the illuminating light, which limits image quality.
- An FC does not produce 3D image data (ie it cannot be used to focus on different depths).
- The SL Ophthalmoscope can work with lower light power levels than the FC.

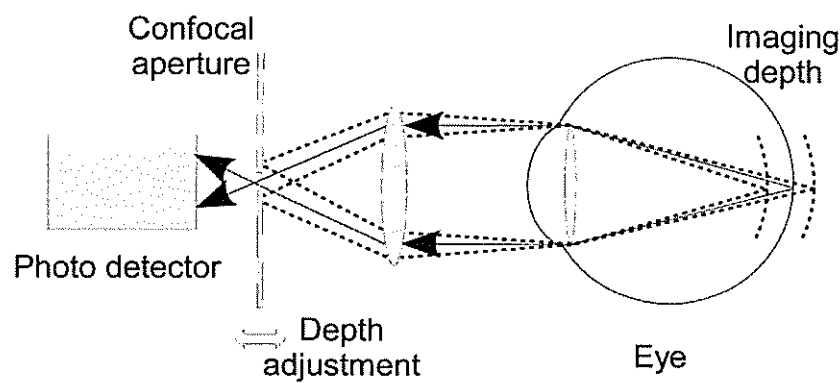
[6]



(ii) The lateral (x) resolution is determined by the spot size of the laser as it hits the back of the eye, which itself is determined by the optics of the instrument and the lens of the eye being imaged. Typically achievable values are  $15\mu m$  for a normal system,  $2\mu m$  if adaptive optics are used. The depth (y) resolution is determined by the confocal system, the most important aspect being the width of the confocal aperture, though the numerical aperture of the imaging lenses will have some impact on this too. Typical values are in the  $100\mu m$  to  $400\mu m$  range. [3]

(iii) See diagram below. Confocal scanning can be described as follows:

- An aperture (plate with a small hole) is placed at or near the focus just before the photo detector, so that only light passing very close to the focus passes through.
- Most of the light reflected from greater or lesser depths into the tissue hits the plate and does not pass through to the photo detector: this reduces the depth of tissue over which the instrument is sensitive.
- A smaller aperture increases the depth resolution (i.e. a narrower tissue section) but it also decreases the amount of light reaching the detector. Conversely larger apertures increase the amount of light but reduce the depth resolution.
- The scanning depth can be changed by moving the confocal aperture axially (along the optical axis). Ideally we want the lens before the aperture to have low numerical aperture (NA) so there is a reasonable range of depths with good optical efficiency.



[6]

19

Answer: Collagen is a protein, a polymer made up of repeating amino acid subunits in the typical pattern Gly-X-Y where Gly is glycine and the other amino acids are commonly proline and hydroxyproline. (Hydroxyproline is unique to collagen and is thus used in collagen quantification assays in vitro.) Collagen is a triple helix, where three individual protein chains coil together to form a repeating subunit 1.5 nm in diameter and 300 nm in length. The three chains can be identical (type II collagen) or differ (type I collagen, with two identical chains and a third different chain). These triple helices then self assemble to form fibrils with a quarter-staggered array of helices giving rise to a measurable gap of 67 nm between consecutive helices, which results in the typical striped appearance of collagen in transmission electron microscopy and atomic force microscopy. There is cross linking both within the triple helix and between adjacent triple helices to stabilize the protein assembly into a mechanically functional rope-like structure with excellent mechanical properties.

Collagen in the cornea is crystalline: it is organized into very regular perpendicular lamellae with uniform spacing between individual collagen fibrils. The lamellar structure is important both mechanically and optically. Mechanically, the lamellar organization provides resistance to intraocular pressure and allows the cornea to serve its critical function of providing two thirds of the optical power of the eye overall. Optically, the regular crystalline structure allows for corneal transparency, again required for unconstructed vision. The collagen in the sclera is much less well-organized and this gives rise to the "white" appearance of the "whites" of the eyes as light is scattered rather than transmitted as in the regular structure of the cornea. In disorders and diseases of the cornea, the tissue can become cloudy due to changes in the collagen organization resulting in a loss of transmissibility of light.

Describe two different ways of mechanically testing the cornea and explain the advantages and disadvantages of each. [5]

Answer: Uniaxial mechanical testing can be done on strips excised from the cornea, or biaxial pressure inflation experiments can be conducted on the entire cornea structure.

