Version CYB/7

EGT1
ENGINEERING TRIPOS PART IB

Friday 7 June 2019 9 to 11.40
9 to 10.40 Foreign Language Option

Paper 8

SELECTED TOPICS

Answer ***all*** *questions from Section A. In addition:*

If you are not taking the Foreign Language option, answer ***four*** *questions, taken from only two of Sections B–H. Not more than* ***two*** *questions from each section may be answered.*

If you are taking the Foreign Language option, answer ***two*** *questions from one of Sections B–H.*

All questions in Sections B–H carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

Answers to questions in each section should be tied together and handed in separately.

STATIONERY REQUIREMENTS

Single-sided script paper; separate answer sheet for Section A

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A: *The Engineer in Business*

*Answer **all** questions from this section, on the special answer sheet.*

1 If one observes that a search ad has high click-through rate and low conversion rate, which of the following inferences is correct:

(a) there are not many people searching for the bid keywords;

(b) the ad is attractive, but the landing page is not aligned with the users' interests;

(c) there are too many competitors bidding for the same keywords;

(d) the landing page is attractive, but the search ad is not aligned with the users' interests;

(e) none of the above? [2.5]

2 Which of the following is the second step in marketing strategic planning:

(a) developing a marketing mix strategy;

(b) 3C analysis;

(c) branding;

(d) market segmentation;

(e) none of the above? [2.5]

3 Which of the following is included in a firm's brand positioning statement:

(a) the competitive set;

(b) the brand associations;

(c) the brand equity;

(d) the brand identity;

(e) none of the above? [2.5]

4 Which one of the following is a customer insight:

(a) Chinese Americans are family oriented;

(b) women shampoo their hair six times a week;

(c) car consumers use colour to express themselves;

(d) baby's skin is the softest;

(e) none of the above? [2.5]

5 Which of the following is a source of rivalry among competitors in the Five Forces model:

(a) capital requirements for entering the industry;

(b) buyers' propensity to substitute;

(c) relative bargaining power of suppliers;

(d) excess capacity and exit barriers to leaving the industry;

(e) none of the above? [2.5]

6 Which of the following is a generic characteristic of a strategic resource for an organisation:

(a) low cost;

(b) hard to substitute;

(c) technologically advanced;

(d) sophisticated;

(e) none of the above? [2.5]

7 Which of the following is a characteristic of a successful differentiation strategy:

(a) easy to imitate;

(b) emphasis on scope economies;

(c) emphasis on experience effects;

(d) meaningful for the shareholders;

(e) none of the above? [2.5]

8 Two bus companies control a local market for bus services. If one company significantly increases the number of buses on its routes such that it incurs a large financial loss, this is likely to be an example of what anti-competitive practice:

(a) price discrimination;

(b) collusive pricing;

(c) competitive pricing;

(d) predatory pricing;

(e) none of the above. [2.5]

9 Sinopec is an oil refiner and petrochemical producer. If Sinopec set up a subsidiary to bid for oil production rights, this is an example of:

(a) growth by diversification;

(b) growth by backward vertical integration;

(c) growth by forward vertical integration;

(d) growth by merger;

(e) none of the above. [2.5]

10 Aerotec made components for Brazilian aircraft manufacturer Embraer in the 1980s. A good reason why Embraer took over Aerotec in 1987 might have been:

(a) Aerotec was a small firm, and small firms are always at a cost disadvantage relative to large firms;

(b) the merged firm had more layers of management and that increases the efficiency of decision making;

(c) Aerotec had a monopoly position in supplying Embraer, and Embraer needed to own Aerotec in order to avoid being exploited;

(d) the demand for air transport was falling in the 1980s and Aerotec was in danger of bankruptcy;

(e) none of the above. [2.5]

