Version SKH/5

EGT0
ENGINEERING TRIPOS PART IA

Thursday 6 June 2019   9 to 12.10

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (**short**) The pin-jointed truss shown in Fig. 1 carries a vertical load of W at joint B. All members are constructed of a uniform section with cross-sectional area A and Young's modulus E. Calculate the horizontal deflection of joint A. [10]

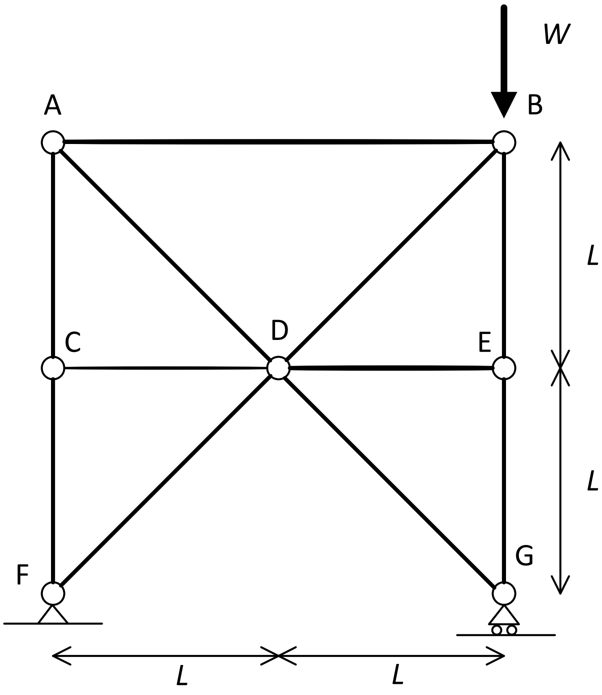


Fig. 1

2 (**short**) A tyre inner-tube of circular cross-section has outer radius r_o and inner radius r_i, as shown in Fig. 2. The tube is manufactured from rubber with a thickness t and is inflated to a pressure of p. Calculate the stresses within the walls of the tube, clearly indicating the directions in which they act. [10]

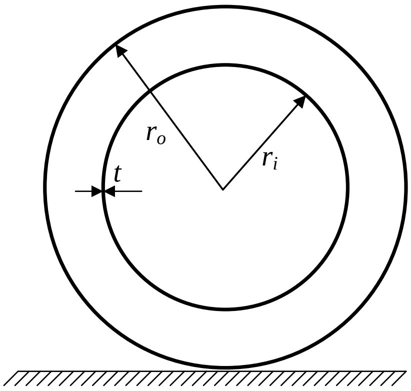


Fig. 2

3 (**short**) An inextensible cable is hung between two fixed supports 100 m apart as shown in Fig. 3. The cable is loaded with a vertical point load of 20 kN at the cable centre and with a distributed load of 1 kN m^{-1} over the right-hand half of the cable.

(a) If the dip at midspan is 1 m, calculate the support reactions. [7]

(b) Calculate the dip at a point 10 m from the right-hand support. [3]

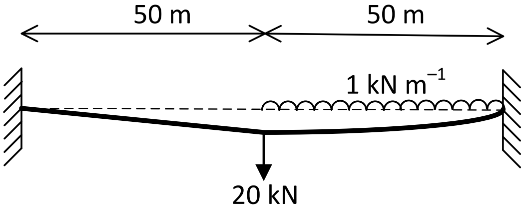


Fig. 3

4 (**short**) The 3-pin arch shown in Fig. 4 has a right-hand side whose shape is given by:

$$y = h\left(\frac{x}{L}\right)^3$$

The left-hand side of the arch is formed of two straight sections, rigidly connected at A. The horizontal section is loaded with a uniform downwards vertical load of *w* per unit width.

