EGT1
ENGINEERING TRIPOS PART IB

Monday 7 June 2021 13:30 to 15.40

Paper 1

MECHANICAL ENGINEERING

This is an ***open-book*** *exam.*

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the top sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
You have access to the Engineering Data Book, online or as your hard copy.

10 minutes reading time is allowed for this paper at the start of the exam.

The time allowed for scanning/uploading answers is 20 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers to both sections.

SECTION A

*Answer not more than **two** questions from this section.*

1 Figure 1 shows a cylinder rolling over the rounded edge of a table. The rounded edge is a quarter-circle of radius $2r$ centred at A. Point Q is where the rounded edge meets the horizontal surface of the table and point R is where it meets the vertical face. The cylinder of radius r, centre B, rolls without slipping over the edge. Point P on the cylinder was originally in contact with the table at Q. At the instant shown, contact between the cylinder and the edge of the table is at C, and the angle QAC is θ. The unit vectors $\mathbf{e}_1$ and $\mathbf{e}_1^*$ are orthogonal and $\mathbf{e}_1$ is aligned with CP. For the instant shown:

(a) show that the velocity of P is given by

$$\mathbf{v}_{\mathrm{P}} = 6r\dot{\theta}\sin\theta\mathbf{e}_1^*;$$

[13]

(b) obtain an expression for the acceleration of P; [6]

(c) obtain an expression for the radius of curvature of the path of P. [6]

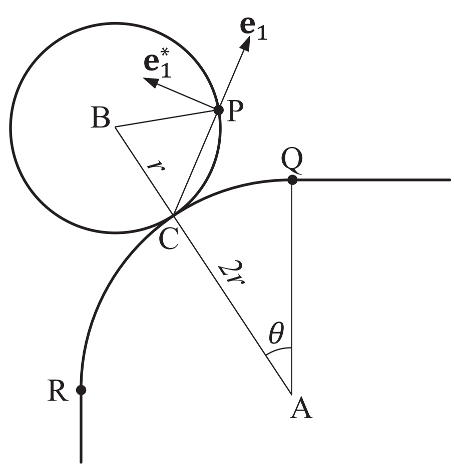

Fig. 1

2 A mechanism is shown in plan view in Fig. 2. The uniform light bar CD of length a is free to slide on the light bar AB via a slider at C. The bar AB is driven at constant angular speed ω, defined positive in the anticlockwise direction. The resulting angular velocity of CD is $\dot{\phi}$, defined positive in the anticlockwise direction. At the instant shown the angles DAC and CDA are both 30° and the distance AC is a. For the instant shown:

(a) find the angular velocity $\dot{\phi}$ in terms of ω; [8]

(b) derive an expression for the sliding speed at C; [7]

(c) assuming that there is a sliding friction force, F, acting at C, and friction torques, Q, acting at each of the pin joints A,C,D, derive the expression for the necessary driving torque, T, at A. [10]

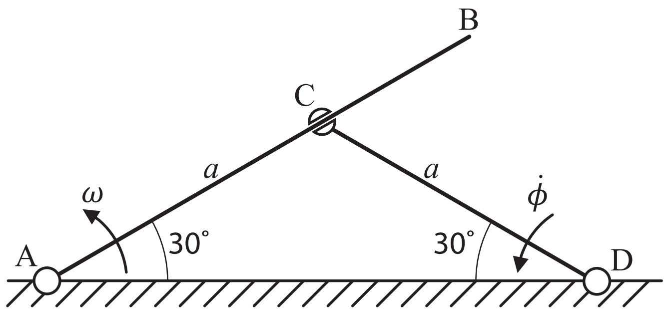

Fig. 2 (not to scale)

3 A slender column has mass m and height L. At the top of the column, a small additional heavy block of mass m is placed. The column falls from rest, rotating as a rigid body around the fixed pivot, O, at the base of the column. The angle of tilt from the vertical is θ.

