Version NRA/3

EGT1
ENGINEERING TRIPOS PART IB

Friday 9 June 2023 9.00 to 11.10

Paper 4

THERMOFLUID MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

Answer not more than ***two*** *questions from this section*

1 A heating device with a humidifier passes a moist air mass flow rate of $\dot{m} = 1$ kg s^{-1} over a heater followed by evaporation coils, as shown in Fig. 1. The device is at constant pressure $p = 1$ bar with an inlet air temperature $T_1 = 10\,°\mathrm{C}$ and relative humidity $\phi_1 = 50\%$. The dry air may be considered a perfect gas with constant pressure specific heat capacity $c_p = 1.1$ kJ kg^{-1} K^{-1}. Superheated water vapour enthalpies may be taken from the saturation curve at the same temperature.

(a) Calculate the inlet specific humidity ω_1 and water vapour mass flow $\dot{m}_{\mathrm{v1}}$. [5]

(b) For a humid air flow temperature $T_2 = 70\,°\mathrm{C}$ downstream of the heater, calculate the required rate of heat transfer $\dot{Q}_{12}$. [4]

(c) At the exit of the evaporation coils the moist air has a relative humidity $\phi_3 = 100\%$ and temperature $T_3 = 28\,°\mathrm{C}$. Calculate the liquid water mass flow rate $\dot{m}_{\mathrm{w}}$. [5]

(d) Explain why there is no change in the total enthalpy flow through the evaporation coils when considering the humid air and liquid water collectively. Calculate the supplied water temperature to the nearest degree. [6]

(e) A second heater is added downstream of the evaporation coils to achieve a final temperature $T_4 = 40°\mathrm{C}$. Sketch a T-s diagram which indicates the thermodynamic states of the water vapour throughout the entire device (you do not need to calculate the entropy values). [5]

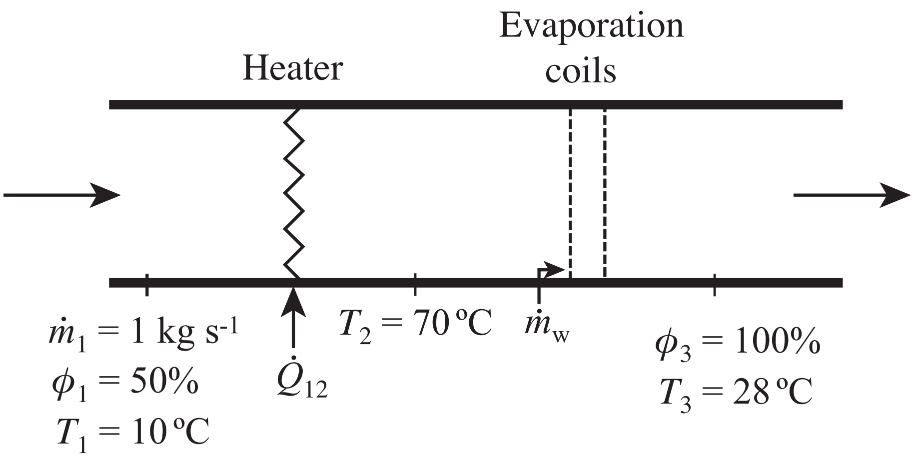
Heater
Evaporation coils
$T_2 = 70\,^\circ\mathrm{C}$
$\dot{m}_w$
$\dot{m}_1 = 1$ kg s^{-1}
$\phi_1 = 50\%$
$T_1 = 10\,^\circ\mathrm{C}$
$\dot{Q}_{12}$
$\phi_3 = 100\%$
$T_3 = 28\,^\circ\mathrm{C}$

Fig. 1

2 An energy storage process compresses a steady mass flow rate $\dot{m} = 1\,\mathrm{kg\,s^{-1}}$ of air from ambient at $T_\infty = 300\,\mathrm{K}$ and $p_\infty = 1\,\mathrm{bar}$ to a pressure $p_3 = 40\,\mathrm{bar}$. Heat is stored in an isothermal storage tank that has a temperature $500\,\mathrm{K}$. The air may be considered a perfect gas with constant pressure specific heat capacity $c_p = 1.1\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$ and heat capacity ratio $\gamma = 1.4$.

(a) During storage, air flows from stations 1a to 3a as shown in Fig. 2a. A reversible adiabatic compressor increases the air pressure to $p_{2\mathrm{a}} = 40\,\mathrm{bar}$. A heat exchanger then transfers heat from the flow to the temperature of the storage tank $T_{3\mathrm{a}} = 500\,\mathrm{K}$.