(a) Calculate the bending moment at point A. [5]

(b) Calculate the location of the maximum bending moment in the right-hand side of the arch. [5]

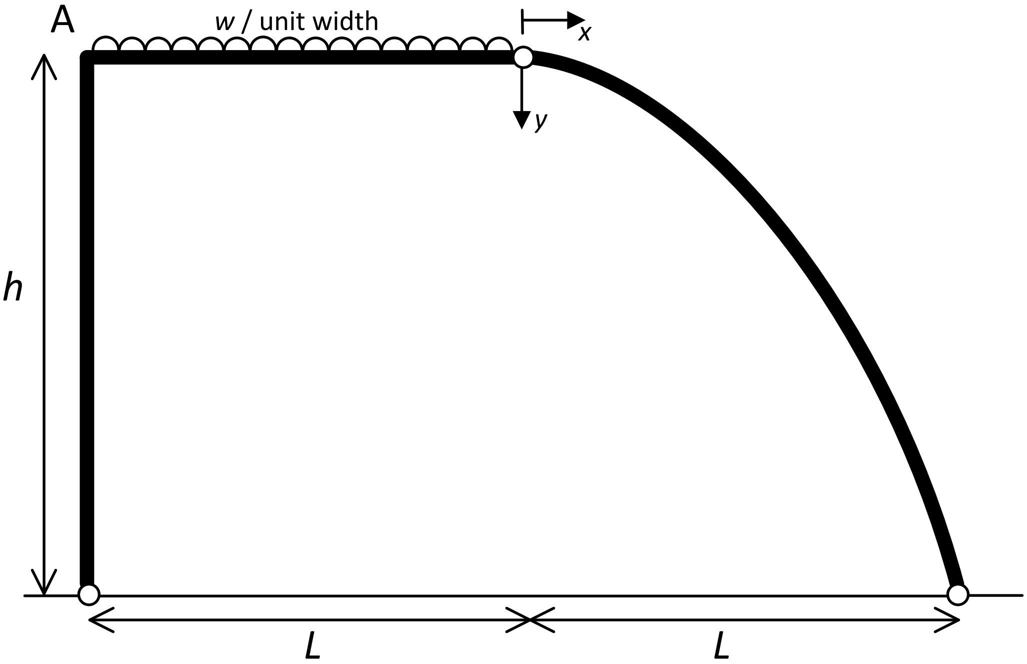


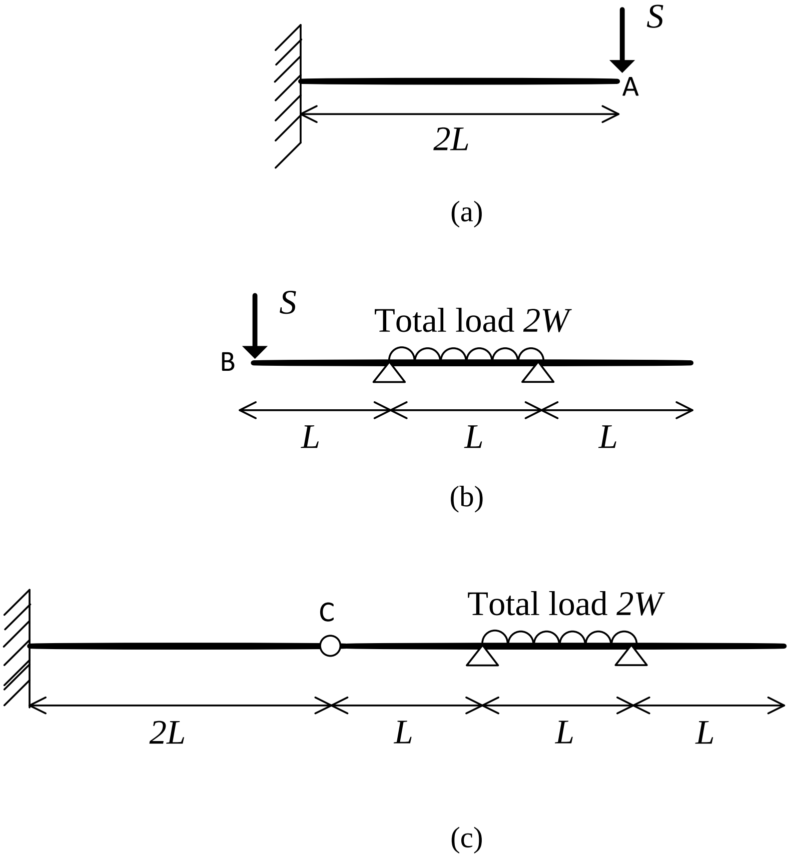
Fig. 4

5 (**long**) Figure 5 shows a set of uniform beams, all with flexural rigidity EI.

