Version MCS/5

EGT1
ENGINEERING TRIPOS PART IB

Thursday 8 June 2023   2 to 4.10

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than **two** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Supplementary pages: two extra copies of Fig. 3 for Question 2
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

SECTION A

Answer not more than **two** *questions from this section*

1 (a) Show that the transfer function relating the output voltage v_o to the input voltage v_i for the operational amplifier circuit shown in Fig. 1 is given by

$$K(s) = -\frac{R_1C_1s + 1}{R_1C_2s}$$

[Assume the amplifier to be ideal.] [6]

(b) An amplifier with constant gain $-k$ in series with the amplifier circuit of Part (a) is used to control the plant

$$G(s) = \frac{1}{(Ts + 1)^3}$$

in the feedback arrangement of Fig. 2 where T is a positive constant. The circuit parameters are selected so that $R_1C_1 = T$ and $R_1C_2 = 1$. Let $L(s) = -kK(s)G(s)$.

(i) Find the frequency $\omega_1 > 0$ where $\angle L(j\omega_1) = -\pi$ rad. [3]

(ii) Find the frequency $\omega_2 > 0$ where $\angle L(j\omega_2) = -5\pi/6$ rad. [4]

(iii) Show that the feedback system of Fig. 2 is asymptotically stable with a phase margin of 30° when

$$k = \frac{4}{3\sqrt{3}T}$$

[4]

(iv) With k chosen as in Part (b)(iii) find the gain margin of the feedback system in decibels to 3 significant figures. [4]

(v) With k again chosen as in Part (b)(iii) a pure time delay D is introduced into the feedback loop. Find the ratio D/T to 3 significant figures for which the feedback system is marginally stable. [4]

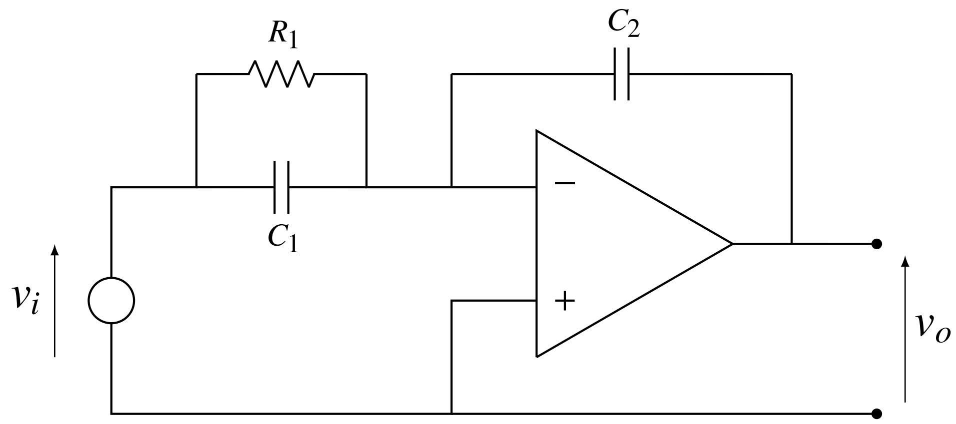


Fig. 1

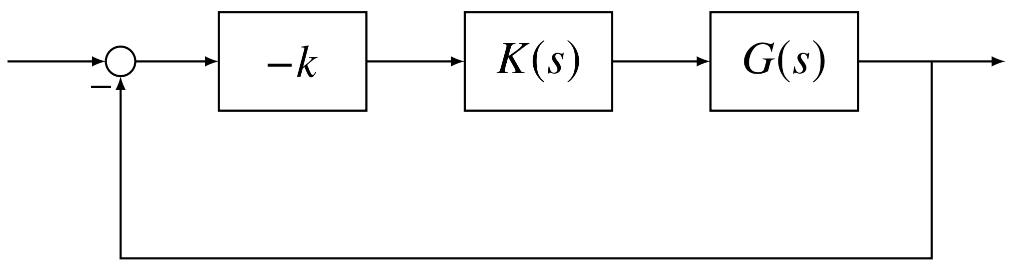


Fig. 2

2 A low-order linear model of a stable lightly-damped structure has a transfer function from an actuator to a sensor in the form

$$G(s) = \frac{a(1 + 0.4T_1 s + T_1^2 s^2)(1 + T_3 s)}{s(1 + 0.2T_2 s + T_2^2 s^2)(1 + T_4 s)}$$

The Bode diagram of $G(s)$ is shown in Fig. 3.

