Version JSB/3

EGT0
ENGINEERING TRIPOS PART IA

Wednesday 5 June 2019   9 to 12.10

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (**short**) Two liquids with densities ρ_1 and ρ_2 are separated by a gate which pivots about the foot of the gate, O, as shown in Fig. 1. Find the ratio of depths h_1 and h_2 in terms of the two densities for zero moment about O. [10]

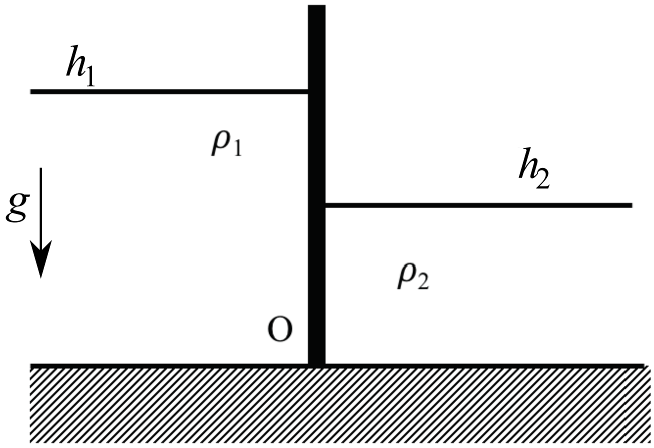

Fig. 1

2 (**long**) Consider the incompressible, steady flow in a pipe with an orifice plate as shown in Fig. 2.

(a) Find the difference in static pressure, $(p_1 - p_2)$, between the entrance and the orifice in terms of the inlet dynamic head and A_1 and A_2. State clearly your assumptions. [10]

(b) Find the difference in static pressure, $(p_2 - p_3)$, between the orifice and the pipe exit in terms of the inlet dynamic head and A_2 and A_3. Again, state clearly your assumptions. [10]

(c) By considering an appropriate control volume, find the force on the orifice plate in terms of the inlet dynamic head and the areas A_1 and A_2. Again, state clearly and justify any assumptions you make. [10]

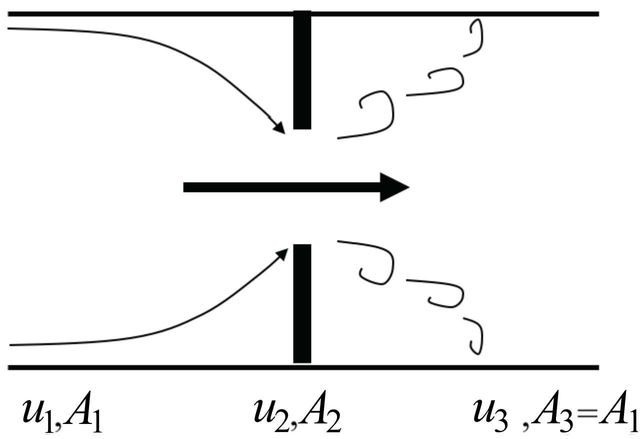

Fig. 2

3 (**short**) A cylindrical container with radius R and height H rotates at angular rate Ω as sketched in Fig. 3. Within the container is an incompressible liquid; gravity is downwards as shown.

(a) Derive an expression for the radial variation of liquid depth, h, in terms of the depth at the centre-line h_o, local radius r, Ω and g. [4]

(b) If the total liquid volume is Q , find the depth at the centre-line h_o in terms of Q , R, g and Ω. [2]

(c) If the liquid depth at radius R is H derive an expression for the volume Q in terms of h_o, H and R. [4]

