Version NRA/5

EGT1
ENGINEERING TRIPOS PART IB

Wednesday 16 June 2021 13.30 to 15.40

Paper 4

THERMOFLUID MECHANICS

*This is an **open-book** exam.*

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the top sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
You have access to the Engineering Data Book, online or as your hard copy.

10 minutes reading time is allowed for this paper at the start of the exam.

The time allowed for scanning/uploading answers is 20 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

SECTION A

*Answer not more than **two** questions from this section*

1 A laminar boundary layer develops on the top surface of a thin stationary plate of length L as shown in Fig. 1. Upstream of the plate, the flow is uniform, with velocity U_∞ in the x-direction. The fluid is incompressible, Newtonian and has density ρ and uniform viscosity μ. Outside of the boundary layer, where $y > \delta$, $u_x = U_\infty$.

Within the boundary layer, the x-component of velocity can be approximated by

$$\frac{u_x}{U_\infty} = A\left(\frac{y}{\delta}\right)^n + B\left(\frac{y}{\delta}\right) + C$$

where A, B, C and n are constants.

(a) State the assumptions made for the flow within a boundary layer and explain why the velocity outside the boundary layer can be considered constant in this case. [3]

(b) Find values of A, B, C and n which match the no-slip condition at the wall and give no shear outside of the boundary layer. In addition, the values must satisfy the following expression for the skin friction coefficient, c_f, based on wall shear stress, τ_w,

$$c_f = \frac{\tau_\mathrm{w}}{\frac{1}{2}\rho {U_\infty}^2} = \frac{10/3}{Re_\delta} \quad \text{where} \quad Re_\delta = \frac{\rho U_\infty \delta}{\mu}$$ [7]

(c) Using mass conservation for an appropriate control volume, find the vertical displacement of a streamline between a point in the uniform upstream region and a point above the plate at $x = L$. Give your answer in terms of δ_L, the boundary-layer height at $x = L$. [5]

(d) By considering momentum conservation, show that the force on the top surface of the plate per unit span due to the fluid is of the form

$$F = K\frac{1}{2}\rho U_\infty^2 \delta_L$$

and determine the value of K. [10]