(a) The beam shown in Fig. 5a is subjected to a downwards vertical load S at A. Calculate the vertical deflection of point A. [5]

(b) The beam shown in Fig. 5b is subjected to a downwards vertical load S at B and a uniformly distributed load over the central span. Calculate the vertical deflection of point B. [20]

(c) For the beam shown in Fig. 5c, calculate the vertical deflection of point C. [5]



(a)

(b)

(c)

Fig. 5

6 (**long**) The steel column shown in Fig. 6 has a solid square cross section of side length 25 mm. The column, of length $L = 1$ m, is subjected to a compressive axial load P. The column is restrained from horizontal movement at its top, bottom and centre and is constrained to only move in the $x-y$ plane.

(a) Using standard formulae, calculate the buckling load P_E of the column. [5]

(b) If the column has an initial sinusoidal imperfection in the y direction given by $v_0(x) = \delta \sin \frac{2\pi x}{L}$, derive the differential equation for the shape of the column and hence calculate the total deflected shape of the beam. [20]

(c) If the magnitude of the initial imperfection δ is 2 mm, calculate the maximum compressive stress in the column at a load $P = \frac{P_E}{2}$ and comment on its magnitude relative to the average compressive stress. [5]

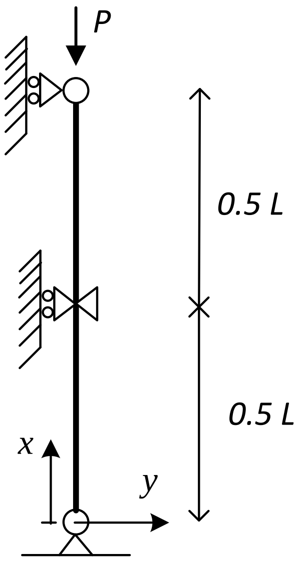


Fig. 6

SECTION B

7 (**short**)

(a) Briefly explain why, for a given applied stress, the survival probability P_s of a brittle material reduces as the volume of material increases. [2]

(b) An axially constrained cylinder of length L has a solid circular cross section of outer radius R. Due to the manufacturing process, it develops a residual tensile stress that acts parallel to the axis of the cylinder, and varies linearly with the radial coordinate r $(0 \leq r \leq R)$ as follows

$$\sigma(r) = \frac{\alpha\, r}{R^4}$$

where α is a constant. The cylinder is made from a brittle material. Using a suitable relationship for the Weibull statistics of fracture from the Materials Databook, derive an expression for the survival probability P_s. [4]

(c) Taking the Weibull modulus $m = 3$ (as defined in the Materials Databook), calculate the value of P_s for a cylinder of radius R = 17 mm, given that P_s = 0.9 for a cylinder of radius R = 15 mm. L is the same in each case. Comment on the result. [4]

8 (**short**) A viscoelastic material can be modelled using the spring-dashpot network shown in Fig. 7 .

(a) For this material, derive a differential equation relating the total applied stress σ, the total strain ε, time t and the material constants. [8]

(b) *Without further calculation*, sketch the equivalent spring-dashpot networks for this material when subjected to:

(i) high frequency deformation, and

(ii) low frequency deformation. [2]

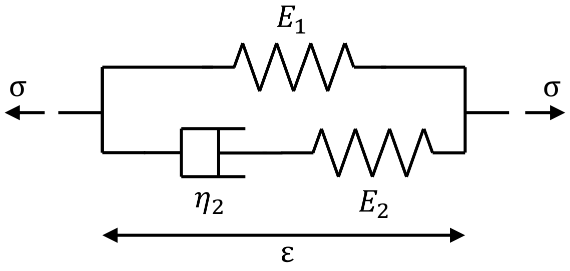


Fig. 7

9 (**short**) The Young's modulus of composite materials may be estimated using lower bound and upper bound formulae, given on p.5 of the Materials Databook.

