Version JAB/4

EGT1
ENGINEERING TRIPOS PART IB

Wednesday 7 June 2023   14.00 to 16.10

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

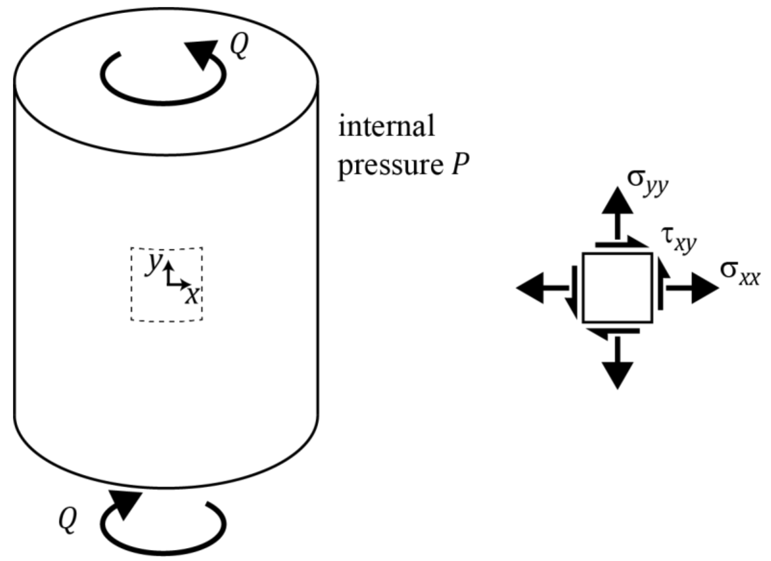
SECTION A

1 Figure 1(a) shows a thin-walled closed-end cylinder of radius r and wall thickness t that is subjected to an applied torque Q and an internal pressure P. The cylinder is unconstrained axially, and is made of material which yields according to the Tresca criterion with a yield stress in uniaxial tension Y.

(a) A representative patch around a point is shown on the surface of the cylinder in Fig. 1(a), and separately in Fig. 1(b). Calculate the stress components σ_{xx}, σ_{yy}, and τ_{xy} at this point, due to the load applied. [3]

(b) Consider the dimensionless variables $q = Q/(2\pi r^2 tY)$ and $p = Pr/(tY)$. In an experiment, the loads Q and P are increased from zero until yield occurs. Assuming that q is much greater than p, find a relationship between p and q at yield. [15]

(c) Find the minimum value of q/p for which the expression derived in (b) is valid. For values of q/p less than this, find an alternative relationship between p and q at yield. [7]



(a) (b)

Figure 1

2

(a) The cantilever shown in Fig. 2(a) takes the form of a quarter of a circle of radius r, and is mounted so that its tip is horizontal. It has a bending stiffness EI. A vertical load P and a couple Q are applied at the tip, as shown.

(i) Find the moment carried at point C, defined by the subtended angle θ shown in the figure. [3]

(ii) Find the rotation at the tip due to the loads applied. [7]

(b) Two copies of the structure shown in Fig. 2(a) are rigidly joined at their tips to give the structure shown in Fig. 2(b), which is 2-fold rotationally symmetric about its centre, the point A, where the beam is horizontal. A vertical load W is applied at A.

(i) Explain why A will not rotate due to the load W. [5]

(ii) Find the vertical deflection of A due to the load W. [10]

