EGT1
ENGINEERING TRIPOS PART IB

Wednesday 9 June 2021 13.30 to 15.40

Paper 2

STRUCTURES

This is an ***open-book*** *exam.*

Answer not more than ***four*** *questions, which may be taken from either section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the top sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
You have access to the Engineering Data Book, online or as your hard copy.

10 minutes reading time is allowed for this paper at the start of the exam.

The time allowed for scanning/uploading answers is 20 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

SECTION A

1 Figure 1 shows a weightless pin-jointed truss subjected to a point load W. The unloaded structure comprises members I to X and is free of stress. There is no connection between bars III and IV or between bars VIII and IX.

(a) Show that there are two redundant bars. [5]

(b) Using the diagonals IV and IX as the redundant bars:

(i) find the particular solution for bar tensions in equilibrium with the applied load W. [5]

(ii) find the possible states of self stress in the structure; [5]

(iii) find the elastic solution for the bar tensions under load W. [10]

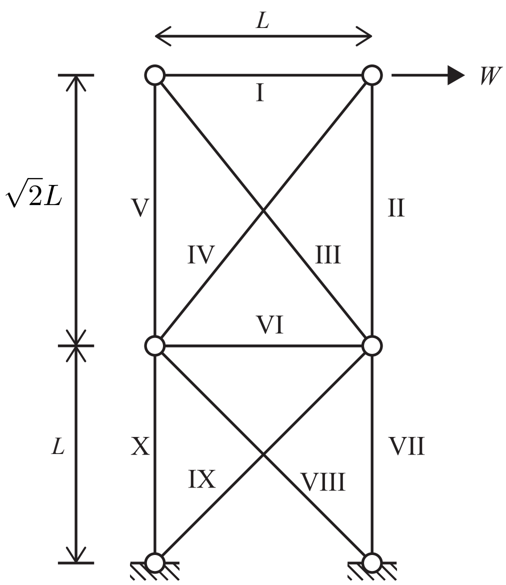


Fig. 1

2 A portal frame with rigid joints is to carry a horizontal load of magnitude F at its upper corner. One column base is fixed and the other is pinned. The frame geometry is shown in Fig. 2. All members have flexural rigidity EI with respect to bending deformations within the plane of the paper, and all behaviour may be assumed to be linear elastic. All members may be considered to be axially incompressible.

(a) Determine the reactions at the pinned base. [12]

(b) Draw the bending moment diagram, indicating important values. [4]

(c) Determine the horizontal deflection at the point of the applied load. [6]

(d) What would be the support reactions if both feet were pinned? [3]

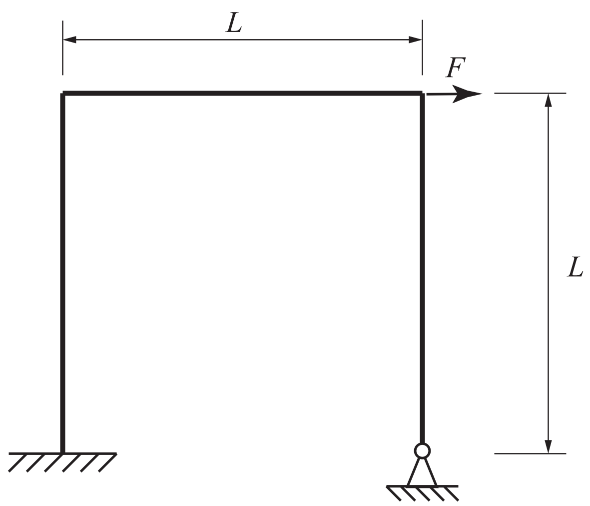


Fig. 2

3 (a) A cantilever beam with a thin-walled rectangular cross-section is to carry a point load of 10 kN applied eccentrically at its tip, as shown in Fig. 3. The line of action of the load is along the mid-wall of the web. The beam is 3 m long, 250 mm wide and 150 mm deep. Each side wall is 6 mm thick whereas the upper and lower surfaces are each 10 mm thick. Assume that all cross-sections remain rectangular, that all behaviour is linear elastic and that self-weight can be ignored. The beam is made of steel with a Young's modulus of 210 GPa and a Poisson's ratio of 0.3.

Stating any assumptions, determine:

(i) the second moments of area of the section relevant for vertical and horizontal bending; [5]

(ii) the torsion constant for the section; [5]

(iii) the maximum axial stress at the point A marked on Fig. 3; [2]

(iv) the maximum shear stress at the point B marked on Fig. 3; [6]

(v) the maximum shear stress on any plane at the point A marked on Fig. 3. [7]