(a) Explain the key assumptions behind the derivation of these formulae, and indicate on a sketch how they would apply to a unidirectional fibre-reinforced material. [2]

(b) A layer of unidirectional fibre-reinforced material contains 50% of fibres with modulus E_f in a matrix of modulus E_m, with $E_f/E_m = 100$. A composite panel is manufactured from an even number of layers of this undirectional material stacked and stuck together. Fibres in alternate layers are oriented at 90° to one another. Derive an expression for the Young's modulus in either of the two directions parallel to sets of fibres. [5]

(c) The layered composite panel in part (b) is manufactured using drawn polymer fibres ($E_f = 120$ GPa) in a thermoset matrix ($E_m = 1.2$ GPa). Calculate the Young's modulus in either of the two directions parallel to sets of fibres. The density of the composite is $\rho = 950$ kg m^{-3}. Using the Young's modulus - density property chart in the Materials Databook, identify the nearest material that competes with this composite. [3]

10 (**short**)

(a) The work hardening contribution to the yield stress $\Delta\sigma_{wh}$ can be assumed to be proportional to $\sqrt{\rho_d}$, where ρ_d is the dislocation density. A hot rolled sheet of pure copper (Cu) has a yield stress of 90 MPa and $\rho_d = 10^{13}$ m·m^{-3}. A cold rolled sheet of the same material has a yield stress of 360 MPa and $\rho_d = 10^{15}$ m·m^{-3}. Find the yield stress of fully annealed pure Cu. [3]

(b) The solid solution hardening contribution to the yield stress $\Delta\sigma_{ss}$ can be assumed to be proportional to $(C_{ss})^n$, where C_{ss} is the concentration in solid solution. An annealed Cu alloy containing C_{ss} = 10 % Ni has a yield stress of 110 MPa, while an alloy containing C_{ss} = 20 % Ni has a yield stress of 131 MPa. Find the exponent n. [3]

(c) A Cu alloy containing 15 % Ni is cold rolled to a dislocation density $\rho_d = 10^{14}$ m·m^{-3}, and has a yield stress of 216 MPa. Assess whether these values are consistent with the theory that the hardening contributions $\Delta\sigma_{wh}$ and $\Delta\sigma_{ss}$ are additive. [4]

11 (**long**) A number of large display boards are required for a conference on the sustainability of materials. Each board consists of a rectangular panel of material whose length L and width b are fixed, and whose thickness d may vary. The panels are most heavily loaded by their self-weight when being carried horizontally, simply supported along their edges of width b. As the boards are to be used as a design case study, the objective is to minimise their environmental impact.

(a) Show that the central deflection δ and the maximum bending stress σ_{max} in the panel, due to its self-weight, are given respectively by

$$\delta = \frac{5\rho g L^4}{32 E d^2} \quad \text{and} \quad \sigma_{max} = \frac{3\rho g L^2}{4d}$$

where ρ is the density, E is Young's modulus, and g is the acceleration due to gravity. [8]

(b) The length and width of one set of panels are 3 m and 2 m respectively, and the thickness must be between 5 and 20 mm. For each of the materials in Table 1, find the minimum thickness required to meet both a stiffness constraint that the maximum allowable deflection is 20 cm, and a strength constraint that the maximum stress must not exceed half the failure strength, σ_f. Eliminate any materials that fail to meet the target range of thickness. [10]

(c) The organisers have imposed upper limits of £30 per panel and 35 kg per panel. Eliminate any of the materials identified in part (b) that fail to meet either constraint. For the remaining shortlist, evaluate the embodied energies, and identify the material with the lowest value. [7]

