Version NRA/5

EGT1
ENGINEERING TRIPOS PART IB

Tuesday 4 June 2019   9 to 11.10

Paper 4

THERMOFLUID MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section*

1 (a) A sphere of radius r_1 is surrounded by a spherical shell with thermal conductivity λ_s and outer radius r_2.

(i) Show that the thermal resistance of the spherical shell is given by the expression

$$R_{\text{th}} = \frac{1}{4\pi\lambda_s}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

[4]

(ii) The outer surface is exposed to a flow with average convective heat transfer coefficient h. Derive an expression for the outer radius of the spherical shell r_2, which would maximise the heat transfer to (or from) the internal sphere in terms of λ_s and h. [6]

(b) Whitaker's correlation for convective heat transfer from a sphere of diameter d is given by

$$Nu_d = 2 + \left[0.4{Re_d}^{1/2} + 0.06{Re_d}^{2/3}\right] Pr^{0.4}$$

(i) By considering the thermal resistance of a spherical shell of stationary fluid with large outer radius (or otherwise) explain the significance and numerical value of the first term of the correlation. [5]

(ii) Given that the correlation is valid for Re_d below 10^5, suggest a physical explanation for the other terms. [5]

(c) Spherical beads of diameter 1 mm are coated with a spherical shell of thickness 0.5 mm, thermal conductivity $0.26\,\mathrm{W\,m^{-1}\,K^{-1}}$ and thermal diffusivity $1.84\times10^{-7}\,\mathrm{m^2\,s^{-1}}$. The beads are cooled by dropping them into a stream of air with relative velocity $20\,\mathrm{m\,s^{-1}}$. Taking the thickness as the characteristic dimension, determine the Biot number of the shell and estimate the characteristic time scale before the inner bead begins to cool. [5]

Data for Air: Density, $\rho = 1.25\,\mathrm{kg\,m^{-3}}$, thermal conductivity, $\lambda_{\text{air}} = 0.026\,\mathrm{W\,m^{-1}\,K^{-1}}$, dynamic viscosity, $\mu_{\text{air}} = 1.8\times10^{-5}\,\mathrm{Pa\,s}$ and Prandtl number, $Pr = 0.71$.

2 A heat pump uses refrigerant R-134a as working fluid. Dry saturated vapour at 3.5 bar exits the evaporator and is compressed to 16.0 bar. The compressed vapour is condensed at constant pressure until it is saturated liquid, before being throttled back to 3.5 bar at the evaporator inlet.

(a) Consider the compressor in the heat pump to be adiabatic and reversible.

(i) Sketch the cycle on *p-h* and *T-s* diagrams. Pay careful attention to the start and end points of each process. [4]

(ii) Calculate the specific work required to compress the vapour and the heat rejected in the condenser. [5]

(iii) Evaluate the Coefficient of Performance (COP) of the heat pump. [2]

(b) Consider the compressor in the heat pump to have an isentropic efficiency of 60%.

(i) Redraw the *p-h* diagram to highlight the effects of non-ideal compression. [3]

(ii) Compare the COP of the non-ideal cycle to that calculated in part (a). [3]

(iii) The heat pump is rated at 2.0 MW heat output. Calculate the mass flow rate of working fluid and power input to the compressor. [4]

(c) The heat pump is being considered to replace a gas boiler which has 90% efficiency. The heat pump will be driven by a 98% efficient motor, supplied with electricity with 10% transmission loss, generated in a gas fired power plant with an efficiency of 62%. Comment on the advantages and disadvantages of the heat pump. [4]

3 (a) A steam power plant with output 600 MW operates on a super heated Rankine cycle. Steam enters the turbine at 650 °C and 150 bar. The condenser pressure is 0.06 bar with saturated liquid at exit. The isentropic efficiency of the turbine is 0.85. The pressure losses in the boiler and the condenser are negligible. You should neglect the feed pump work.

