Version MO/5

EGT1
ENGINEERING TRIPOS PART IB

Monday 10 June 2019 2 to 4.10

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (a) You are designing the mast of a cable stayed bridge for a model railway, shown in Fig. 1. A steel mast, inclined at 45º to the horizontal, supports a horizontal bridge deck. The mast cross section is triangular and all of the 19 equally spaced cables are inclined at 45º to the eccentrically-positioned deck, as shown. In the critical load case, each of the 19 cables carries the same tension force of P = 335 N. Ignoring the self-weight of the structure:

(i) determine the longitudinal normal stresses at points A and B on the mast cross section shown in Fig. 1(c); [6]

(ii) calculate the shear stresses at points A and B. [5]

(b) The mast is to be fabricated in steel with a minimum yield strength of 275 MPa. Considering only points A and B, calculate a factor of safety against yielding:

(i) using the Tresca yield criterion; [5]

(ii) using the von Mises yield criterion. [5]

(c) Comment on your result from part (b). [2]

(d) The cost of the mast is proportional to the volume of steel used. Suggest one way in which this cost might be reduced. [2]

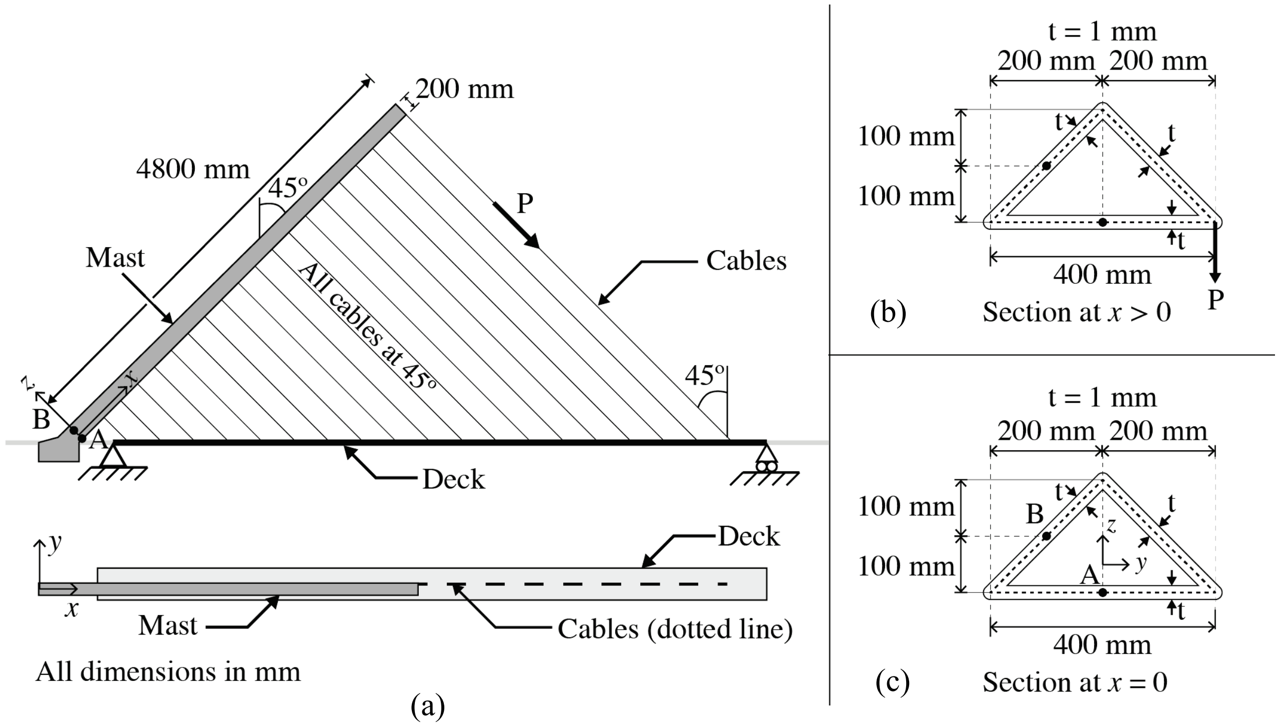
200 mm
4800 mm
45°
P
Cables
Mast
All cables at 45°
45°
z
x
B
A
Deck
y
x
Deck
Mast
Cables (dotted line)
All dimensions in mm
(a)
t = 1 mm
200 mm 200 mm
100 mm
100 mm
t
t
t
400 mm
P
(b) Section at x > 0
t = 1 mm
200 mm 200 mm
100 mm
100 mm
B
z
y
A
t
t
t
400 mm
(c) Section at x = 0