(a) Estimate the values of a, T_1, T_2, T_3 and T_4, illustrating your reasoning on a copy of Fig. 3. [6]

(b) A proportional gain $k > 0$ is employed in the standard negative feedback configuration. Using Fig. 3:

(i) explain why the feedback system is asymptotically stable with any $k > 0$; [3]

(ii) find the two values of k for which the system has a phase margin of 45°. [4]

(c) A controller of the form

$$K(s) = \frac{10(s + 1)}{s + 10}$$

is selected.

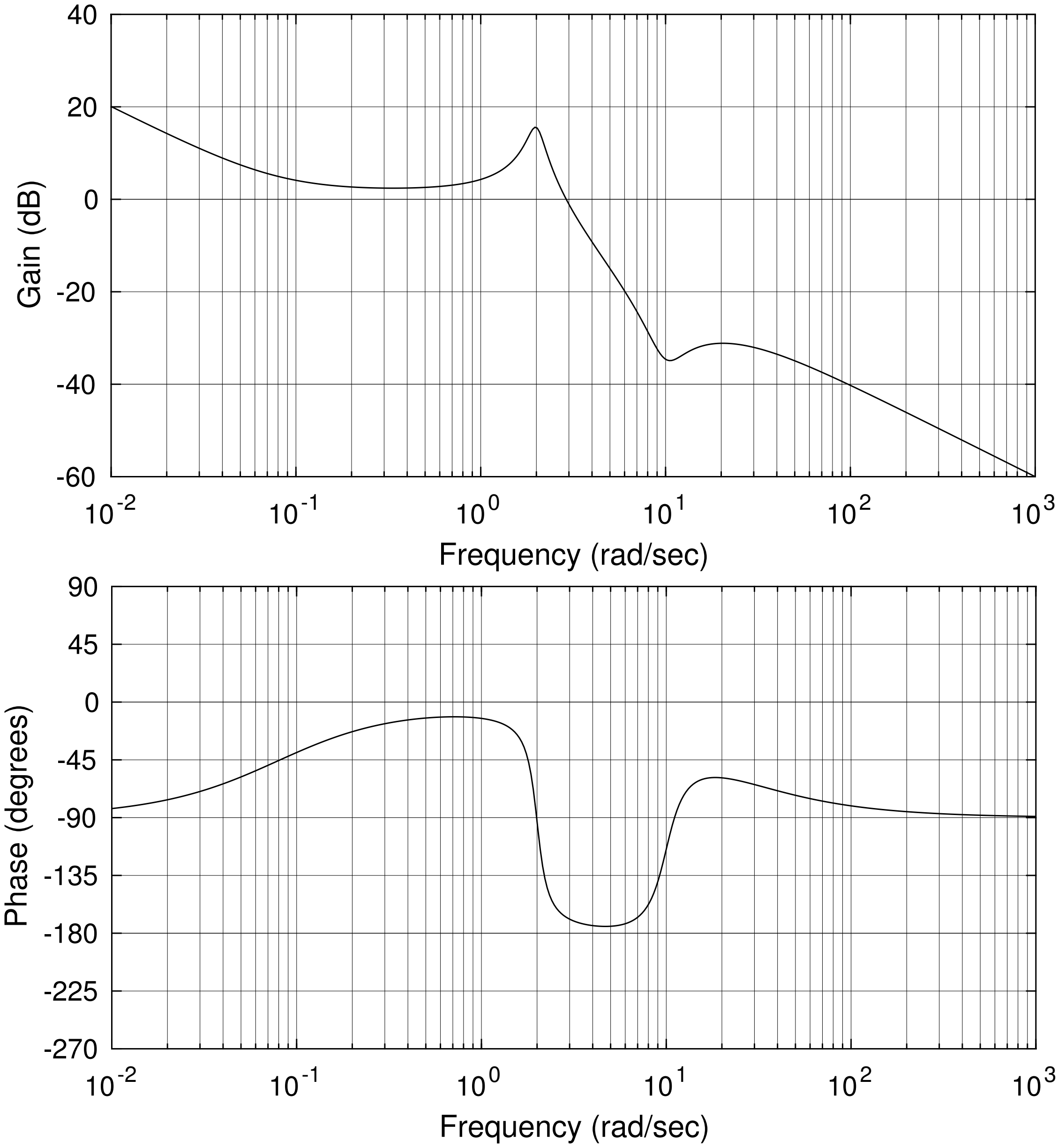
(i) Sketch the Bode diagram of both $K(s)$ and $G(s)K(s)$ on a further copy of Fig. 3. [5]