(i) Draw the storage process from station 1a to 3a on a T-s diagram. [4]

(ii) Calculate the temperature, $T_{2\mathrm{a}}$ at the compressor exit and the power supplied, $\dot{W}_{12\mathrm{a}}$. [4]

(iii) Calculate the rate of heat $\dot{Q}_{23\mathrm{a}}$ extracted from the flow by the heat exchanger. [3]

(iv) Calculate the change in available power $\Delta\dot{B}_{23\mathrm{a}}$ between inlet and exit of the heat exchanger. Discuss why the change in available power differs from the change in the rate of enthalpy flow $\Delta\dot{H}_{23\mathrm{a}}$ across the heat exchanger. [5]

(b) During power generation, $\dot{m} = 1\,\mathrm{kg\,s^{-1}}$ of air flows from stations 3b to 1b as shown in Fig. 2b. The stored compressed air is at ambient temperature $T_{3\mathrm{b}} = 300\,\mathrm{K}$, while the pressure remains $p_{3\mathrm{b}} = 40\,\mathrm{bar}$. At the heat exchanger exit, the temperature of the flow is the same as the thermal storage $T_{2\mathrm{b}} = 500\,\mathrm{K}$. A reversible adiabatic turbine extracts power from the flow to an exhaust pressure $p_{1\mathrm{b}} = 1\,\mathrm{bar}$.

(i) Calculate the change in available power $\Delta\dot{B}_{32\mathrm{b}}$ between inlet and exit of the heat exchanger and compare it to the change in available power $\Delta\dot{B}_{23a}$ during storage. [5]

(ii) Calculate the power generated by the turbine, $\dot{W}_{21\mathrm{b}}$, and determine the total storage efficiency of the process, $\eta = \dot{W}_{21\mathrm{b}}/\dot{W}_{12\mathrm{a}}$. Comment on how a change in storage temperature T_s would improve the efficiency. [4]