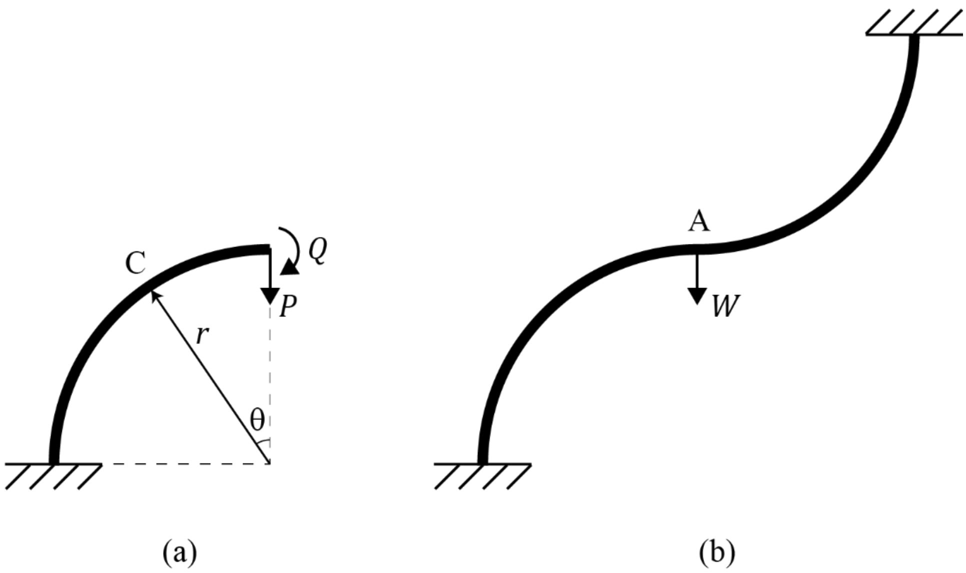


Figure 2

3 Figure 3(a) shows a three-span beam with bending stiffness EI that is initially stress free, and is then subjected to a uniformly distributed load w.

(a) Calculate the bending moment in the beam over the interior supports. Plot a bending moment diagram for the beam. Find the reaction forces at the interior supports. [8]

(b) Show that, if the interior supports now settle by a distance δ, the reaction force from these supports drops to zero when

$$\delta = \frac{11}{12}\frac{wL^4}{EI}$$ [9]

(c) The interior supports are replaced with two vertical struts, as shown in Fig. 3(b), where these struts have axial stiffness EA. Calculate the displacement of the interior supports of the beam due to the applied load. [8]

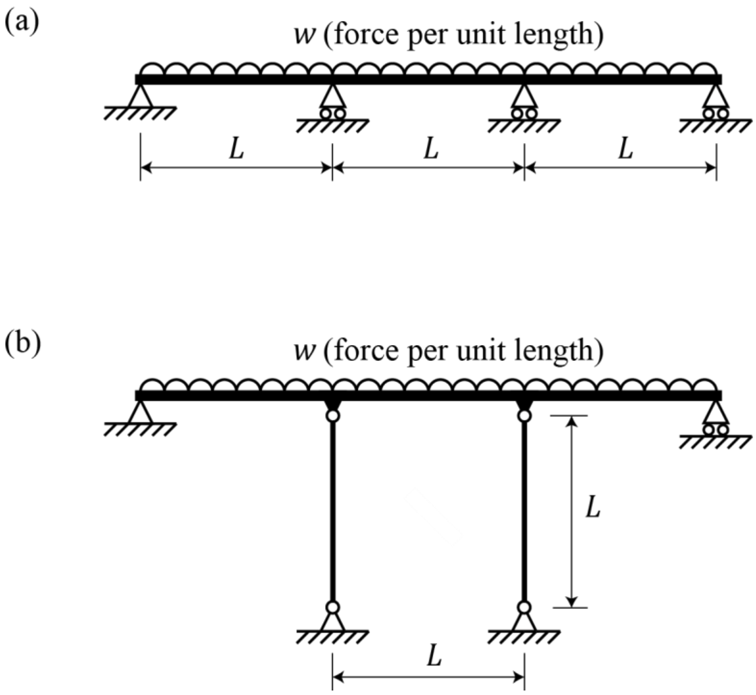


Figure 3

SECTION B

4 A semi-circular arch is fixed at the supports and loaded with a point load P at the top, as shown in Fig. 4(a). The arch has a constant cross-section along its length, with a fully plastic moment capacity M_p.

(a) Assuming a symmetric plastic collapse mechanism develops with the hinge locations shown in Fig. 4(b), obtain an upper bound estimate for the collapse load P. [12]

(b) Obtain an estimate of the collapse load P using the lower bound theorem of plasticity. [13]