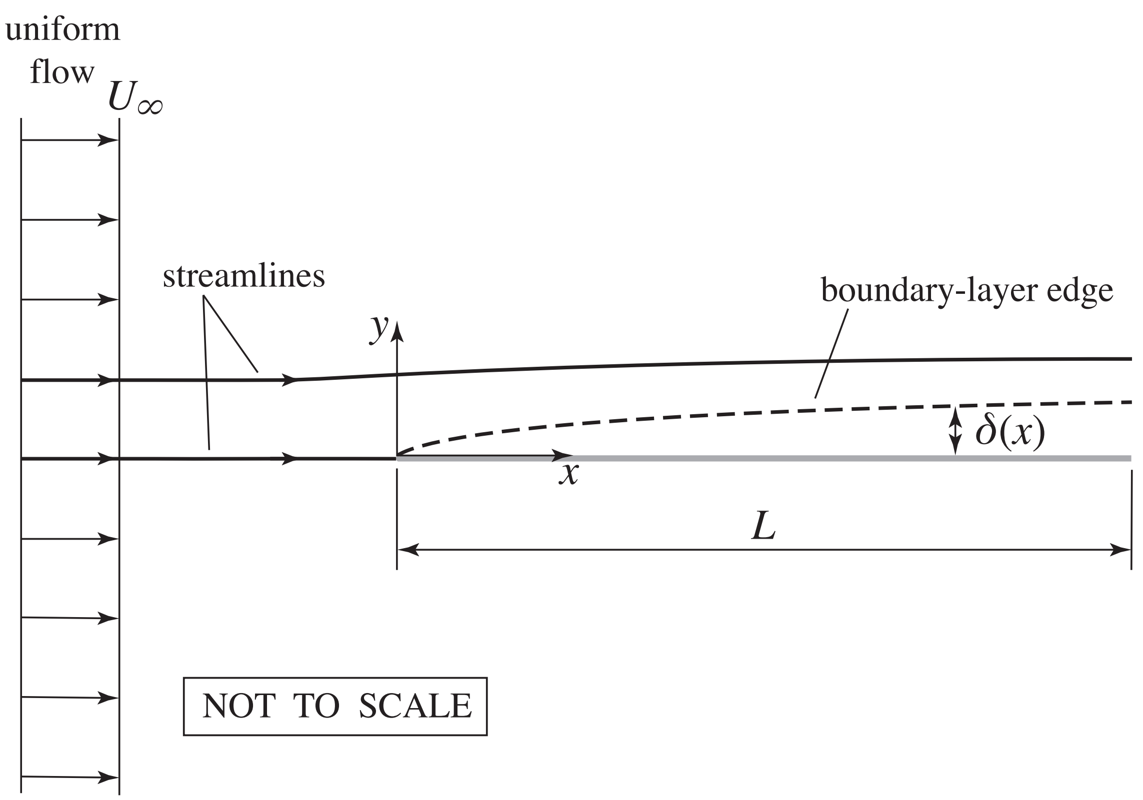

Fig. 1

(TURN OVER

2 Figure 2 shows an ejector pump, where high-velocity air is ejected into a duct by means of a thin-walled pipe of diameter d. Atmospheric air is drawn into the circular duct and mixes with the flow from the pipe. At position 1, the velocities of the streams entering from the atmosphere, V_{a}, and exiting the pipe, V_{p}, are uniform. The pressures at positions 1 and 2 are also uniform. Between positions 1 and 2, the duct has a constant diameter D. The flow mixes to a uniform state by the exit of the duct at position 2. The flow is incompressible with uniform density ρ. Friction on the walls of the duct and pipe can be neglected.

(a) Show that

$$V_2 = \alpha(V_{\mathrm{p}} - V_{\mathrm{a}}) + V_{\mathrm{a}} \quad \text{where} \quad \alpha = \left(\frac{d}{D}\right)^2$$

[3]

(b) Find the pressure, p_1, at position 1 in terms of the atmospheric pressure, p_{atm}, the velocity V_{a} and the density ρ. [3]

(c) By considering momentum conservation show that

$$V_2{}^2 = \alpha\left(V_{\mathrm{p}}^2 - V_{\mathrm{a}}{}^2\right) + \frac{V_{\mathrm{a}}{}^2}{2} \qquad [7]$$

(d) For the case where $\alpha = 0.5$, find an expression for the force applied by the flow on the duct in terms of ρ, V_{p} and D. Identify the direction in which this force acts. [10]

(e) Explain how and where this force is applied to the duct. [2]

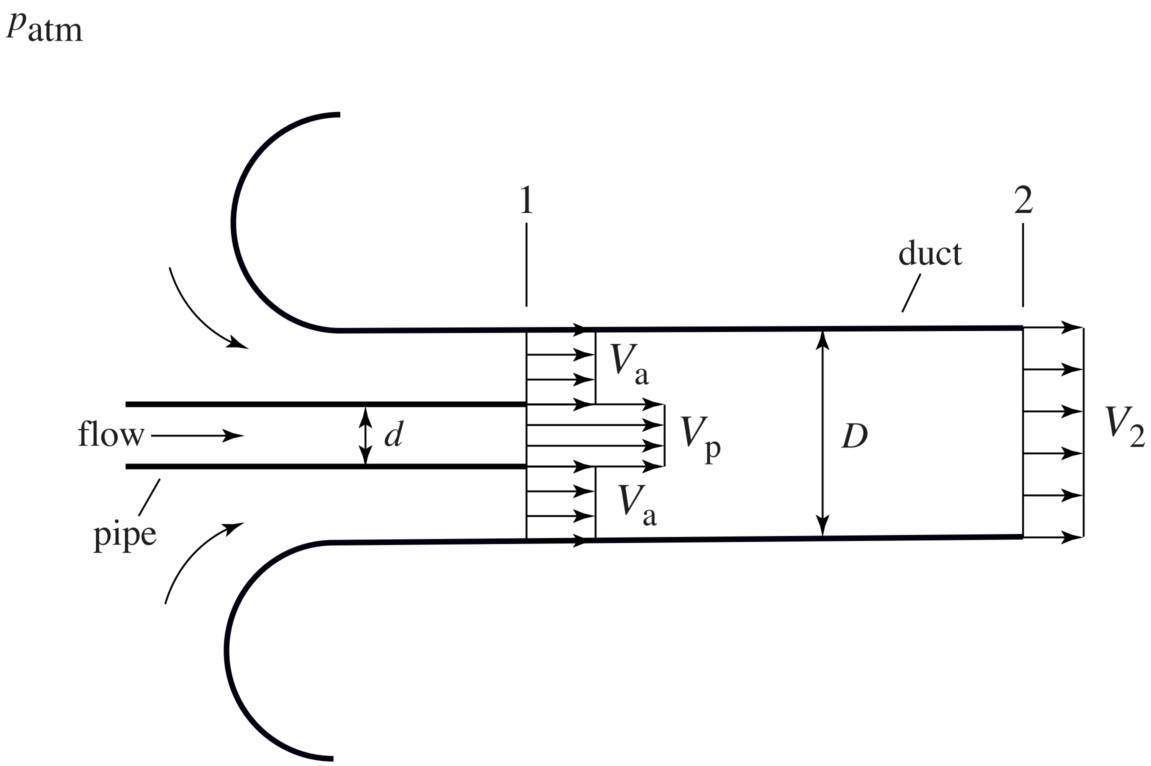

Fig. 2

(TURN OVER

3 Air of density ρ is drawn into a pipe by means of a pump, as shown in Fig. 3. This inlet pipe has a length $L = 1\,\mathrm{m}$, diameter $d = 0.05\,\mathrm{m}$ and is hydraulically rough. The flow leaving the pump exits to the atmosphere through a short pipe of the same diameter with negligible friction. The pipes are horizontal. Use air properties at standard atmospheric pressure and temperature for this question.