Fig. 1

2 The weightless pin-jointed structure shown in Fig. 2 is initially stress free. Each member has cross sectional area A, Young's Modulus E, and all behaviour is linear elastic.

(a) Calculate the number of redundancies in the structure. [1]

(b) A vertical load P is applied to node D.

(i) Find a particular equilibrium solution for the bar forces. [3]

(ii) Calculate any states of self-stress in the structure. [3]

(iii) Find the elastic solution for the bar forces due to the applied load. [4]

(c) With the load still applied, the temperature of bar AD is increased by T ˚C. It has a coefficient of thermal expansion of α.

(i) Find the new bar forces. [5]

(ii) Find the total displacement of node D due to the combined effects of heating and loading. [9]

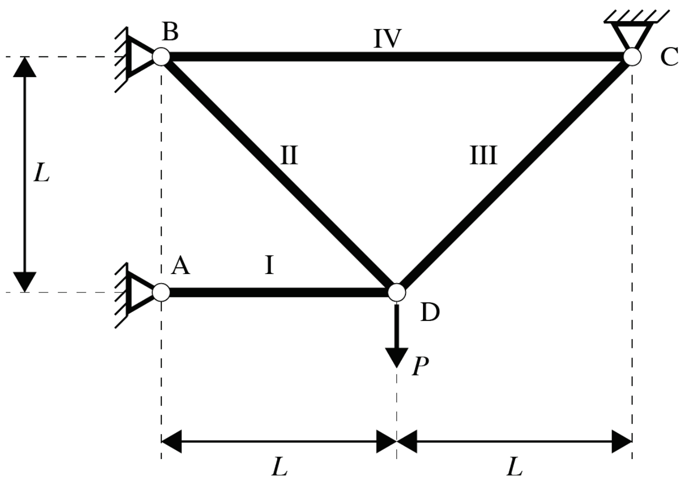

Fig. 2

3 (a) The weightless frame shown in Fig. 3 is pinned at points A and D. The frame has a uniform cross-section and flexural rigidity EI, is axially rigid, and behaves elastically. The unloaded frame is unstressed before the external forces are applied as shown and buckling does not occur.

(i) Sketch the deflected shape of the frame. [3]

(ii) Find an expression for the vertical deflection at the midpoint of BC. [6]

(iii) Find an expression for the horizontal deflection of point C. [7]

(b) After installation it is found that the load closest to B is reversed, and now acts upwards.

(i) Sketch the revised deflected shape of the frame. [3]

(ii) Calculate the reaction forces at A and describe how the resultant force at the support has changed under the revised loading. [6]

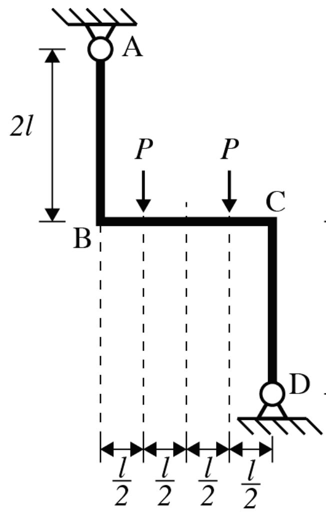

Fig. 3

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SECTION B

4 (a) Figure 4 shows a thin-walled cylinder of radius R, wall thickness t and a uniform bending moment M along its length. By considering suitable free body diagrams, show that the longitudinal stress in the cylinder, measured at a point (R, θ) from the centre of its cross-section, is given by:

$$\sigma_l = \frac{M\cos\theta}{\pi R^2 t}$$ [6]