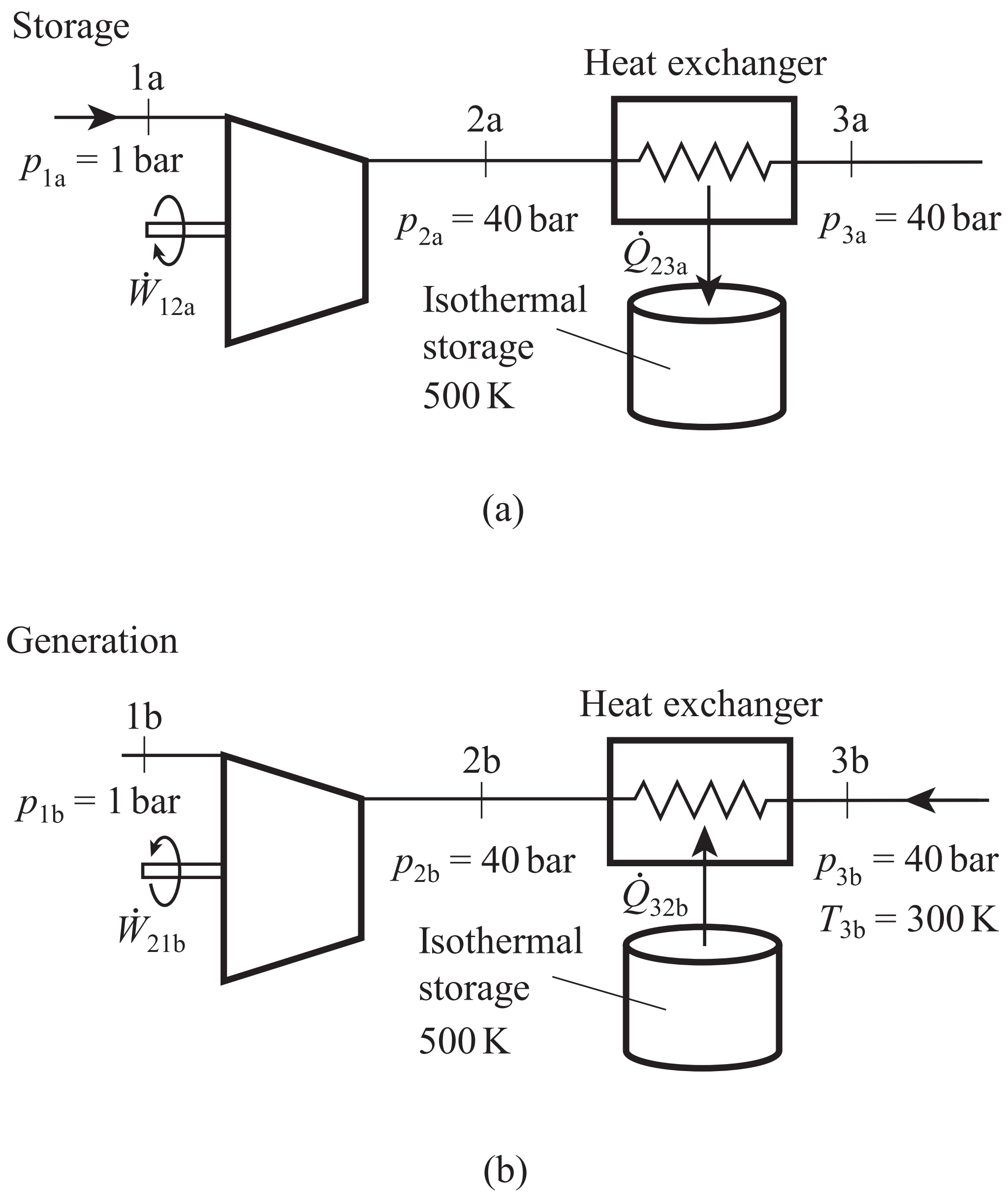

Fig. 2

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3 (a) Show that the resistance of radial heat flow through a tube of unit length, thermal conductivity λ, inner radius r_1 and outer radius r_2 is

$$\frac{1}{2\pi\lambda} \ln\left(r_2/r_1\right).$$ [3]

(b) A fluid of thermal conductivity $0.05\,\mathrm{W\,m^{-1}\,K^{-1}}$, dynamic viscosity $0.3\,\mathrm{mPa\,s}$ and heat capacity $4.23\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$ flows through a tube with a mass flow rate of $0.01\,\mathrm{kg\,s^{-1}}$. The tube has inner radius $4\,\mathrm{mm}$, outer radius $7\,\mathrm{mm}$ and thermal conductivity $0.1\,\mathrm{W\,m^{-1}\,K^{-1}}$. The outer surface heat transfer coefficient is $7.5\,\mathrm{W\,m^{-2}\,K^{-1}}$.

(i) Determine an appropriate internal heat transfer coefficient using the Databook, assuming constant wall temperature. [4]

(ii) Calculate the overall thermal resistance of a $1\,\mathrm{m}$ length of tube. [4]

(c) An engineer considers increasing the outer radius of the tube in part (b) to increase the thermal resistance. Assuming a constant external heat transfer coefficient:

(i) explain why this may not work and determine the worst case; [5]

(ii) use an iterative method to estimate the minimum outer radius required to increase the resistance. [5]

(d) Consider the assumption of constant external heat transfer coefficient in part (c). Neglecting all other changes, what error in external heat transfer coefficient would arise from a doubling in radius when the external flow field is driven by:

(i) natural convection with a Nusselt number, Nu, which scales with Grashof number, Gr, as

$$Nu \sim Gr^{1/3};$$ [2]

(ii) forced convection with Nusselt number which scales with Reynolds number, Re, as

$$Nu \sim Re^{4/5}.$$ [2]

SECTION B

*Answer not more than **two** questions from this section*

4 Euler's equations, in vector form and excluding gravity, can be written as

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\, \mathbf{v} = -\frac{1}{\rho} \nabla p.$$

(a) Find an expression for the gradient of total pressure $\nabla \left(p + \frac{1}{2} \rho \, (\mathbf{v} \cdot \mathbf{v}) \right)$ in a steady, inviscid, incompressible flow in terms of the vorticity vector $\boldsymbol{\omega} = \nabla \times \mathbf{v}$. You should use the identity $(\mathbf{v} \cdot \nabla)\, \mathbf{v} = (\nabla \times \mathbf{v}) \times \mathbf{v} + \frac{1}{2} \nabla \, (\mathbf{v} \cdot \mathbf{v})$. [5]

(b) A two-dimensional, steady, inviscid, incompressible flow has velocity components:

$$u(x, y) = A \sin(2\pi x/L) \cos(2\pi y/L);$$
$$v(x, y) = -A \cos(2\pi x/L) \sin(2\pi y/L);$$

where u and v are the x and y components of velocity respectively and A is a constant.