(a) find an expression for $\ddot{\theta}$ in terms of the angle of tilt, θ; [5]

(b) for $\theta = \frac{\pi}{2}$ find the shear force at distance $x = L/3$ from the pivot and show that the magnitude of the bending moment at this point is $mgL/18$; [12]

(c) for $\theta = \frac{\pi}{2}$ the bending moment has largest magnitude at a distance $x = 0.4L$. Roughly sketch the shear force and bending moment diagrams using the usual sign convention, and giving values at $x = 0$ and $x = L$. [8]

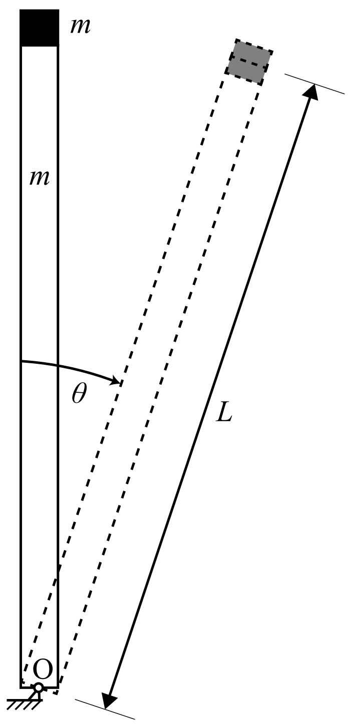

Fig. 3

SECTION B

Answer not more than ***two*** *questions from this section.*

4 Figure 4 shows two frictionless pendulums both with length L and mass m hanging on a freely rolling trolley also of mass m. The acceleration due to gravity is g.

(a) Write down the Lagrangian of the system, using the angles θ_1, θ_2 and the horizontal position of the trolley, x, as variables, and hence find the equations of motion. [6]

(b) Linearize the equations of motion and show that they can be written in the form

$$\begin{bmatrix} 3m & Lm & Lm \\ Lm & L^2m & 0 \\ Lm & 0 & L^2m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & Lmg & 0 \\ 0 & 0 & Lmg \end{bmatrix} \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \end{bmatrix} = 0$$

[7]

(c) Find the angular frequency and mode shapes of the normal modes of the system. [7]

(d) Describe in words the motion corresponding to each normal mode. [5]

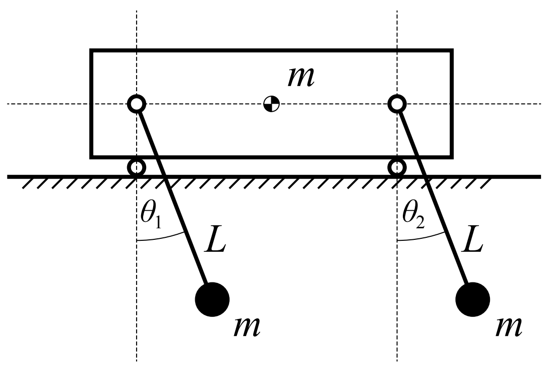

Fig. 4

5 Figure 5 shows a simple pendulum consisting of a point mass m and a light rod of length l connected to a horizontal arm of length R. The arm is attached to a vertical shaft that rotates at constant speed ω. The pendulum, arm and shaft are coplanar. The angle of the pendulum to the vertical direction is θ. The acceleration due to gravity is g.

(a) Show that the kinetic energy T of the system is given by

$$T = \frac{1}{2}m\left[\left(R^2 + 2Rl\sin\theta + l^2\sin^2\theta\right)\omega^2 + l^2\dot{\theta}^2\right]$$

and also derive an expression for the potential energy of the system. [5]

(b) Use the Lagrangian to find the equation of motion of the system. [8]

(c) Show that equilibrium can exist if and only if

$$\frac{R}{l} + \sin\theta = \frac{g}{\omega^2 l}\tan\theta.$$

[5]

(d) For the condition

$$\frac{R}{l} \gg 1 \gg \frac{g}{\omega^2 l}$$

find the equilibrium states. For each equilibrium angle θ_0, consider a small perturbation from equilibrium $\theta = \theta_0 + \delta\theta$, and hence determine the equilibrium's stability. [7]