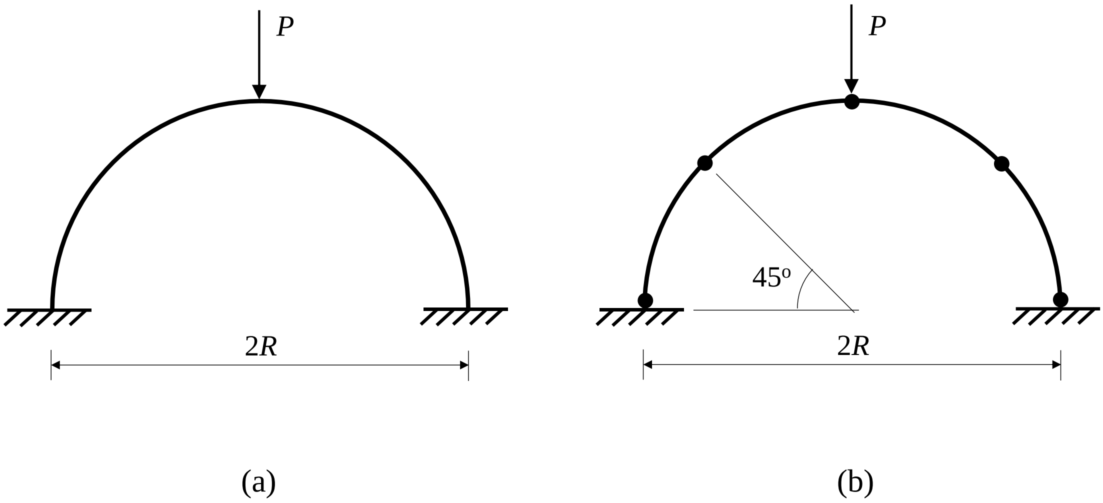


Figure 4

5 A horizontal square plate with side b has fixed boundary conditions along two edges, with the other two edges free, as shown in Fig. 5(a). A vertical plate is welded to the horizontal plate, and a moment M is applied to the vertical plate. The horizontal plate has a fully plastic moment capacity per unit length given by m. The thickness of the vertical plate is negligible compared to the length b. The self-weight of the plates can also be neglected.

(a) Considering the yield line mechanism shown in Fig. 5(b), prove that when point C deflects by a small amount δ, the work done by the moment M is $M\delta/b$. [3]

(b) Estimate the value of the moment M causing collapse, using the yield line mechanism shown in Fig. 5(b). [11]

(c) Investigate whether the yield line mechanism in Fig. 5(c) is critical over that in Fig. 5(b). [11]