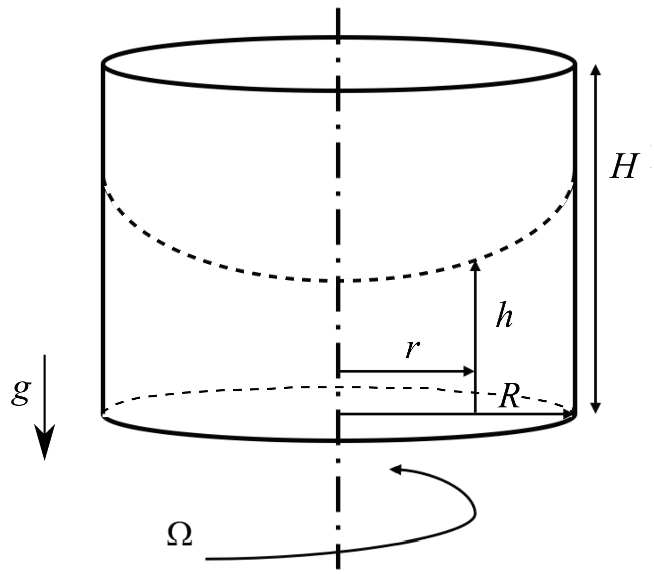

Fig. 3

4 (**short**) A quantity of air enclosed in a cylinder by a piston undergoes a fully resisted expansion from a temperature $T_1 = 280$ K and a pressure $p_1 = 4$ bar to a final pressure $p_2 = 1$ bar. The expansion can be modelled by the polytropic equation $pv^n = \text{constant}$, where $n = 1.2$. Treat air as a perfect gas with $c_\text{v} = 718$ J kg^{-1} K^{-1} and $R = 287$ J kg^{-1} K^{-1}.

(a) Calculate the work done per kg of air. [6]

(b) Determine the final temperature T_2 and the heat transferred per kg of air. [4]

5 (**short**)

(a) Starting from the Clausius Inequality, show that the Coefficient of Performance for a cyclic refrigerator operating between two constant temperature reservoirs can be expressed as

$$COP_R \leq \frac{T_L}{T_H - T_L}$$

where T_L is the temperature of the low temperature reservoir and T_H is the temperature of the high temperature reservoir. [6]

(b) A cyclic refrigerator operates between a cold space at -10°C and the environment at 20°C. Calculate the minimum power input required to extract heat from the cold space at a rate of 0.4 kW. Why, in practice, is significantly more power required? [4]

6 (**long**)

(a) State whether each of the following statements is true or false, giving explanations for each of your answers:

(i) When a mass m of a perfect gas is heated at constant pressure such that its temperature rises by ΔT, the increase in its internal energy is given by $\Delta U = mc_{\mathrm{p}}\Delta T$ where c_{p} is the isobaric specific heat capacity.

(ii) For the low speed flow of an ideal gas through an insulated throttle, the temperatures upstream and downstream of the throttle are approximately equal.

(iii) When a fluid flows steadily through a pipe at high speed, such that frictional effects are significant, the entropy of the fluid necessarily increases in the downstream direction. [9]

(b) Air flows steadily through a horizontal insulated pipe with a uniform cross-sectional area of 0.05 m^2 as illustrated in Fig. 4. At a point A in the pipe the pressure, temperature and velocity of the air are 2.5 bar, 325 K and 400 m s^{-1} respectively. At point B (which may be either upstream or downstream of A) the pressure is 2.0 bar.

(i) Calculate the mass flow in the pipe. [3]

(ii) Calculate the velocity of the air at B. [7]

(iii) Using a thermodynamic approach and explaining your reasoning, determine whether B is upstream or downstream of A. [6]

(iv) Calculate the force exerted by the pipe on the air in the section between A and B and explain how the result may be used to confirm your answer to Part (b)(iii). [5]

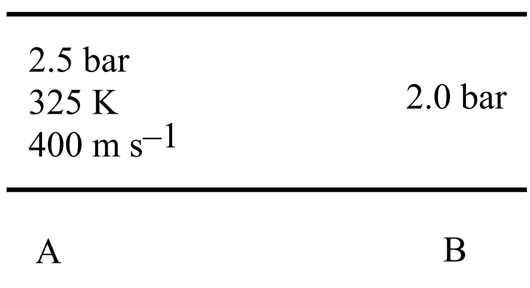

Fig. 4

SECTION B

7 (**short**) A satellite is in an elliptical orbit around the earth. The eccentricity of the orbit is 0.7 and the radius of the earth is R. The altitude of the satellite above the earth's surface at the apogee is $11R$ and the speed of the satellite at the apogee is V.

(a) Find the altitude at the perigee. [5]

(b) Find the speed of the satellite at the perigee. [5]

8 (**short**)

(a) A ball of mass M and initial speed u collides, head-on, with a stationary ball of mass m. Assuming the collision is perfectly elastic, find the velocity, V, of the second ball after the collision. [6]

(b) A third ball of mass m_3 is placed, initially at rest, between the two original balls. The collision now proceeds in two steps: first M strikes m_3, then m_3 strikes m. Again assuming the collisions are head-on and elastic, find the value of m_3 which results in the largest final speed for m. [4]
[Hint: Maximizing a quantity, y, is equivalent to minimizing its reciprocal, $1/y$.]