Uniaxial tests are simple and fast to execute and analyse. By using strips in different orientation, one can obtain some anisotropy information. However, uniaxial deformation does not reflect cornea biaxial loading in vivo and there can be difficulties due to the nonuniform thickness of the specimens and their intrinsic curvature. The results from strip tests overestimate the tissue stiffness when compared with biaxial inflation tests.

Biaxial inflation tests can give more physiological information as well as information about the overall 3D deformation of the cornea under normal and abnormal pressures. However, the tests are extremely difficult to execute and they require the use of models to interpret the obtained data, which is normally both mechanical measurements and still or video images of the experiments.

(c) Describe the structure of the crystalline lens. Explain the process of lens accommodation and how it changes with ageing. [6]

Answer: The crystalline lens sits behind the iris and contributes 1/3 of the total focussing power of the eye. The lens is about a cm across and half a cm thick. The transparent, biconvex lens structure changes shape

to change focus. There is an exterior capsule that contains the lens, which is in two parts, the nucleus and the cortex. The nucleus is older lens fibers and the cortex is the newer lens fibers; the capsule is the source of new lens fiber cells. The "lens fibers" are specialised elongated epithelial cells surrounded by unusual proteins called crystallines (30% by mass). The overall structure of the lens is complex and "onion-like" in terms of being in layers. There are no blood vessels in the lens.

Lens accommodation is the process of lens shape change that allows the eye to adjust for focus on objects nearer or further away. Lens curvature is controlled by ciliary muscles, and by changing curvature, one can focus the eye on objects at different distances. "Amplitude of accommodation" is the max amount that the lens can accommodate in diopters (D), equal to the reciprocal of the focal length measured in metres.

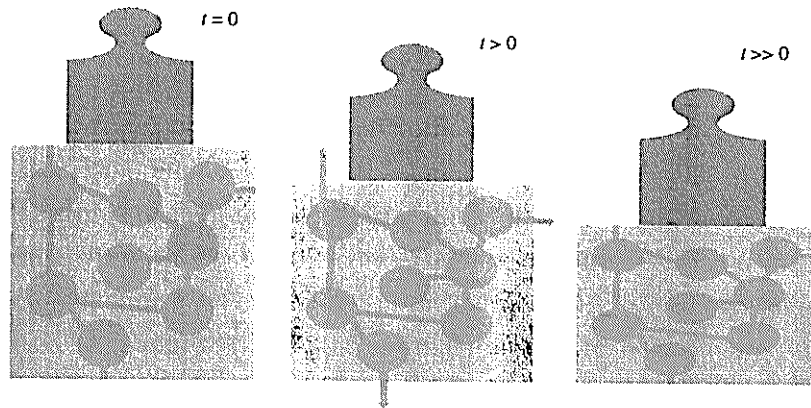
The lens continually grows throughout life, laying new cells over the old cells, which results in stiffening of the lens as well as growth of the lens size. It then becomes more difficult for the lens to change shape under the action of the ciliary muscles, and thus the lens gradually loses accommodation ability with age. This is called Presbyopia and it is part of the natural ageing process and happens to nearly everyone. The nucleus stiffens more than the cortex with ageing. The near point is the closest object that can be brought into focus naturally. This ranges from a few cm in children to an arm's length in advancing middle age to beyond an arm's length in old age.

(d) Describe the transport mechanism in poroelasticity. If a cornea has a thickness  $h$  of  $400 \mu\text{m}$ , an elastic modulus  $E$  of  $500 \text{ kPa}$ , and an intrinsic permeability  $k$  of  $2 \times 10^{-16} \text{ m}^2$ , what is the time constant for poroelastic transport through the cornea, assuming the viscosity of water  $\eta$  is  $1 \text{ mPa s}$ ? If the cornea thickness decreases to  $200 \mu\text{m}$ , what effect does this have on the transport behaviour? If the permeability stayed constant, how would the modulus have to change to restore the original transport response in this thinner tissue?

[8]

Answer:

Poroelectricity is the pressure-induced flow of fluid through a porous elastic (or viscoelastic) network, like a mechanical analog of chemical diffusion where the driving force is physical instead of chemical. When a load is applied to a poroelastic material there is an elastic deformation in the solid skeleton that results in a build up of pore pressure. The fluid then flows to decrease the pressure and a new equilibrium state is eventually reached with a decreased fluid volume.