(ii) Estimate the phase margin when unity gain negative feedback is used to control the system together with this controller. [4]

(iii) Explain why there might be practical drawbacks to using a substantially larger loop gain to control this sytem. [3]

Two additional copies of Fig. 3 are attached to the back of this paper. These should be detached and handed in with your answers.

(cont.



40
20
0
-20
-40
-60
Gain (dB)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)
90
45
0
-45
-90
-135
-180
-225
-270
Phase (degrees)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/sec)

Fig. 3

3 A hydraulic control system consists of three components: a servovalve, an actuator and a load. The control system is to be analysed in terms of its incremental deviations from an operating point. The operation of the servovalve is described by the equation

$$q(t) = k_1 u(t) - k_2 p(t)$$

where $u(t)$ is the valve position input, $p(t)$ is the hydraulic pressure of fluid delivered to the actuator, $q(t)$ is the flow rate, and k_1, k_2 are positive constants. The actuator obeys the equation

$$q(t) = A\frac{dy}{dt} + k_3\frac{dp}{dt}$$

where $y(t)$ is the position of the load, and A (piston area) and k_3 are positive constants. The dynamics of the load satisfy

$$m\frac{d^2y}{dt^2} + c\frac{dy}{dt} = Ap(t) - f_L(t)$$

where m and c are positive constants and $f_L(t)$ is a disturbance force on the load.

(a) Draw a block diagram of the control system. [Hint: you may find it helpful to first eliminate q from the equations, to take p as the output of the actuator block, and to indicate y and $\dot{y}$ separately in the diagram.] [6]

(b) Let $v(t) = \dot{y}(t)$. Find the transfer function from u to v (with $f_L = 0$) and the transfer function from f_L to v (with $u = 0$) and explain why they are asymptotically stable. [6]

(c) Without performing detailed calculations describe qualitatively the response of v and y to a step input at u or f_L after the initial transients have decayed. [4]

(d) A proportional feedback control is applied to the system in the form

$$u(t) = k(r(t) - y(t))$$

where $r(t)$ is the reference input and k is a constant gain. Without performing detailed calculations explain why the closed-loop system is asymptotically stable for sufficiently small $k > 0$. [Hint: you may find it helpful to consider the form of the open loop transfer function from u to y at low frequency with reference to the Nyquist stability criterion.] [4]

(e) With the feedback law of Part (d) for a stabilising k find the steady state gain from the disturbance f_L to the load position y. [5]

SECTION B

Answer not more than **two** *questions from this section*

4 (a) A function $f(t)$ has Fourier transform $F(\omega)$. Show that $\frac{df(t)}{dt}$ has Fourier transform $j\omega F(\omega)$. [5]

(b) Let $f(t) = \exp(-t^2/a^2)$ for a constant a. Determine $\frac{df(t)}{dt}$ and hence, using the definition of $F(\omega)$, show that the Fourier transform $F(\omega)$ satisfies the following relationship:

$$\frac{dF(\omega)}{d\omega} = -\frac{a^2\omega}{2}F(\omega)$$

[9]

(c) Hence show that:

$$F(\omega) \propto \exp(-a^2\omega^2/4)$$

[5]

(d) The function $f(t)$ in (b), with $a = 1$, is to be low-pass filtered prior to digital sampling. Determine the bandwidth of the ideal low-pass filter if 95% of the energy of $f(t)$ should be preserved at the filter output. [6]

(TURN OVER

5 (a) A continuous time signal $x(t)$ is multiplied by a periodic train of δ-functions $\delta_s(t)$:

$$\delta_s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

(i) The Fourier spectrum of $x(t)$ is known to be:

$$X(\omega) = \begin{cases} 1, & |\omega| < \pi\alpha/T \\ 0, & \text{otherwise} \end{cases}$$