(i) Sketch the cycle on a *T-s* diagram. [4]

(ii) Calculate the dryness fraction at the turbine exit, the cycle efficiency and mass flow rate of steam. [8]

(b) In order to allow more renewable energy onto the power grid it is desirable to vary the power output of the steam power plant. Consider the cycle in part (a) with a 25% reduction in heat input. Assuming that the condenser pressure, boiler pressure, mass flow rate and turbine efficiency remain constant, calculate the new turbine inlet temperature, power output and cycle efficiency. Comment on the viability of this cycle. [7]

(c) Explain how a throttle valve at turbine inlet could improve the viability of the cycle suggested in part (b). Considering an isentropic turbine only, estimate the pressure drop required to bring the turbine exit flow to the same state as that in the cycle of part (a). Comment on the result. [6]

SECTION B

Answer not more than ***two*** *questions from this section*

4 Figure 1 shows two flows of differing velocity and density, mixing within a duct of area A. The velocities of the two streams at position 1 are ϕV_1 and V_1, and the densities of the streams are $\beta\rho_1$ and ρ_1 respectively. The cross sectional areas are αA and $(1-\alpha)A$ respectively. The flow mixes to a uniform state at position 2. The pressure can be assumed to be uniform at positions 1 and 2. Friction on the walls can be neglected, and you should assume incompressible flow.

(a) Using conservation of both mass *and* volumetric flow rate, find expressions for the density ρ_2 and velocity V_2 at position 2. [5]

(b) Hence, show that the pressure difference across the mixing region is given by

$$\frac{p_1 - p_2}{\rho_1 {V_1}^2} = -\alpha(1-\alpha)\left[\phi^2\beta - \phi(\beta+1) + 1\right]$$ [10]

(c) For a given density ratio β, what range of values of ϕ lead to a reduction in pressure across the mixing process? [5]

(d) Find the rate of change of mechanical energy due to mixing when $\phi = 2$, $\beta = 1$ and $\alpha = 0.5$, in terms of V_1, ρ_1 and A. Give a physical explanation for this behaviour. [5]

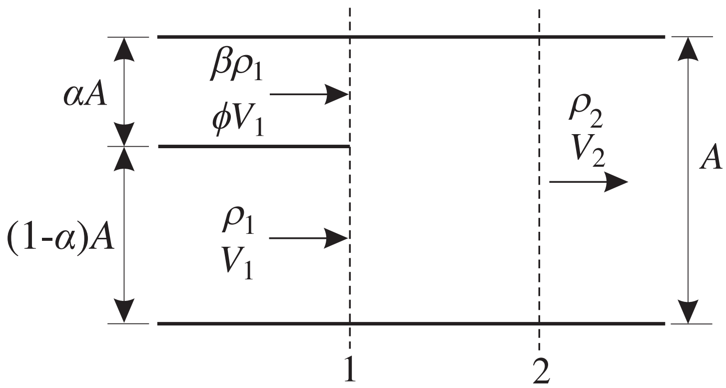

Fig. 1

5 A two-dimensional hydraulic damper is shown in Fig. 2(a). It consists of a rectangular solid block of length l and height h moving at a constant speed V between two stationary parallel walls. As shown in Fig. 2(b), the size of the clearance between the moving block and walls is c. The damper is filled with oil which can be assumed to be an incompressible Newtonian fluid with uniform density and uniform viscosity μ. The velocity and shear stress distributions within the clearance are $u(y)$ and $\tau(y)$, which can be assumed to be steady and independent of x. The clearance can be assumed to be small compared to the height, i.e. $c \ll h$.

(a) Considering the force balance on a suitable fluid element within the clearance, show that

$$\frac{\mathrm{d}\tau}{\mathrm{d}y} = \frac{(p_2 - p_1)}{l}$$

where y is the distance from the block. Justify the use of perfect derivatives in this expression. [5]

(b) Show that the velocity profile within the clearance is given by

$$u = \frac{(p_2 - p_1)}{2\mu l} y^2 + \left(\frac{\tau_w}{\mu}\right) y + V$$

where τ_w is the shear stress on the moving block i.e. when $y = 0$. [5]

(c) Sketch the velocity profile within the clearance. Find an expression relating the speed of the block V and the volumetric flow rate per unit width (into the page) within the clearance. [5]