The time constant for transport is:  $\tau = h^2 / (E\kappa) = h^2 / (Ek/\eta)$  (This expression was shown in class but can also be nearly worked out from scaling arguments given the information provided).

Plugging in the values gives:

$$\tau = 1.6 \text{ s}$$

A decreased thickness would decrease the time constant since  $h$  is on the top of the equation and squared, such that the time constant becomes

$$\tau = 0.9 \text{ s}$$

In order to compensate and keep  $\tau$  fixed, the modulus would have to decrease as  $E_2 = E_1 (h_2^2 / h_1^2)$  to a value of 280 kPa to restore the transport time constant to 1.6 s.

- 20 (a) Write short notes on
- (i) rods and cones;
  - (ii) the retinotopic arrangement of neurons in the primary visual cortex;
  - (iii) orientation maps in the primary visual cortex.

[6]

(i) Rods and cones are two types of photoreceptors: they have photopigments that absorb photons, resulting in light-dependent changes in membrane potential. In total, there are ~ 126 million photoreceptors in the human retina (cones: 6m; rods: 120m). Cones are used for high-resolution colour vision during the day, because i) they are densely packed in the fovea, ii) selective to a specific wavelength (blue, green, or red), and iii) require tens/hundreds of photons (daylight condition) to produce a response. In contrast, rods are much more photosensitive and concentrate at the outer edges of the retina: they are useful for night vision in the periphery.

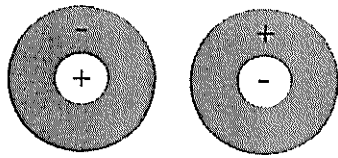
(ii) Retinotopy refers to the fact that two neighbouring neurons of the primary visual cortex (that is, two neurons that are physically close to each other) have spatial receptive fields that strongly overlap. In other words, they respond to patterns of light in roughly the same small area of the visual field.

(iii) Within its retinal (spatial) receptive field, a neuron of the primary visual cortex (V1) responds best to small oriented edges with a certain "preferred orientation". Moreover, neighbouring neurons tend to have the same preferred orientation, thus forming a smooth map of orientation selectivity.

- (b) Sketch and explain the receptive field properties of a typical retinal ganglion cell. Label each relevant component of your sketch clearly.

[3]

Answer: Retinal ganglion cells (RGCs) respond only to light in a particular location of the visual field (i.e. they respond only to the activation of a small cluster of neighbouring photoreceptors). Within this small region (few minutes of arc in the fovea, few degrees in the periphery), receptive fields have a roughly circular, center/surround structure (true for the vast majority of RGCs). In some RGC types, activity is boosted (relative to spontaneous action potential emission) by light in the center, but suppressed by light in the surround (left drawing below). In some other types of RGCs, this structure is reversed (right drawing).



+: light in this area enhances the cell's response  
-: light in this area suppresses the cell's response

(c) In what form does the brain receive visual information from a retinal ganglion cell? Explain whether this information is mostly about mean luminance or luminance contrast, and why this information is computationally useful?

[3]

Answer: Retinal ganglion cells convey information to the brain in the form of trains of action potentials. This information is mostly about luminance contrast, because of the center-surround structure of the receptive fields described in (b). This is computationally advantageous, because spatial patterns of contrast are more informative about the identity of objects in the visual scene, compared to mean luminance which undergoes changes due to variations in ambient illumination that affect all objects.

(d) In the context of vision in vertebrates, explain what the “physiological blind spot” is, and the mechanism underlying it. Describe an experiment with which you could demonstrate the existence of the blind spot in your left eye. [3]

Answer: The physiological blind spot of, say, the left eye, is a region of visual field that this eye cannot sense. It emerges from the relative spatial arrangement of retinal nerve fibers and photoreceptors in vertebrates. The axons of the retinal ganglion cells, which bundle up together to form the optic nerve, must leave the eye one way or another. In vertebrates, retinal ganglion cells are found closer to the light source (closer to the surface of the eye) and photoreceptors are at the back. Thus, there must be a hole in the sheet of photoreceptors to make space for the optic nerve to leave the eye through the back and reach the brain. This hole underlies the physiological blind spot. To demonstrate its existence, close your right eye, and stare at the right dot below. Hold the paper at arm’s length, and progressively move it closer to your eyes. At some point, the dot on the left will “disappear” - it has entered your left eye’s blind spot.