9 (**short**) A car moving at speed v enters a corner with radius of curvature R centred at O. As shown in Fig. 5, the car's centre of mass is a height h above the ground, and the wheels are a width $2w$ apart. It can be assumed that R is large compared to w and h.

(a) If the car enters the corner too fast it will start to tilt. Will the wheel marked A or the wheel marked B leave the ground? [2]

(b) Show that the wheel will leave the ground if the speed of the car is greater than

$$v_t = \sqrt{\frac{gRw}{h}}.$$

[3]

(c) The car has moment of inertia $I_G = mk^2$ around a horizontal front-back oriented axis through its centre of mass. If the car actually enters the corner at $2v_t$, what is its (instantaneous) angular acceleration as it starts to tilt? [5]

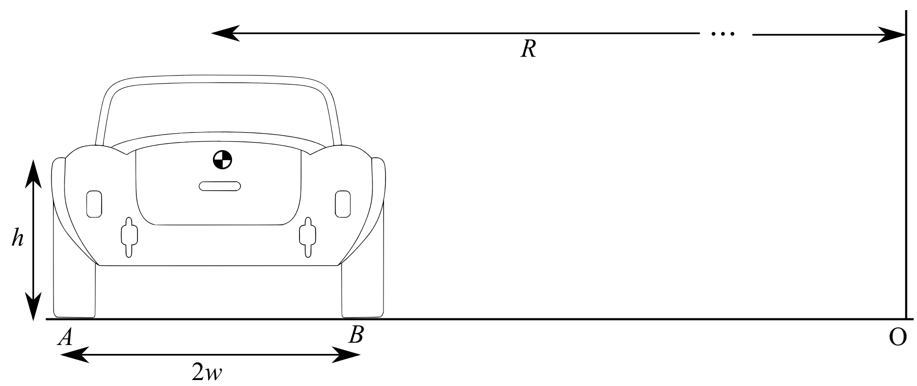

Fig. 5

10 (**long**) In a proposed fairground ride, an unpowered trolley rolls, without friction, along a U-shaped track, as illustrated in Fig. 6. The height of the track has the form $h = ks^4$, where s is the distance along the track measured from O.

The trolley is released at rest from $s = s_0$, and there is a gravitational field g.

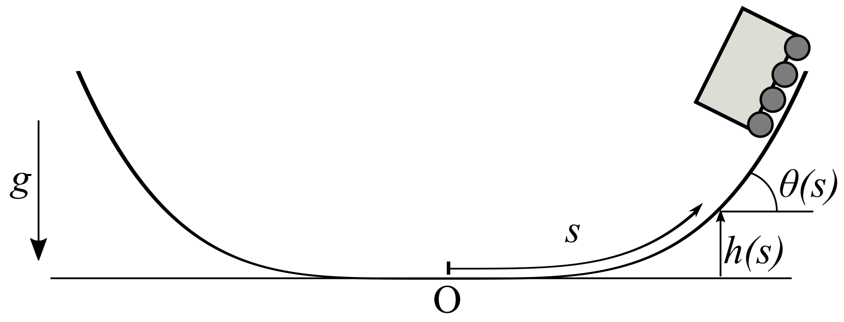

Fig. 6

(a) Show that the track makes an angle with the horizontal $\theta(s) = \sin^{-1}\left(\frac{\mathrm{d}h}{\mathrm{d}s}\right)$. [2]

(b) Find the velocity of the trolley as a function of s. [4]

(c) Find the tangential acceleration of the trolley as a function of s. [3]

(d) Using the approximation $\sin\theta \approx \theta$, find the acceleration of the trolley normal to the track as a function of s. [5]

(e) Find the locations, s, on the track where the trolley's velocity, tangential acceleration and normal acceleration are maximum. For normal acceleration you may still assume $\sin\theta \approx \theta$. [8]

(f) (i) The trolley returns to its original position a time T after it is released. Explain why T can only depend on g and k via their product, gk. [2]

(ii) If the trolley is released at $s_0 = 1\,\mathrm{m}$, it takes 1 second to return to its original position. Use dimensional analysis to calculate the return time if $s_0 = 5\,\mathrm{m}$. [6]

11 (**short**) Two rigid discs with moment of inertia J are mounted on three light elastic shafts with torsional stiffness k as shown in Fig. 7. The outer ends of the system are clamped.