(a) The inlet pipe has a constant skin friction coefficient (based on bulk velocity V) of $c_f = \tau_\mathrm{w}/(\frac{1}{2}\rho V^2) = 0.015$, where τ_w is the wall shear stress. Neglecting inlet losses into the pipe and assuming fully developed flow, derive, from first principles, the value of the stagnation pressure loss coefficient given by the following expression

$$K_\mathrm{pipe} = \frac{p_{0\mathrm{A}} - p_{0\mathrm{B}}}{\frac{1}{2}\rho V^2}$$ [5]

(b) The velocity in the inlet pipe is of the order of $10\,\mathrm{m\,s^{-1}}$. Describe the type of flow expected and explain why the skin friction can be considered constant. [3]

(c) Show that if the streamlines at C are straight and parallel, then

$$\frac{p_{0\mathrm{C}} - p_\mathrm{atm}}{\frac{1}{2}\rho V^2} = 1.0$$ [2]

(d) The stagnation pressure rise across the pump is given by,

$$\frac{p_{0\mathrm{C}} - p_{0\mathrm{B}}}{\frac{1}{2}\rho V^2} = a + \frac{b}{V} + \frac{c}{V^2}$$

where $a = -1.13$, $b = -6.53\,\mathrm{m\,s^{-1}}$ and $c = 653\,\mathrm{m^2\,s^{-2}}$. Find the volumetric flow rate through the pipe. [10]

(e) Compute the rates of change of mechanical energy across the inlet pipe and pump. Is the change of mechanical energy in the pipe equal to the change across the pump? Give a physical explanation for your results. [5]

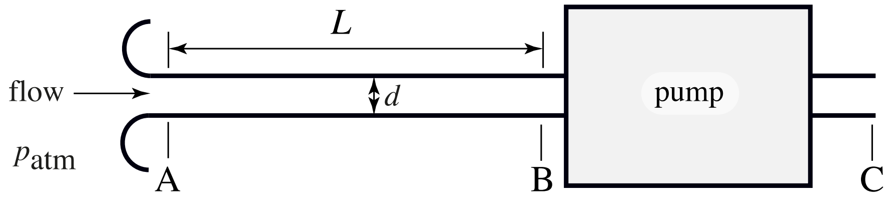

Fig. 3

SECTION B

*Answer not more than **two** questions from this section*

4 The cycle in Fig. 4 takes air from the environment at 1.0 bar and 288 K. The air is compressed to 20.0 bar before entering a combustor. The combustor has no pressure drop and an exit temperature of 1400 K. After the combustor, the air is expanded back to 1.0 bar in a turbine which is coupled to the compressor. The turbine and compressor have isentropic efficiencies of 0.85.

You should ignore the kinetic energy of the air and assume it has specific heat at constant pressure $c_p = 1.005\,\mathrm{kJ\,kg^{-1}K^{-1}}$ and a ratio of specific heats $\gamma = 1.4$ throughout the cycle.

(a) (i) Calculate the temperatures at compressor and turbine exit and sketch the cycle on a temperature-entropy (*T*-*s*) diagram. [4]

(ii) Calculate the 2$^{\text{nd}}$ Law efficiency of the cycle. [3]

(iii) Calculate the available power per unit mass flow that is lost in the turbine exhaust. [2]

(b) The pressure ratio of the cycle is doubled, but the heat input is held constant.

(i) Add the modified cycle to the *T*-*s* diagram of part (a)(i). [3]

(ii) Calculate the available power per unit mass flow that is lost in the turbine exhaust and explain the physical reason for the change compared to the original cycle. [3]

(iii) Calculate the 2$^{\text{nd}}$ Law efficiency of the cycle. [3]

(c) Explain, with the aid of a *T*-*s* diagram, how the increase in pressure ratio changes the loss of available power in the compressor and turbine. Comment on the physical mechanisms which cause this change. How would the effect of increasing pressure ratio change if the compressor and turbine efficiencies were raised? [7]

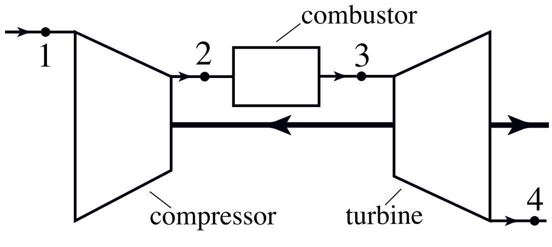

Fig. 4

5 (a) An opaque, diffuse, grey surface is shown in Fig. 5. The surface has area A, reflectivity ρ and emissivity ε. It is subject to incident irradiation G [$\mathrm{W\,m^{-2}}$]. It radiates to the surroundings with radiosity J [$\mathrm{W\,m^{-2}}$], with a net heat loss by radiation of $\dot{Q}$ [W].