(e) Write short notes on

- (i) everted versus inverted retinas;
- (ii) spherical aberration.

[4]

Answer:

(i) In an everted retina, the layer of receptors is closest to where light enters the eye, and the layer of neurons whose axons form the visual nerve is behind it, so the visual nerve can leave the eye without crossing other layers of the retina. In contrast, in an inverted retina, the layer of receptors is furthest from where light enters the eye, and the layer of neurons whose axons form the visual nerve is in front of it, so the visual nerve can only leave the eye by going through other layers of the retina (thus forming a blind spot).

(ii) For a (homogeneous) lens to collect all beams parallel to its axis into a single focal point, it would need to be aspherical. However, in practice, most lenses (both natural and man-made) are spherical, which causes parallel beams to scatter around the focus. This can be compensated for by making the material of the lens inhomogeneous (such as in the lens of the rat eye) or by using additional lenses (such as in *Pontella*, or the scallop).

(f) What is the distribution,  $P(n)$ , of (integer) spike counts,  $n \geq 0$ , in a receptor that maximises its response entropy with the constraint that its average spike count is  $\bar{n}$ ? [6]

Answer: For a continuous non-negative variable, the maximum entropy distribution with a constraint on its mean is the exponential distribution. By analogy, the maximum entropy distribution for a discrete variable with a constraint on its mean is the geometric distribution:  $P(n) = (1 - p) p^n$ , with parameter  $p$ . The mean of a geometric distribution is  $\frac{p}{1-p}$ . By setting this to  $\bar{n}$ , we obtain  $p = \frac{\bar{n}}{1+\bar{n}} = 1 - \frac{1}{1+\bar{n}}$ , and so the answer is

$$P(n) = \frac{1}{1 + \bar{n}} \left( \frac{\bar{n}}{1 + \bar{n}} \right)^n$$

## 2015-2016 IB Paper 8, Section H

### Crib

- 21 (a) What is the maximum length of time for which a U.K. patent can be granted?

[1]

*20 years*

- (b) Describe the three tests that an invention must satisfy in order for it to be considered patentable.

[4]

1. *it must be novel;*
2. *involve an **inventive step**: i.e. not be obvious to someone in the light of what has been done before (the 'prior art');*
3. *have a **practical application**: be capable of being made or used in some kind of industry; and*  
*(there are also issues of **exclusion** that need to be considered (e.g. scientific theory or mathematical method, method of doing business, perpetual motion machine))*

- (c) A team of students is planning to commercialise a new tracking technology that helps managers locate key items of equipment in hospitals, factories and airports. The technology combines small, low-cost wireless tags with software based upon a novel algorithm. The technology is going to be marketed using the name 'AssetTracker'.

(i) Discuss the Intellectual Property (IP) issues that the team will need to consider when they commercialise this technology.

(ii) Discuss the possible business models that could be used to commercialise this technology.

[20]

(i) *There are numerous issues that would need to be considered, including:*

**Who owns the IP?** *The team will need to decide who actually owns the technology – will all members of the team own the IP equally? What happens if they form a business to commercialise this and one member of the team leaves?*

**What form of IP applies here?** *Answers should consider: the software, the hardware and the brand. **Software**: would be protected under copyright and could, in the code is embedded within a system, be considered patentable in some circumstances (more likely if a US Patent is applied for.) **Hardware**: if the functionality of the tag passes the four patentability tests, then patenting should be considered. If so, they would need to consider the trade offs between cost and benefit of worldwide coverage. The*



design of the tag might be protectable via Design Rights. **Brand:** The name of the technology is quite descriptive, so the team would need to take advice on whether the name could be trademarked.

**What will influence the choice of IP strategy?** There are strong links between the resources available to the firm (e.g. investment, partnerships, etc), the strategy of the firm, the business model(s) they could consider using, and the choice of IP strategy.

**Post exam comment:** Weaker answers (a) just talked about patents and the patenting process and /or (b) didn't discuss the factors that are likely to affect the choice of IP strategies.

(ii)

The basic options available to the 'AssetTracker' team – and the relative merits/costs for each option – are as follows:

**Sell the idea to someone else**

Pros = A quick route to cash, no responsibility for building the business / achieving success, no further funding needed/capital investment.