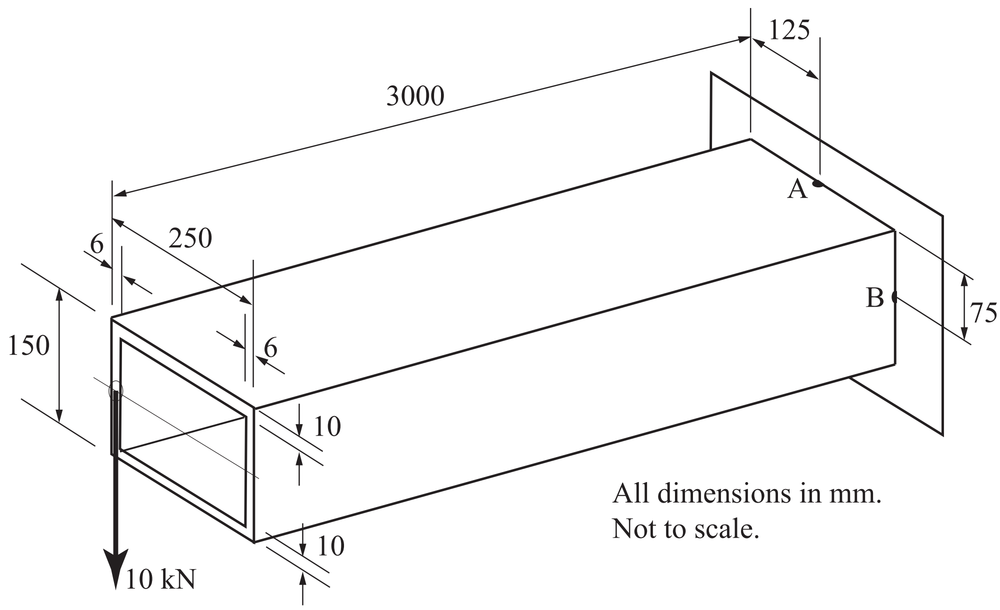


Fig. 3

SECTION B

4 A long circular cylindrical pressure vessel of external diameter 2000 mm and wall thickness of 10 mm is sketched in Fig. 4. It is made of steel with a uniaxial yield stress of 220 MPa, a Young's modulus of 210 GPa and a Poisson's ratio of 0.3. Longitudinal and rotational movements are prevented at one end of the vessel. It is supported on rollers along its length such that there is no restraint against longitudinal expansion or contraction. There are hemispherical closed ends whose details need not concern us. Self-weight may be ignored throughout.

The cylinder is subject to an internal pressure of 2 MPa. For a typical location in the vessel wall that is situated well away from the end caps:

(a) draw the Mohr's circles of stress; [4]

(b) calculate the principal strains. [6]

(c) Whilst maintaining the internal pressure at 2 MPa, a torque is applied about the longitudinal axis of the vessel at the end away from the support. At the typical location considered above, determine the value of the torque that would cause yield according to:

(i) Tresca's criterion; [6]

(ii) von Mises' criterion. [9]

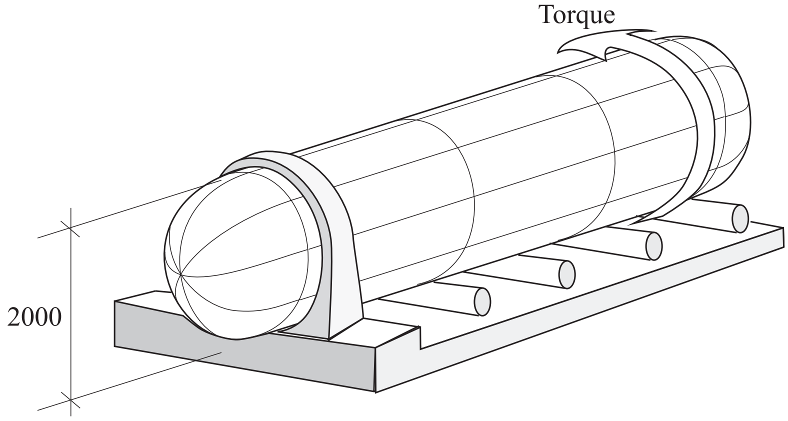


Fig. 4

5 The asymmetric frame illustrated in Fig. 5 has fully-rigid connections and is built-in at the supports. The columns each have a fully-plastic moment of resistance M_p whilst that of the beam is twice this. The frame is to support a vertical load V at the midspan of the beam and a horizontal load H at the upper right-hand corner. Assume all behaviour is rigid plastic and that all members are axially inextensible.

(a) Find three limiting combinations of V and/or H which will guarantee collapse, and draw these limits on an interaction diagram of VL/M_p against HL/M_p (where L is the length shown in the figure). [11]

(b) If $V = 2H$, find the corresponding point (H, V) on the limiting boundary of your upper-bound interaction diagram. Show that this point is also a lower bound, and draw the corresponding bending moment diagram, clearly indicating all important values. [14]