Sketch the Fourier transform of the product $y(t) = \delta_s(t)x(t)$, for the range $|\omega| < 4\pi/T$, for (i) $\alpha = 0.75$ and (ii) $\alpha = 1.5$. Determine the range of values of $\alpha > 0$ for which $x(t)$ may be perfectly reconstructed from $y(t)$. You may assume ideal sampling and reconstruction filters. [6]

(ii) The sampled signal $y(t) = \delta_s(t)x(t)$ is passed through a linear filter whose impulse response is

$$\Lambda(t) = \begin{cases} (1 - |t|/T), & |t| \le T \\ 0, & \text{otherwise} \end{cases}$$

Describe how the resulting filtered output waveform $z(t)$ is related to the digital samples $x(kT)$ (for integer k) and sketch its Fourier transform for the range $|\omega| < 4\pi/T$, with $X(\omega)$ as in Part (i) and $\alpha = 0.75$. [7]

(b) An information signal $m(t)$ is modulated with carrier $\sin(2\pi f_c t)$ as

$$s(t) = m(t)\sin(2\pi f_c t)$$

The waveform $s(t)$ is transmitted. Assume that there is no noise, so that the received waveform $y(t)$ equals $s(t)$.

(i) Draw a block diagram of the receiver to recover $m(t)$ from $y(t)$ assuming that $m(t)$ is a baseband signal with bandwidth W, with $W \ll f_c$. Specify the cut-off frequencies and the gain of any filter used. You may assume that the receiver has a perfect copy of the carrier wave available. [6]

(ii) Now suppose that, unknown to the receiver, its version of the carrier wave has a *frequency deviation* of $\Delta = 0.1W$ from the transmitter's carrier, i.e., the receiver's copy of the carrier is $\sin(2\pi(f_c + \Delta)t)$. Determine the recovered signal if we use the receiver of Part (b)(i), assuming that the gain of the low-pass filter is constant in the frequency band $[-1.2W, 1.2W]$. [6]

6 Consider a baseband transmission system using Pulse Amplitude Modulation (PAM). The transmitted waveform is:

$$x(t) = \sum_k X_k\, p(t - kT)$$

where the information symbols $X_0, X_1, \ldots$ are drawn from a constellation, and $p(t)$ is a unit energy pulse waveform. At the receiver, the demodulator filters the received waveform using a filter with impulse response $p(-t)$ and samples the filter output $r(t)$ at times mT for $m = 0, 1, 2, \ldots$

(a) Assume that the received waveform is $x(t)$.

(i) Show that the signal at the filter output is $r(t) = \sum_k X_k\, g(t - kT)$, where [5]

$$g(t) = \int_{-\infty}^{\infty} p(u + t)p(u)\, du$$

(ii) What property should the pulse $p(t)$ have so that the sampled filter output $r(mT) = X_m$, for $m = 0, 1, 2, \ldots$ [3]

(iii) State two other desirable properties for the pulse $p(t)$. [3]

(b) Let $T = 10^{-6}$ s, and assume that the information symbols are drawn from the constellation $\{-7A, -5A, -3A, -A, A, 3A, 5A, 7A\}$, where $A > 0$ is a constant.

(i) What is the rate of transmission, in bits per second? [1]

(ii) Suppose that the received waveform is noisy, and the sampled filter output is given by

$$Y_m = X_m + N_m\,, \qquad \text{for } m \geq 0$$

where N_m is independent Gaussian noise, with mean zero and variance σ^2. For the optimum decision rule, compute the probability of detection error for each of the symbols, and hence compute the overall probability of error. Express your answer in terms of the ratio $\frac{A}{\sigma}$ and the Q-function, where $Q(u) = 1 - \Phi(u)$, and $\Phi(u)$ is the Gaussian cumulative distribution function. You may assume that all the constellation symbols are equally likely. [9]