Cons = Only get a small % of potential value, need to find someone to buy the idea.

**Licence/leasing the idea to someone else**

Pros = Get the cash quite fast, little responsibility for building the business/achieving success, little further funding /capital investment needed.

Cons = Get relatively small % of value, need to find and manage licensees.

**Form partnership with someone else**

Pros = Use experience and resources of another business, increased speed to market, shared responsibility for building the business/achieving success, shared risk, reduced level of investment/capital.

Cons = Share the value with other people / organisations, need to work with (and 'get-on with') other people / organisations.

**Do it all yourself**

Pros = You get 'all' the value generated, and are (in theory) in full control of what happens.

Cons = You need to raise all the investment/capital, entrepreneur takes responsibility for all that goes wrong, you have to work within the limits of the resources you can access.

**Sell a product:** Pros = Increase revenue with higher volumes; Con = Need to develop, make, distribute, support, etc.

**Sell a service:** Pro = No manufacturing costs; Con = Can be hard to scale up the business (in many cases can only grow through recruiting and training lots of people).

**Sell a product plus services:** Pro = Long-term revenues on the back of the sale of each product, Con = Need infrastructure to provide services.

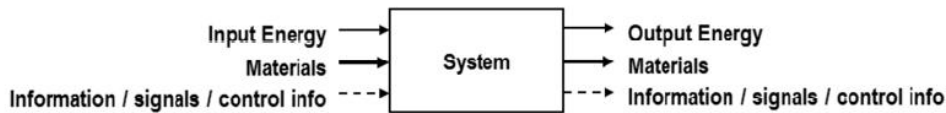
For this particular project, the example application areas given in the question (hospitals, factories, airports) provide some hints for structuring the discussion of the options available. For example, you could discuss the relative merits of delivering this

technology in an airport through a product + service business model, and could then discuss some of the challenges of deploying this when the team are so inexperienced.

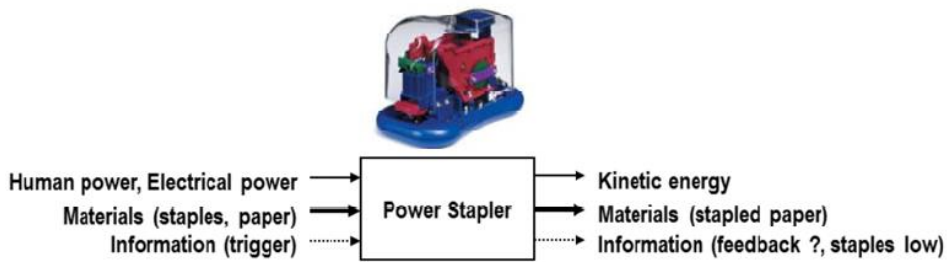
***Post exam comment: Weaker answers just listed the options without commenting on their relative strengths and weaknesses in this context (i.e. inexperienced team, technology deployment would be complicated with multiple stakeholders, etc). Stronger answers drew on the hints in the question and used these to structure a balanced discussion of different options.***

- 22 (a) Explain what is meant by:  
 (i) functional decomposition;

*Consider the transformations that are needed to achieve the desired outputs; consider the functions within the system (or 'Black Box') that are needed to achieve these functions.*



### Basic functions for each sub-system



- (ii) product specification.

[6]

*A statement describing in detail what the product has to do. A good specification should be qualified and quantified, deal with the whole design mix, use visual (system pictures, module pictures, etc) and written information, avoid trivia, put the designer in the position of the user, etc.*

## The detailed specification ...

Machine tool company		Page 1 of 23						
Project: New Machine tool		Issue: 1.1		Last revised: 1 April 2007				
REQUIREMENTS								
Need No.	Functional requirement		Desirable	Acceptable	Source	D/W	Weight	Date Changed
1.1	The machine	Is lightweight	5Kg	<6Kg	JM	W	H	1-1-07
1.2	The machine	Is compact	10x10	10x15	JM	W	M	1-1-07
1.3	The machine	Is insensitive to temperature changes	-20 to 20 deg C	-15 to 20 deg C	SG	D	-	1-1-07
1.4	The machine	Is safe to operate	CE mark	CE	KWP	D	-	1-4-07
1.5	The machine	Is quick to install	<2 minutes	<5 minutes	JM	W	L	1-1-07
1.6	The machine	Can be easily accessed for maintenance	No moving to maintain	Front & side access	JM	D	-	1-1-07
2.1	The user interface	Provides reliable results	Rep >97%	Rep >95%	ANO	D	-	1-1-07
2.2	The user interface	Is simple and intuitive	Use within 10 mins	30 mins training	ANO	D	-	1-1-07