(b) A thin-walled steel cylinder of radius R = 1.5 m and wall thickness t = 10 mm has closed ends and is subjected to an internal gauge pressure of 1200 kPa. The steel has a Young's Modulus E = 210 GPa and a Poisson's ratio ν = 0.3. Calculate the components of stress and strain in the longitudinal and hoop directions. [6]

(c) A thin-walled steel cylinder with internal gauge pressure of 1200 kPa spans 40 m between simple supports. The densities of the steel and the fluid in the cylinder are 7840 kg m^{-3} and 815 kg m^{-3}, respectively. Considering self-weight and internal pressure, identify the location of maximum bending moment, and for this location:

(i) calculate the total longitudinal and hoop stresses; [7]

(ii) use the von Mises yield criterion to calculate the minimum yield strength required to prevent inelastic behaviour. [6]

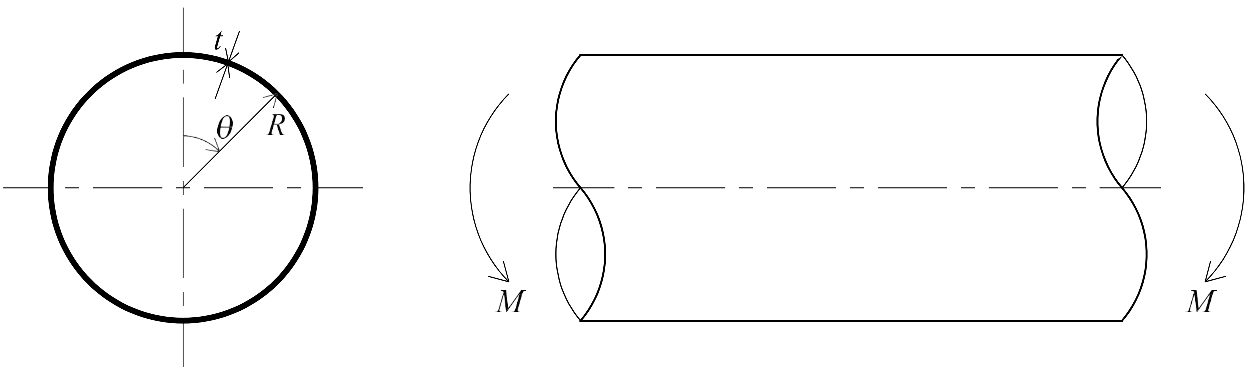

Fig. 4

5 Figure 5 shows a propped cantilever beam of length L and fully plastic moment capacity M_p. A uniform load of total magnitude W is applied over its length as shown and the self-weight of the cantilever is negligible.

(a) Perform a *lower bound* analysis to find the maximum safe value of the load W in terms of L and M_p. [7]

(b) Use the *upper bound* method to calculate the collapse load W in terms of L and M_p. Compare your result to your lower bound estimate and comment. [7]

(c) Perform a linear elastic analysis to find an expression for the deflection at mid-span in terms of W, L and bending stiffness EI. In your answer you may assume that the bending moment at the clamped end of the beam is $WL/8$. [5]

(d) As the load W increases, the first plastic hinge forms at the clamped support and is followed by a second plastic hinge along the beam span. Use your answers to parts (a), (b) and (c) to sketch the variation of load against mid-span deflection of the beam. Label salient values. [6]

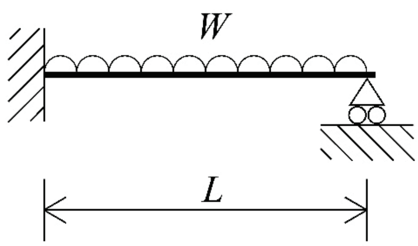

Fig. 5

6 The ductile isotropic plate ABCD shown in Fig. 6 has clamped supports along sides AB and CD, is simply supported along BC, and has a free edge along AD. The plate has a fully plastic moment capacity of m_p per unit length and its self-weight is negligible. It is subjected to a concentrated load of magnitude P acting normal to the plate, at E, mid-way between A and D. Two collapse mechanisms are proposed in Fig. 6(a) and Fig. 6(b).