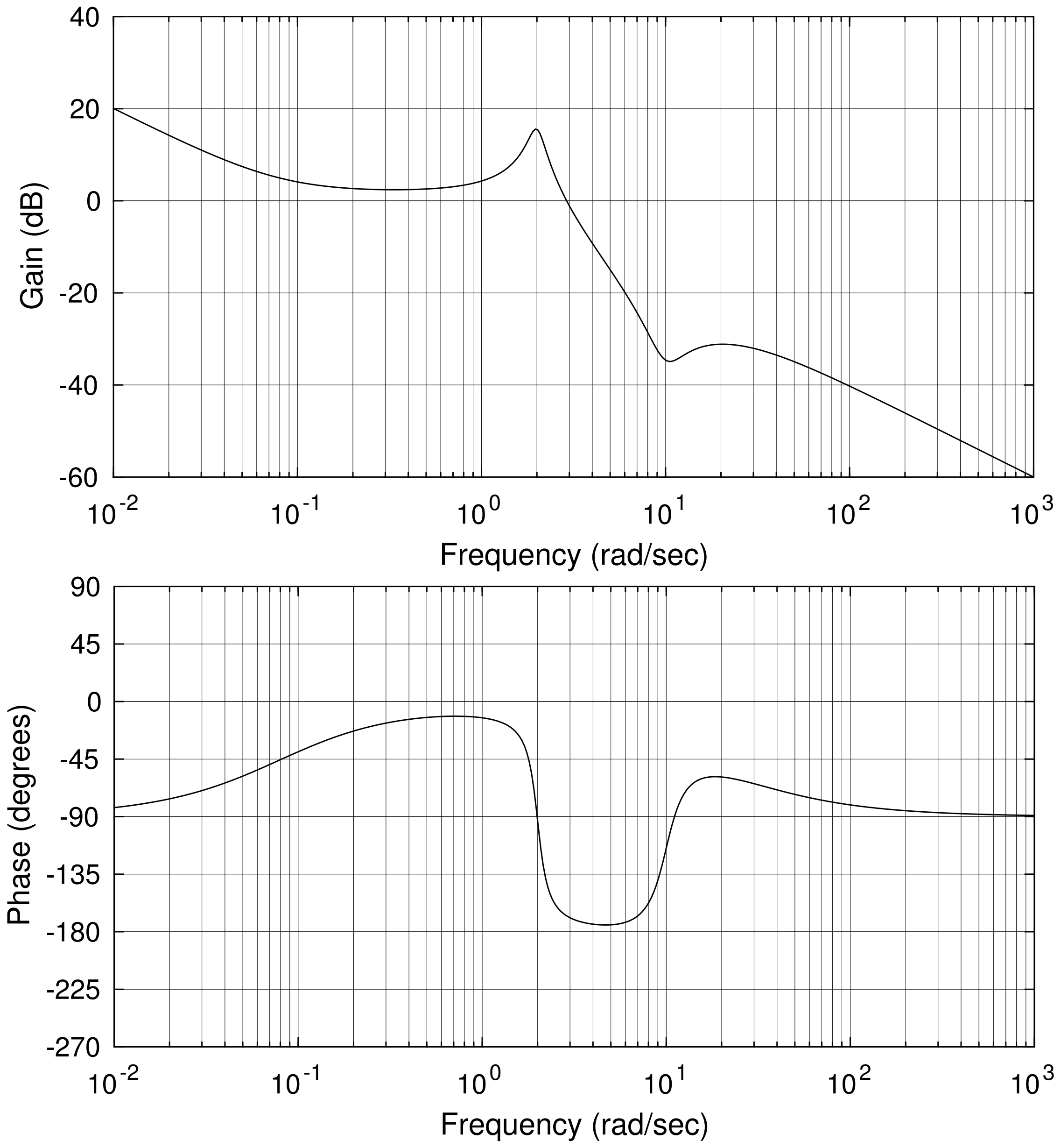
(iii) Briefly discuss the tradeoff (costs versus benefits) when the constellation parameter A is increased. You may use the bound $Q(u) \leq \frac{1}{2}e^{-u^2/2}$ for $u > 0$. [4]

END OF PAPER

THIS PAGE IS BLANK

Version MCS/5 Candidate Number:

EGT1
ENGINEERING TRIPOS PART IB
Thursday 8 June 2023, Paper 6, Question 2.



Extra copy of Fig. 3 for Question 2.