(d) Stating any necessary assumptions, find an expression for the force per unit width (into the page) on the moving block in terms of V, c, h, μ and l. [10]

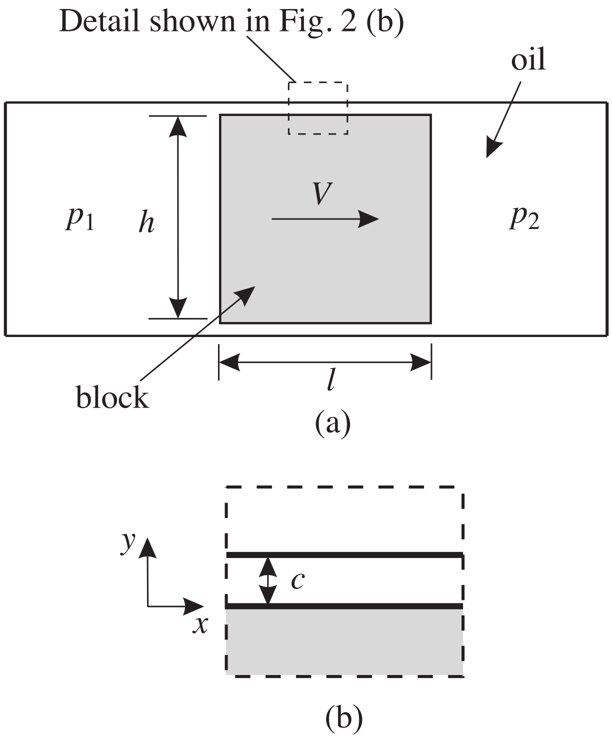

Fig. 2

(TURN OVER

6 A water tank shown in Fig. 3 consists of an inlet pipe and an outlet pipe. Both pipes have a diameter d and length l. Water leaves the tank through the exit pipe. As water leaves the tank, air is drawn from the atmosphere into the inlet pipe. The pipe diameters are small enough that the flow can be considered to be quasi-steady. You should assume incompressible flow and use databook values for the density and viscosity of air and water at $1.01325\,\text{bar}$ and $15\,°\text{C}$.

(a) Stating any assumptions, find an expression relating the bulk velocity in the inlet and outlet pipes. Hence determine the ratio of the Reynolds number based on diameter in the inlet pipe Re_{a} to the Reynolds number in the outlet pipe Re_{w}. [5]

(b) Neglecting frictional effects within each pipe, derive estimates for Re_{a} and Re_{w} when the height of the water above the exit pipe is $h = 0.4\,\text{m}$ and $d = 0.01\,\text{m}$. [5]

(c) The roughness in the pipes is $k = 0.1\,\text{mm}$. Using the 'Moody Chart' in Fig. 4 with your Reynolds number estimates from part (b), estimate the skin-friction coefficient in each pipe. Describe the different flow regimes expected within each pipe. [5]

(d) Using the values obtained above, determine the pressure drop in each pipe if $l = 0.25\,\text{m}$. Does the inclusion of frictional effects significantly alter your answers to part (b)? [5]

(e) Hence, determine the bulk velocity in each pipe. [5]