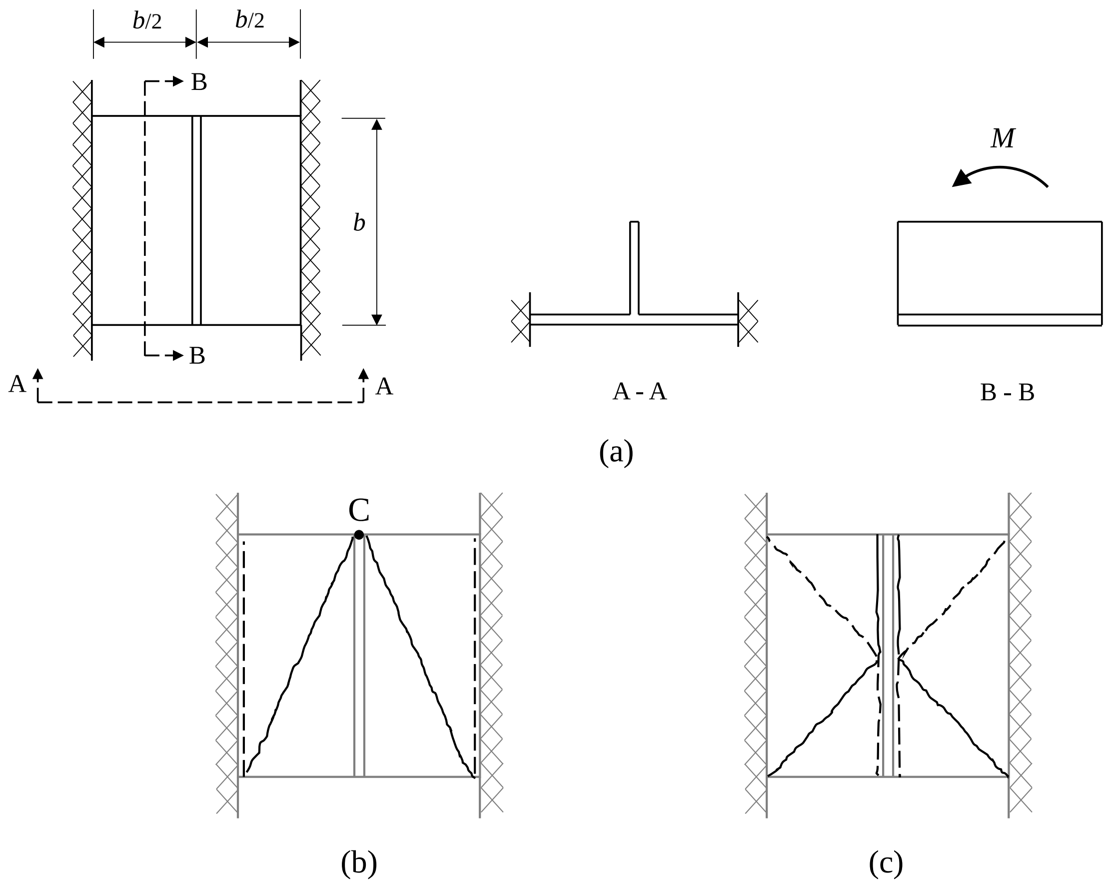


Figure 5

6 A load P is placed at a distance L from an excavation with depth $3d$, as shown in Fig. 6. The soil has a layered structure. A stronger layer with depth d and plastic shear strength $k_2 = 6k_1$ is sandwiched between two layers with plastic shear strength k_1. The soils are assumed to behave with rigid-plastic properties. The weight of the soil may be neglected. Note that the load P is a line load in the out-of-plane direction.

(a) Considering the failure mechanism shown in dashed lines in Fig. 6, draw a displacement diagram of the various blocks of soil comprising the mechanism. [5]

(b) Derive an expression for an upper bound estimate of the failure load P as a function of k_1 and the distances d, L, L_1 and L_2. [10]

(c) Determine the lengths L_1 and L_2 so that the most accurate estimate for the failure load P is obtained. [10]

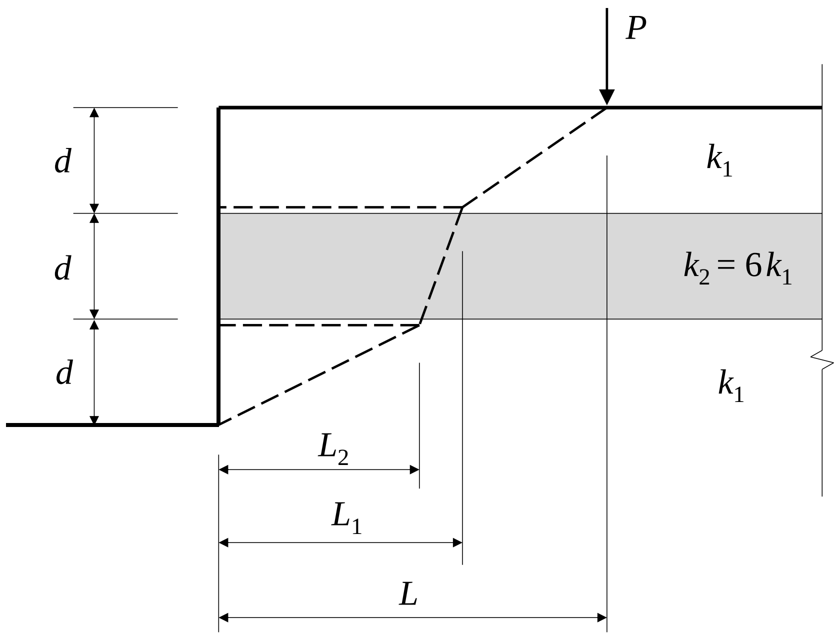


Figure 6

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ANSWERS

Part A

1. a. $\sigma_{xx} = pR/t$; $\sigma_{yy} = pR/(2t)$; $\tau_{xy} = T/(2\pi r^2 t)$; b. $p^2 + 16q^2 = 4$; c. $q/p = 1/\sqrt{2}$; $q^2 - p^2/2 + 2p/3 = 1$

2. a. (i) $M = Q + PR \sin\theta$; (ii) $\theta = (R/EI)(\pi Q/2 + PR)$; b. (ii) $v = (wR^3/EI)(\pi/8 - 1/\pi)$

3. a. $M = wL^2/10$; $R = 11wL/10$; c. $v = (11/2)(wL^4/(6EI + 5EAL^2)$

Part B

4. a. $P = 9.66\ M_p\ /\ R$; b. $P = 9.66\ M_p\ /\ R$

5. a. $W = M\delta\ /\ b$; b. $M = 9m$; c. $M = 6m$ (critical)

6. b. $Pd = k_1\ (L^2 - L_1{}^2 + 3L_2{}^2 - LL_1 + 2d^2) + k_2\ ((L_1 - L_2)^2 + d^2)$; c. $L_1 = L/2$, $L_2 = L/3$