(a) For the mechanism shown in Fig. 6(a), show that the collapse load is $P = 8.6\ m_p$. [7]

(b) In the alternative mechanism shown in Fig. 6(b), the yield lines intersect the free edge at αL from the clamped edges. Calculate the lowest upper bound of the collapse load in terms of m_p. [7]

(c) The plate is made of steel with uniform thickness of 15 mm and yield stress of 350 MPa. By considering your answers to (a) and (b) above, what is your best estimate of the collapse load of the plate in kN? [7]

(d) Without performing further calculations assess whether the collapse load would change for longer and shorter versions of the plate, in which edges AD and BC are elongated or shortened by equal amounts, respectively. [4]

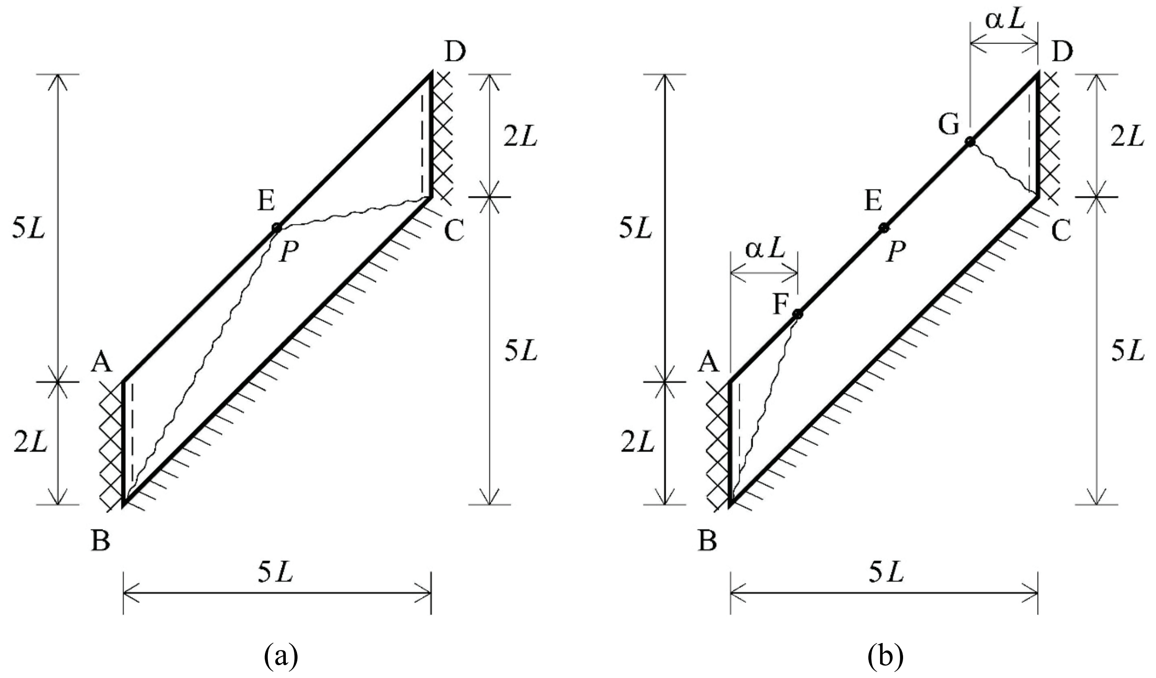

Fig. 6

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