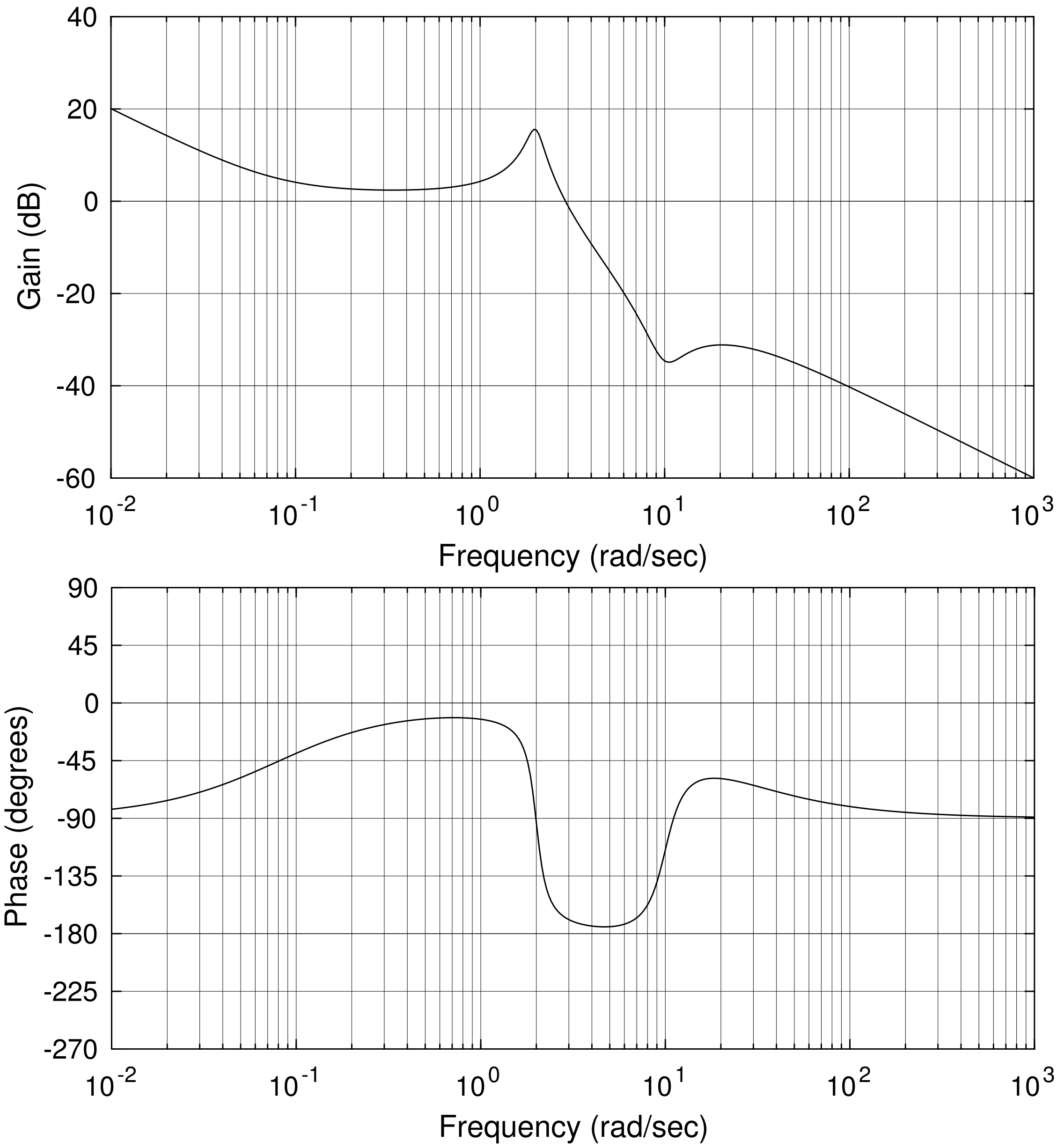
THIS PAGE IS BLANK

Version MCS/5 Candidate Number:

EGT1

ENGINEERING TRIPOS PART IB

Thursday 8 June 2023, Paper 6, Question 2.



Extra copy of Fig. 3 for Question 2.

THIS PAGE IS BLANK