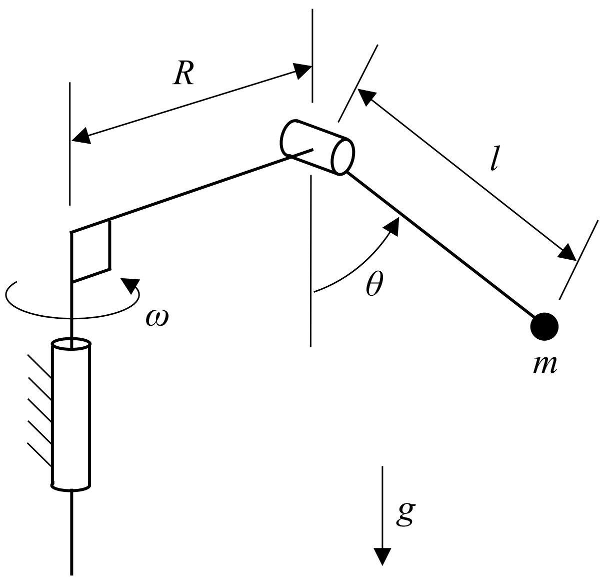

Fig. 5

6 Figure 6 shows the plan view of a point mass m on a frictionless horizontal plane. The mass is connected to one end of a spring of stiffness k. The other end of the spring is pivoted to a stationary point on the plane. The angular position of the mass is θ and the distance from the pivot is r. The force in the spring is zero when $r = 0$.

(a) Show that the Lagrangian of the system is

$$L(r, \theta, \dot{r}, \dot{\theta}) = \frac{1}{2}\left[m\left(r^2\dot{\theta}^2 + \dot{r}^2\right) - kr^2\right]$$

and show that the Lagrangian equation of motion for θ gives conservation of the angular momentum h. [7]

(b) Use the Lagrangian to show that the equation of motion for r can be written in the form of one-dimensional motion in an effective potential:

$$m\ddot{r} = -V'_{\text{eff}}(r)$$

$$V_{\text{eff}}(r) = \frac{h^2}{2mr^2} + \frac{kr^2}{2}$$

[6]

(c) Sketch the form of $V_{\text{eff}}(r)$ and, for a given angular momentum h, derive expressions for the equilibrium point r_0 and the corresponding $\dot{\theta}$. [6]

(d) By considering a small perturbation δr from the equilibrium r_0, such that $r = r_0 + \delta r$, find the frequency of small oscillations of r and sketch the pattern of motion as seen from above. [6]

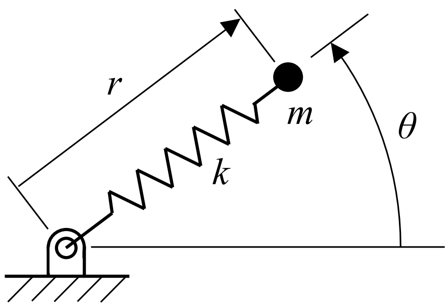

Fig. 6

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Answers

1. b) $(6r\ddot{\theta}\sin\theta + 6r\dot{\theta}^2\cos\theta)\mathbf{e}_1^* - 12r\dot{\theta}^2\sin\mathbf{e}_1$; c) $3r\sin\theta$
2. a) 2ω ; b) $\sqrt{3}\omega a$; c) $6Q + \sqrt{3}aF$
3. a) $\frac{9}{8}\frac{g}{L}\sin\theta$; b) $-mg/24$
4. c) $\omega^2 = 0, [1, 0, 0]; \omega^2 = g/l, [0, 1, -1]; \omega^2 = 3g/l, [1, -3/2L, -3/2L]$
5. d) $\theta_0 = \pi/2$ (stable); $\theta_0 = -\pi/2$ (unstable)
6. c) $r_0^4 = h^2/mk; \dot{\theta} = \sqrt{k/m}$; d) $2\sqrt{k/m}$