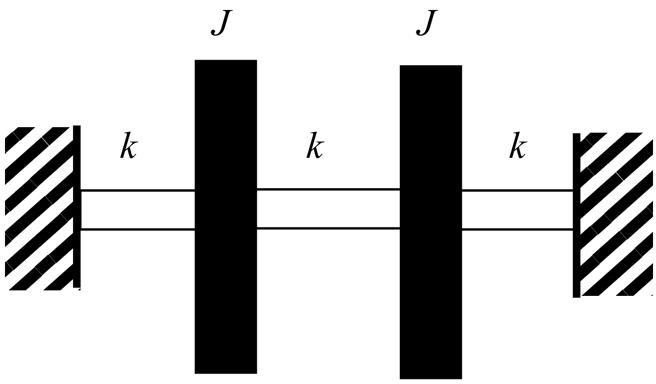

Fig. 7

(a) Determine the natural frequencies and normal modes of this system by inspection. [5]

(b) Describe the behaviour of the system for the initial conditions $\theta_1 = 1$ rad, $\theta_2 = 0$ rad, $\dot{\theta}_1 = 0$ rad s^{-1}, $\dot{\theta}_2 = 0$ rad s^{-1}, where θ_1 and θ_2 are the angular positions of the two disks. [5]

12 (**long**) Figure 8 shows an electrical circuit comprising an ideal current source i_s in combination with a capacitor C in parallel with a series combination of a resistor R and inductor L.

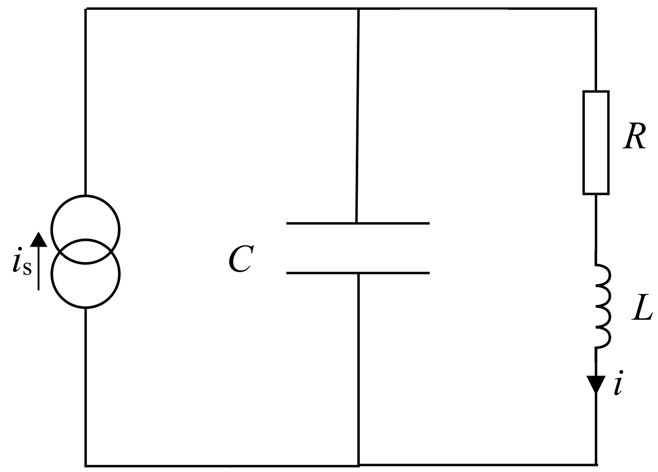

Fig. 8

(a) Show that an equation for the inductor current i can be derived as [5]

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{1}{LC} i_s.$$

(b) Obtain expressions for the natural frequency and damping factor for the system assuming $\frac{1}{LC} = 10^{12}\ \mathrm{s}^{-2}$ and $\frac{R}{L} = 10^6\ \mathrm{s}^{-1}$. [5]

(c) Obtain an expression for the steady-state voltage across the resistor if the current source is given by $i_s = i_0 \sin \omega t$. Estimate a value for the maximum voltage across the resistor for all input frequencies ω if R is 100 Ω and i_0 is 10 mA. [10]

(d) Using the databook or otherwise, estimate the maximum value of the voltage across the resistor if the input source is a unit step function. [5]

(e) What value would R have to be increased to in order to limit the maximum voltage excursion in (d) to 10% of the nominal final value? [5]

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Numerical Answers:

4(a) 82.9 kJ kg^{-1}
(b)T_2=222K, q=41.4 kJ kg^{-1}
5(b)46W
6 (b)(i) 53.6 kg s^{-1}
(ii) 460.3 ms^{-1}
(iii) -19.1 J kg^{-1} K^{-1}
(iv) -0.732 kW
7(a)1.12R
(b)5.67V
10(f)(ii) 0.2s
12) (b)10^6rad s^{-1}, 0.5
c) 1.16V
d) 117V
(e) 120Ω