**Post exam comment:** There were some very waffly answers to this part of the question. Stronger answers were those that drew a simple picture / table for each, thus demonstrating a basic understanding of what each means, and the differences between them

(b) Describe the different types of prototyping and the role each type can play in the design process.

[10]

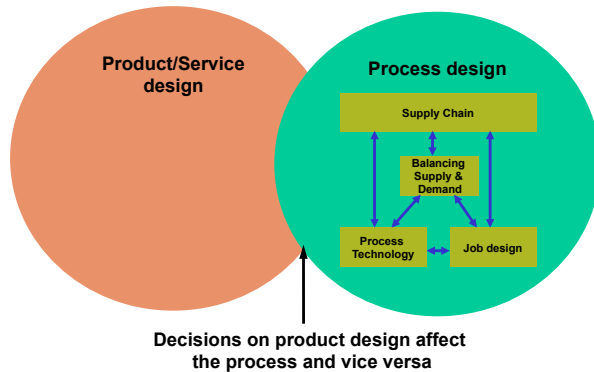
There are a wide range of different roles applicable at different stages of the design process, and each role can be achieved with different types of prototype. This table summarises the key points:

TYPE	USES
Simple Sketch	The simplest, cheapest and quickest way of evaluating lots of ideas for form, technical arrangement and usability. Often highly under-utilised.
Block model	Primarily for early testing of usability, ergonomics and form. Also useful to quickly evaluate a product's physical arrangement. Mainly use easy to work and cheap materials.
Visual (physical) model	Enables evaluation of visual and form aspects. Produced to look as realistic as possible. Good for testing product feel and form.
3D CAD model	Evaluation of overall form, assembly sequence, component fit and production issues. Can also plug into a range of complex analytical models.
Functional (technical) model	To test specific performance aspects. Not necessarily representative of production processes. Good for evaluating reliability, durability, performance, failure etc.
Production prototype	To evaluate all elements of performance, function, form, use and producibility. Made with processes representative of the final production method. Fully functional.
Analytical / virtual models	Mathematical models to support component and assembly optimisation. Often used for safety critical elements. Can be costly and answers are always approximations.

(c) Discuss the links between the design of a product and the design of the manufacturing process for making that product.

[9]

The two are strongly linked (as shown in the figure below).



The output of the product design process should provide clear guidance on the design of the manufacturing operations. However, this is not a one-way process: during the product design process, consideration has to be given to the way in which the product could be made (which also links to consideration of the business model design).

Key issues include:

- How many components were necessary?
- How would the components be joined?
- What shape of component was easy/difficult to manufacture and by what process?
- Are automated or manual processes best for each component, and for assembly;
- What skills are required for each possible manufacturing process? Are these available to the your firm?
- What raw materials / sub-assemblies are required? Are these available to your firm?
- How close is your customer / end user? Will you need to establish a long supply chain?

These issues are all related to the design of the product, and also depend on it. The choice of materials, component shapes, components and the processes required to join them are coupled.

What this question hints at is a demonstration of awareness of the possible manufacturing processes (as shown here) and how the design of the product itself naturally constrains the choice of process (to some degree).



***Post exam comment: Some of the best answers used the examples used in the lectures to show how these two things are linked: Millau Viaduct, Smartphone, and A380.***

- 23 (a) Describe what is meant by:
- (i) technology push; and
  - (ii) market pull.

Give four examples each for (i) and (ii).