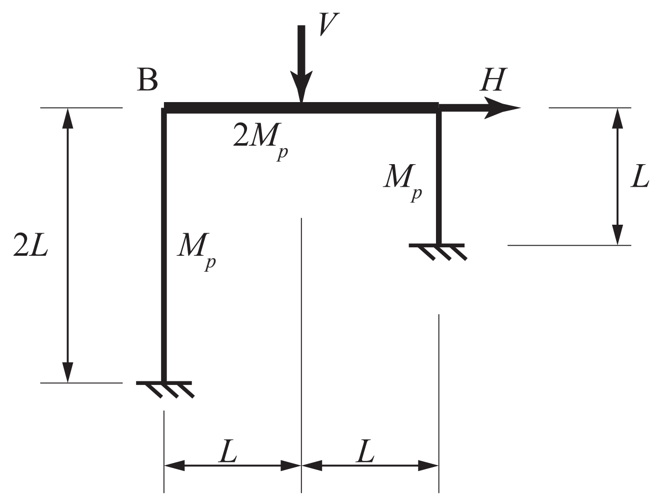


Fig. 5

6 (a) A triangular plate has side lengths of 500 mm. It is clamped along one edge and simply-supported along another, with the long edge being unsupported. The plate is 10 mm thick and is made of steel with uniaxial yield stress of 250 MPa. The plate is to carry a total load W which is uniformly distributed over its whole upper surface. The plate is shown in plan view in Fig. 6(a), and is loaded normally to the plane of the page in Fig. 6(a).

The strength of the plate is to be investigated by considering a yield line collapse mechanism of the form shown in Fig. 6(a). The mechanism has only two yield lines. One is a sagging yield line oriented at an angle α to the simply-supported edge and the other is a hogging yield line which runs alongside the clamped edge.

(i) Determine the plastic moment of resistance per unit length of a yield line in the plate. [2]

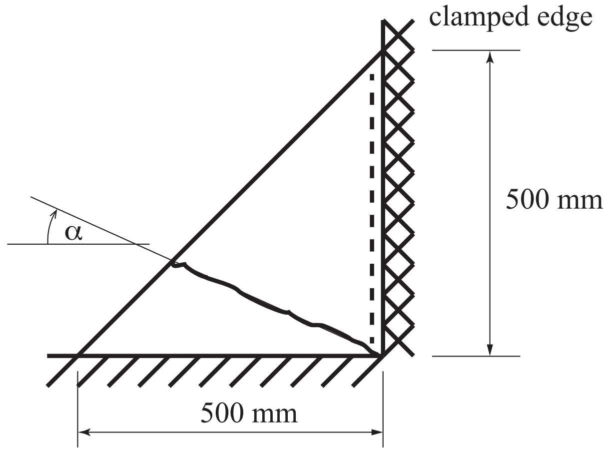
(ii) Derive an expression for the collapse load W in terms of the angle α. [6]

(iii) Estimate the value of α that will give the lowest collapse load W for mechanisms of this form. State the value of α in degrees and the corresponding W in kN. [8]

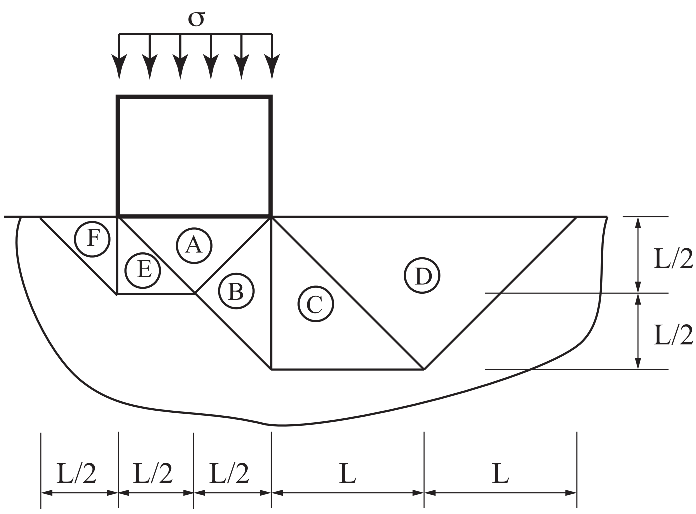
(b) Figure 6(b) shows a cross-section through a metal-forming indentation process in a rigid-plastic material with a yield stress in shear of k. The indentation tool applies a stress σ to the surface. For the mechanism shown:

(i) draw the displacement diagram of the various blocks; [3]

(ii) determine an upper bound on the stress σ needed to cause indentation. [6]



(a) Plan View



(b) Cross Section View

Fig. 6

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2P2
2021
Numerical answers

Q1 (b) (iii)

$t = [0.47\ \ -0.75\ \ -0.82\ \ 0.91\ \ 0.67\ \ -0.025\ \ -1.91\ \ -0.71\ \ 0.70\ \ 1.92]^T$

3(a)(i)	$I_{xx} = 26.74\text{x}10^6$ mm^4
	$I_{yy} = 49.27\text{x}10^6$ mm^4
3(a)(ii)	$J = 48.89\text{x}10^6$ mm^4
3(a)(iii)	$\sigma = 84.14$ MPa
3(a)(iv)	$\tau = 3.26$ MPa
3(a)(v)	$\tau_{max} = 42.11$ MPa

4(b)	$\varepsilon_{hoop} = 802 \text{ x } 10^{-6}$
	$\varepsilon_{long} = 186 \text{ x } 10^{-6}$
	$\varepsilon_{ttt} = -423 \text{ x } 10^{-6}$
4(c)	$T_{max,Tresca} = 3261$ kNm
	$T_{max,VM} = 4997$ kNm

6(a)(i)	$M_p = 6250$ N/m
6(a)(iii)	W = 71.78 kN, a = 35.26^o