(i) Prove that the velocity field is consistent with conservation of mass. [5]

(ii) Determine if the flow is irrotational. Note that the vorticity is given by

$$\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$ [5]

(iii) Sketch the streamline pattern for this flow. Suggest whether the Bernoulli equation can be applied between any two points in this flow. Explain your reasoning. [3]

(c) The pressure for the flow described in (b) is given by

$$p(x, y) = B + \rho C \left[\cos(4\pi x/L) + \cos(4\pi y/L) \right],$$

where B and C are constants.

(i) Determine the units of A, B and C. [2]

(ii) By considering the x-momentum component of Euler's equations, find a relationship between A and C. [5]

5 A closed-loop duct includes a pump and a porous screen, as illustrated in Fig. 3. A bleed pipe is connected at an angle θ to the duct close to the pump inlet and exit.

The duct has a constant cross-sectional area A. The volumetric flow rate through the pump is Q_0. Downstream of the pump the bleed pipe extracts a small amount of flow αQ_0 and returns it to the duct upstream of the pump. A valve on the bleed pipe controls the amount of bleed that is extracted. The main flow is re-circulated via a porous screen in the duct.

The pressures and velocities at a, b, c and d are uniform across the duct. The flow is incompressible and gravity can be neglected. Friction on the duct walls can also be neglected (but not elsewhere).

(a) Find the velocities V_a and V_d as a function of Q_0, α and A. [3]

(b) The bleed pipe has a constant area. By considering the two control volumes indicated in the diagram show that

$$p_d - p_a = p_c - p_b.$$ [5]

(c) The porous screen has a total pressure loss coefficient, K, based on dynamic pressure at the screen. Express your answers below in terms of α, K, Q_0, ρ and A.

(i) Find a relationship for the total pressure rise across the pump Δp_t. [5]

(ii) Find the power delivered by the pump to the flow and the rate of loss of mechanical energy across the porous screen. [5]

(iii) Find the rate of loss of mechanical energy due to the bleed flow. At what value of α is the loss due to the bleed maximised if the pump operates at a constant flow rate? [4]

(iv) Identify, with appropriate sketches of the flow, the mechanisms driving the loss in each component and where they occur. [3]

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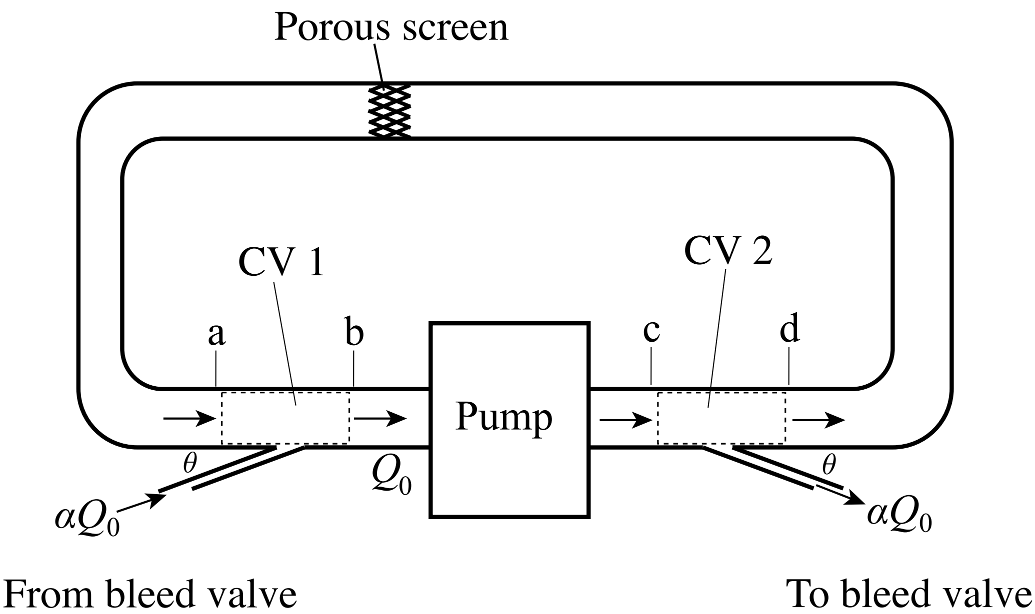

Porous screen
CV 1
CV 2
a
b
c
d
Pump
θ
θ
Q_0
αQ_0
αQ_0
From bleed valve
To bleed valve

Fig. 3

6 The flow in a stationary pipe is incompressible and steady, with uniform density ρ and dynamic viscosity μ. The pipe is held vertically and gravity g acts on the flow. The flow is laminar, fully developed and parallel to the axis of the pipe as indicated in Fig. 4a.