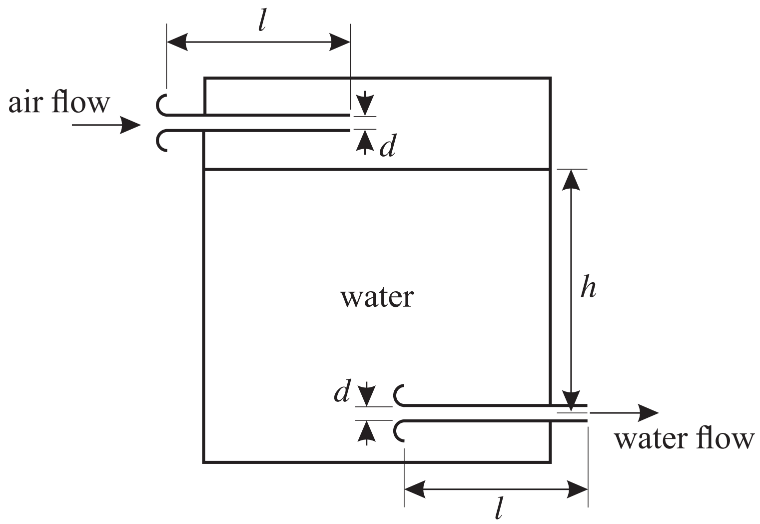

Fig. 3

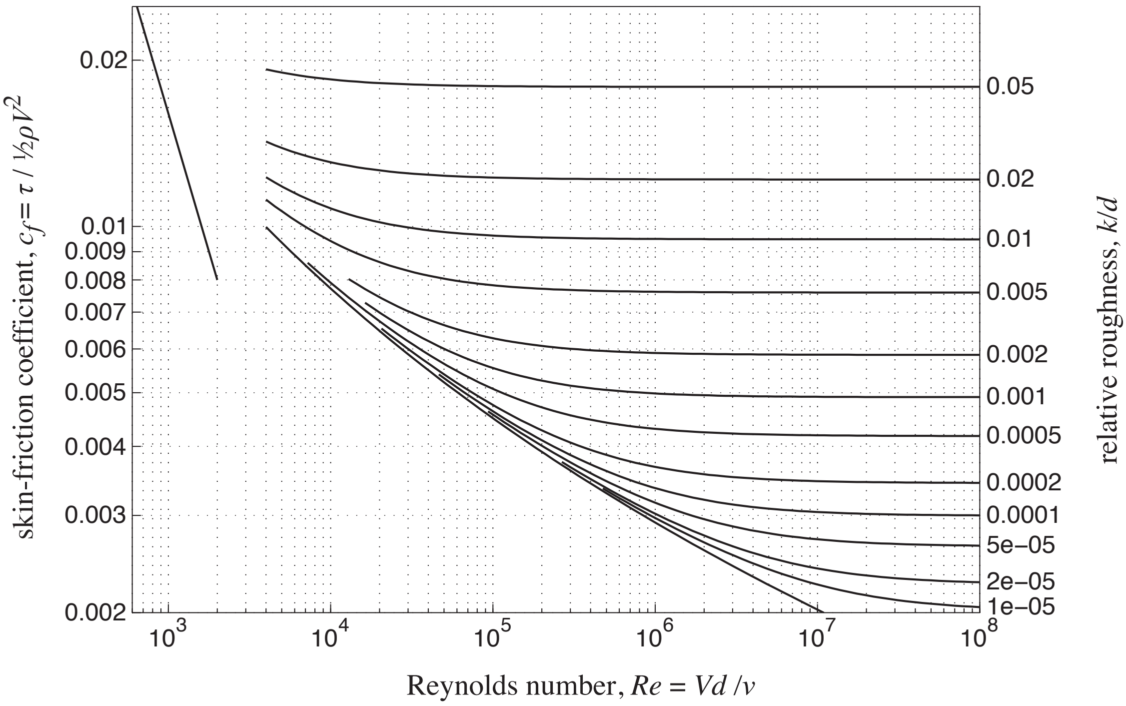

Fig. 4

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1 (a) –

(b) –

(c) $Re_d = 2778$, $Nu_d = 30.7$, $Bi = 0.8$ and $\tau = 1.4$ s

2 (a)(ii) 30.5 kJ kg^{-1}, 150 kJ kg^{-1}, (iii) $CoP_{HP} = 4.92$

(b) (ii) $CoP_{HP} = 3.35$, 32% drop (ii) 11.74 kg s^{-1}, 597 kW

3 (a) 0.91, 38.5%, 438 kg s^{-1}

(b) 370 °C, 403 MW, 34.5%

(c) ~149 bar

4 (a) –

(b) –

(c) –

(d) $-\frac{3}{16}A\rho_1 V_1^3$

5 (a) –

(b) –

(c) –

(d) –

6 (a) 0.078

(b) 1917, 24579

(c) 0.008, 0.01

(d) 3.8 Pa, 3920 Pa

(e) 1.98 m s^{-1}