(i) Considering the *electrical analogy* for radiative heat transfer, show that for a surface, the *resistance* between its equivalent black body emission, E_b, and radiosity, J, for a net heat loss, $\dot{Q}$, is given by

$$R_{\text{surface}} = \frac{1-\varepsilon}{A\varepsilon}$$ [2]

(ii) What are the analogous electrical circuit conditions for an insulated surface, and a black surface? [2]

(iii) Considering the net exchange of radiation, $\dot{Q}_{1-2}$ between two surfaces (i.e surface 1 and surface 2) of areas A_1 and A_2, show that the *resistance* between their radiosities, J_1 and J_2, is given by the expression

$$R_{\text{space}} = \frac{1}{A_1 F_{1-2}} = \frac{1}{A_2 F_{2-1}}$$

where F_{1-2} is the view factor between surface 1 and 2. [2]

(b) The floor of a rectangular cuboid shaped room has area $5\,\mathrm{m^2}$ and is to be held at $320\,\mathrm{K}$. The temperature of the floor is maintained by heating the opposite face (i.e. the ceiling) of the room. The floor and ceiling are opaque, diffuse, grey emitters and are perfectly insulated from behind, with emissivities of 0.2 and 0.8 respectively. The vertical walls of the room are held at a constant $300\,\mathrm{K}$ and can be considered as black bodies.

(i) The view factor between the floor and ceiling is 0.65, calculate the view factor between the ceiling and the vertical walls. [3]

(ii) Considering radiative heat transfer alone, using an analogous electrical circuit (or otherwise), calculate the required ceiling temperature and the necessary input power. [13]

(iii) A passing salesperson offers you a floor covering with a higher emissivity, of 0.95, comment on this suggestion. [3]

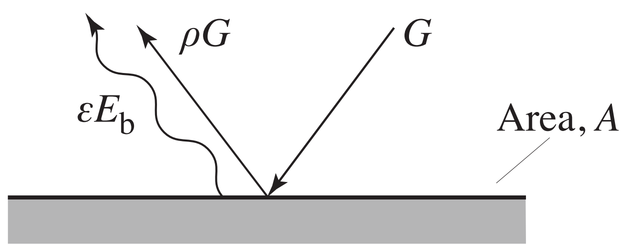

Fig. 5

(TURN OVER

6 A Rankine cycle consisting of a feed pump, boiler, turbine and condenser is used to recover waste heat from an industrial process. The cycle operates with a boiler pressure of 40 bar and condenser pressure of 0.05 bar. Steam exits the boiler at 500 °C and water leaves the condenser wet saturated. The feed pump can be approximated as reversible and adiabatic. The mass flow rate of steam is $10\,\mathrm{kg\,s^{-1}}$.

(a) (i) For a turbine isentropic efficiency of 80%, calculate the turbine power output, the dryness fraction at turbine exit, and the thermal efficiency of the cycle. [8]

(ii) Sketch the temperature-entropy (T-s) diagram of the cycle. [2]

(iii) Discuss, using sketches to illustrate your answer, the ways in which the thermal efficiency of the cycle could be improved. [5]

(b) The heat transferred to the boiler is taken from the industrial process at a constant temperature of 800 °C. The condenser rejects heat to the environment at a constant temperature of 20 °C.

(i) Determine the destruction of available power due to irreversibility in the boiler and the condenser. [7]

(ii) Discuss how this lost power could be extracted. [3]

END OF PAPER

Numerical Answers

1 (b) $A = -2/3$, $B = 5/3$, $C = 0$, $n = 5/2$
(c) $\Delta h = 5/14\ \delta_L$
(d) $K = 155/567$

3 (a) $K_{pipe} = 1.2$
(d) Volumetric flow rate = 0.0256 m^3/s
(e) pipe -3.15 W, pump 5.76 W

4 (a) (i) Compressor exit temperature 746.6 K, turbine exit temperature 715.6 K
(ii) $\eta_{2nd} = 0.478$
(iii) 165.9 kJ/kg/K

(b) (ii) 158.6 kJ/kg/K
(iii) $\eta_{2nd} = 0.478$

5 (b) (i) 0.35
(ii) 330.0 K, 600.9 W

6 (a) (i) Output power 10.29 MW, dryness fraction 0.941, $\eta_{thermal} = 0.31$
(b) power lost in boiler 104 kJ/kg, condenser 96.9 kJ/kg