[6]

*Many inventions arise from the realisation that ‘we can do it’ = **technology push**. Possible examples could include:*

1. *The Post-It note arose by accident*
2. *The DVD player arose by analogy and the Dyson was a transfer of an existing industrial technology to a new domestic applications*
3. *The domestic breadmaker arose from a structured search for new kitchen appliances*
4. *The ‘inertor’ arose due to a gap in an existing map of possibilities*
5. *New materials allowed the hair dryer to move from an expensive metal body to a cheaper plastic body.*

*The difficulty of inventions of this type is that they may reflect the inventors belief that “this ought to be useful” rather than a group of customers’ statement that “we want that.” So an alternative source of inventions is driven by customers = **market pull**. Possible examples could include:*

1. *The ‘aural’ thermometer for babies arose from the difficult experience of using conventional mercury thermometers measuring babies temperatures with*
2. *The chopper bicycle arose from modifications to existing bikes by enthusiastic users*
3. *Fridges and washing machines are now sold as fashion items as the kitchen has become the main entertaining room*
4. *The ink-jet printing industry around Cambridge has grown due to legislation on sell-by dates for food*
5. *The model-T Ford was successful because Ford found ways that by making cars cheaper he could turn a luxury product into a common one*

(b) Compare, using examples, the relative benefits of using business models based on:

- (i) licensing of IP;
- (ii) manufacturing and selling a product; and

(iii) selling a service.

[9]

(i) *Licensing of IP*

*Pros = Get the cash quite fast, little responsibility for building the business/achieving success, little further funding /capital investment needed. Cons = Get relatively small % of value, need to find and manage licensees.*

*Examples = ARM (IP for semiconductors; Microsoft and Windows; Pilkington and the float glass process).*

(ii) *Manufacturing and selling a product*

*Pros = Increase revenue with higher volumes, focus on single, repeatable activity, costs required can make barriers to prevent others entering the market; Cons = Need to develop, make, distribute, support; high capital costs (unless outsource manufacturing – but then have problems of managing contract manufacturers), rigidity of operations.*

*Examples: Ford, Toyota, Boeing, Airbus, Apple, etc.*

(iii) *selling a service*

*Pros = No manufacturing costs – hence lower capital requirements to get started, easy to launch, flexible – easy to pivot (in some cases); Cons = Can be hard to scale up the business (in many cases can only grow through recruiting and training lots of people), how to cope with peaks and troughs in demand, how to deal with low barriers to entry.*

*Examples: McKinsey, Uber, AirBnB, Expedia.*

***Post exam comment:*** *The section of the question that stated the phrase ‘using examples’ was ignored by many students.*

(c) Compare the challenges of managing innovation in a small start-up company with those of managing innovation in a large, long-established firm.

[10]

*To set the scene, start by characterising the differences between the two types of companies (see table below).*

	<i>• Start-up company</i>	<i>• Established company</i>
<i>• Processes</i>	<i>• Informal; ad hoc; rapid</i>	<i>• Formal processes; slow paced (e.g., design review; document control)</i>
<i>• Systems</i>	<i>• Few</i>	<i>• Many systems, tried and tested (e.g., technical database, financial)</i>

		systems)
<ul style="list-style-type: none"> <li>• <i>Activities</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Heroic individual efforts; chaotic; initiative based</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Cross-functional teams; managed tasks; delegated authority; coherence</i></li> </ul>
<ul style="list-style-type: none"> <li>• <i>People</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Many creator / innovator types; role flexibility</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Managed balance between types; clear job descriptions</i></li> </ul>
<ul style="list-style-type: none"> <li>• <i>Management style</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Hands-on, informal; bold decisions taken on incomplete information</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Delegated, professional style; risk assessment; staff development</i></li> </ul>
<ul style="list-style-type: none"> <li>• <i>Communication and documentation</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>High dependence on verbal communication and memory; 'everyone knows everything'</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Greater use of written communication; controlled dissemination; 'need to know'</i></li> </ul>
<ul style="list-style-type: none"> <li>• <i>Market information</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>From intuition, insights and belief; reliance on feedback from small sample of (potential) customers</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>From experience and market research; statistical sampling of customer needs and price sensitivity</i></li> </ul>
<ul style="list-style-type: none"> <li>• <i>Competitors and IPR</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Limited competitor awareness; limited IPR protection</i></li> </ul>	<ul style="list-style-type: none"> <li>• <i>Very aware of competitors; careful and strategic use of IPR.</i></li> </ul>