(a) Draw a control volume for a small annular element of fluid within the pipe, labelling the forces due to both pressure p and shear stress τ. Show that

$$\frac{\mathrm{d}(r\tau)}{\mathrm{d}r} = r\left(\frac{\mathrm{d}p}{\mathrm{d}z} - \rho g\right).$$ [5]

(b) The flow enters and leaves the pipe as a parallel stream as indicated in Fig. 4b. The surrounding atmospheric pressure p_{atm} is uniform. Explain why $\mathrm{d}p/\mathrm{d}z = 0$. [2]

(c) Find the wall shear stress τ_{w} in terms of the pipe radius R, as well as ρ and g. [3]

(d) Find the velocity profile $u(r)$ in terms of r, R and the velocity at the centre of the pipe U_0. [5]

(e) The pipe has radius $R = 1\,\mathrm{mm}$ and carries water with density $\rho = 1000\,\mathrm{kg\,m^{-3}}$ and dynamic viscosity $\mu = 1.14 \times 10^{-3}\,\mathrm{kg\,m^{-1}}\,s^{-1}$. Find the bulk flow speed U_{b} and the value of the skin friction coefficient $c_f = |\tau_{\mathrm{w}}|/(\frac{1}{2}\rho U_{\mathrm{b}}^2)$. [4]

(f) The pipe length is $10\,\mathrm{m}$. Neglecting the mass of the pipe, find the force necessary to hold the pipe in place. [2]

(g) Confirm whether or not the Reynolds number is consistent with flow in the laminar regime. Explain how the analysis above could be changed to model a turbulent flow. [4]

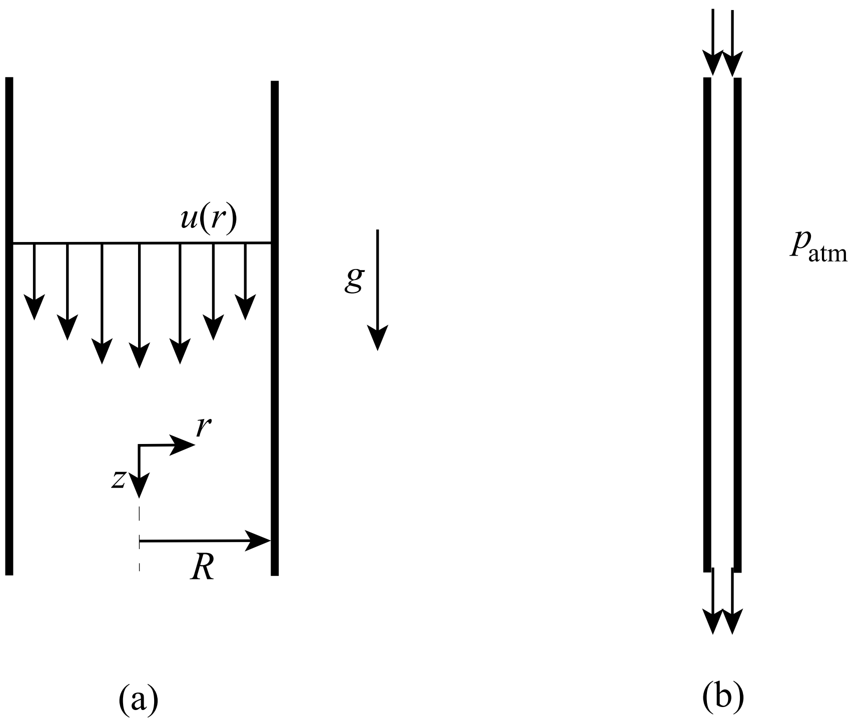

Fig. 4

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2P4 2023 Numerical Answers

Q1 (a) 0.00383, 0.00382 kg/s
(b) 66.15 kW
(c) 0.0205 kg/s
(d) 70 °C

Q2 (a) (ii) 860.7 K, -616.8 kW
(iii) -396.8 kW
(iv) -217.4 kW
(b) (i) 51.4 kW
(ii) 358.3 kW, 58%

Q3 (b) (i) h = 500 W/(m^2 K)
(ii) R_{th} = 4 K/W (per unit length of tube)
(c) (i) 13.3 mm
(ii) 30.3 mm
(d) (i) no error
(ii) 13% reduction

Q4 (c) (ii) $C = A^2/4$

Q5 (c) (iii) $\alpha = 1/3$

Q6 (e) U_b = 1.075 m/s, c_f = 0.0085
(f) F = 0.30 N
(g) Re = 1886