(d) All the materials are imported, and travel a distance of 1000 km. Evaluate the transportation energies for truck freight, for which the energy consumption is 0.94 MJ tonne^{-1} km^{-1}. Does the ranking of materials change, based on the total embodied and transportation energy? [5]

Material	Density, ρ (kg m^{-3})	Young's modulus, E (GPa)	Failure strength, σ_f (MPa)	Cost / kg, C_m (£ kg^{-1})	Embodied energy / kg, C_e (MJ kg^{-1})
Al foam	360	0.9	1.3	9	370
Biocomposite	530	3	10	0.8	7
Pine	600	10	40	0.8	12
Rigid polymer foam	320	0.3	6	8	81
Fibreboard	750	4	10	0.3	18

Table 1

12 (**long**)

(a) Sketch a plot of cyclic stress amplitude against fatigue life, indicating the endurance limit, and the regimes of low- and high-cycle fatigue. Explain what is meant by the endurance limit. Briefly describe the main difference between low cycle fatigue and high cycle fatigue. For each mechanism, explain how the number of cycles to failure N_f can be modelled. [8]

(b) Table 2 shows high cycle fatigue data for an aluminium alloy, from a series of tests in which the stress had a peak-to-peak variation $\Delta\sigma$ about a mean stress of zero.

$\Delta\sigma$ (MPa)	200	300	400
N_f	6.15×10^8	5.06×10^6	1.68×10^5

Table 2

Use a suitable plot of the data to confirm that Basquin's law is applicable. Estimate the number of cycles to failure if a specimen of the same alloy is subjected to a fatigue test where the stress varies from -175 MPa to 175 MPa. [6]

(c) Using sketches and equations as appropriate, explain how you would predict the fatigue life in the following situations:

(i) loading cycles at constant stress range with a constant non-zero mean stress;

(ii) loading cycles of varying stress range about a zero mean stress. [6]

(d) Bicycle spokes are manufactured from the aluminium alloy in part (b). The alloy has an ultimate tensile strength of 600 MPa. In a completed wheel, the spokes are pre-tensioned to a stress of 300 MPa. The training regime of a mountain bike rider subjects a typical spoke in the rear wheel to a number of loading regimes, with maximum and minimum stresses σ_{max} and σ_{min}. The proportion of stress cycles spent in each of four loading regimes are shown in Table 3.

Given that the outside diameter of a mountain bike tyre is 640 mm, and the rider trains for 120 km per day, calculate the fatigue life in days of the spokes in the rear wheel. [10]

(cont.

σ_{max} (MPa)	σ_{min} (MPa)	Proportion of stress cycles (%)
300	250	80
300	200	12
300	150	6
300	100	2

Table 3

END OF PAPER

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ENGINEERING TRIPOS PART IA, 2019
Paper 2

Numerical Answers

Section A (Structures)

1. 2WL/AE to right

2. $P(r_o-r_i)/4t$ and $P(r_o-r_i)/2t$

3. a) Vertical: 22.5 kN, 47.5 kN, Horizontal 1125 kN

(b) 0.38m

4. (a) $M = wL^2/4$

(b) $x= L/\surd 3$

5. (a) $8SL^3/3EI$ downwards

b) $(8S-W)L^3/12EI$ downwards

c) $WL^3/15EI$ upwards

6. a) 270 kN

Section B (Materials)

7. (b) $P_S = \exp\left[-\left(\frac{\alpha}{R^4\sigma_o}\right)^m\left(\frac{2\pi L}{V_o}\right)\left(\frac{R^{m+2}}{m+2}\right)\right]$, (c) $P_S = 0.96$

9. (b) $E_{comp} \sim 0.262\, E_f$, (c) ~31 GPa

10. (a) $\sigma_o = 60$ MPa, (b) $n \sim 1/2$

11. (c) biocomposite, ~234 MJ

12. (b) $N_f \sim 8.1 \times 10^5$ cycles

(d) life = 3471 days