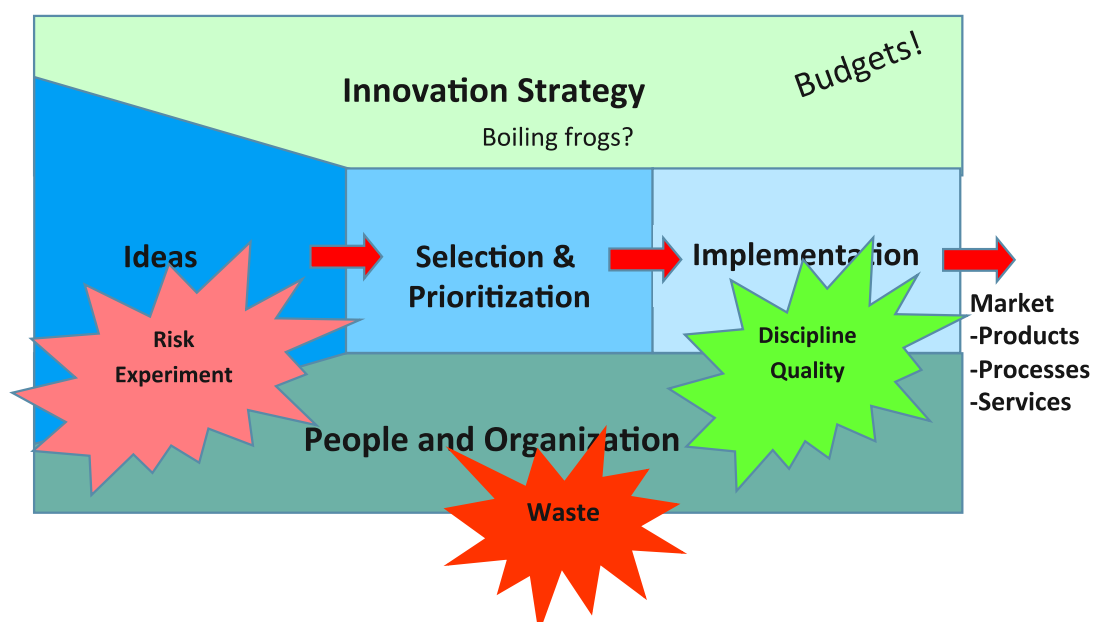
Then, answers should consider the ways in which these two different contexts stimulate or constrain different types of innovation (radical, incremental, disruptive were covered in the lecture course).

*Start-up:* by having few resources and limited 'path dependencies', start-ups can be viewed as being better able to take on risks, and hence are in a better position than large firms to deliver innovations that are radically new and/or disruptive to the market. However, small firms face the 'liability of newness' – why would anyone invest or buy from a firm that they have never heard of, that has no track record? As a result, it can be hard for new firms to access the resources they need to grow the market. They can get stuck in a loop: they need customers to show that the idea is good and worth investing in, but they can't get investment because they haven't got any customers.

*Large firms:* firms with long histories and substantial assets typically find it difficult to change. This is due to inertia – the analogy with a supertanker is very apt: large firms tend to be very good at what they are currently doing – they have optimised all their systems and processes for their current markets and, due to their scale, can do this very efficiently. When the market changes, or new technologies come along, it is very difficult for them to change – in the same way that it is difficult for a supertanker to change direction quickly. As a result, large firms tend to focus on incremental

*innovations – small, low risk changes that do not require major disruptions to current operations. Large firms will also typically have a wide range of stakeholders, all who will have to ‘buy-in’ to any major changes. Such stakeholders include suppliers, investors, current employees, etc. These stakeholders will need to see a clear ‘upside’ to any changes or they will not support them. But change often requires short term discomfort (having to learn new things, having to accept smaller dividends on shares, etc) in order to reap longer term reward. The ‘Pentathlon’ model shown in the lectures provided a great structure around which discussion of large company innovation could take place:*

## Managing Innovation: “Pentathlon Framework”



Goffin, K. and R. F. Mitchell (2010). *Innovation Management: Strategy and Implementation Using the Pentathlon Framework*, Palgrave.

*To overcome the inherent weaknesses in both types of firm, and enable them each to play to their strengths, one option is for start-ups and large firms to collaborate. This strategy of ‘open innovation’ is not without its own challenges but has become increasingly popular. You could then describe some of the challenges of getting start-ups and large firms to work together.*

***Post exam comment:*** *Some students chose to just write about the differences between small and large firms. This was part of the required answer, which also needed a demonstration of how innovation works in these differing contexts.*