SECTION B: ***Civil and Structural Engineering***

Answer not more than ***two*** *questions from this section.*

11 Figure 1 shows a rectilinear reinforced concrete beam over a basement. The beam overhangs at one end in order to pick up a point load from a column close to the site boundary. The soil beneath the overhanging end of the beam has negligible load carrying capacity, so all design should assume that the beam is supported by the reactions R_1 and R_2 only. The beam supports a uniformly distributed load of 48 kN m^{-1} and a point load of 288 kN. These loads include the self-weight of the beam and all relevant partial safety factors for the load. The beam has a concrete depth of 900 mm and a breadth of 325 mm. The minimum required concrete cover to the longitudinal reinforcement is 40 mm. The concrete characteristic compressive strength f_{ck} is to be 40 MPa and all steel reinforcement is to have a characteristic yield strength f_{yk} of 500 MPa.

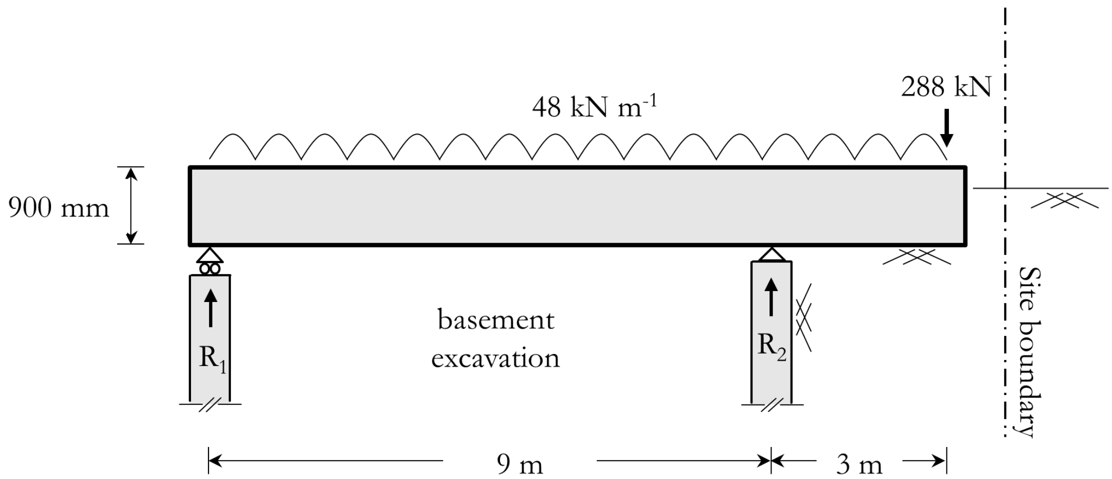

Fig. 1

(a) Sketch the bending moment and shear diagrams for the beam, marking salient values. [5]

(b) Assuming a singly reinforced section, design the required longitudinal reinforcement at the location of maximum hogging moment. Sketch the singly reinforced cross-section, indicating the size, number and arrangement of the bars. [7]

 (cont.

(c) For the region of highest shear, show whether shear reinforcement is or is not required. If shear reinforcement is required, design this reinforcement and sketch the cross-section, indicating the size, number and arrangement of the bars. [8]

(d) A less experienced engineer has helped you by performing the longitudinal reinforcement design for the region of maximum sagging. You discover later that their sagging reinforcement design used the load case given in Fig. 1. You know these loads to be made up of approximately half transient (i.e. live) load and half permanent load. Why might there be a problem with the sagging reinforcement design? What should you do? [5]

12 A sheet pile wall with one level of temporary props at its top supports a 6 m deep excavation in sand, as shown in Fig. 2a. The specific gravity of the sand is $G_S = 2.76$ and it has a void ratio $e = 0.85$. The critical state friction angle of the sand is $\varphi' = 30°$. It is assumed that the wall moves sufficiently to mobilise full active pressure on the retained side, while the lateral earth pressure coefficient K_{mob} on the excavation side is assumed to be a constant value.

(a) Compute the unit weight of the sand in dry and saturated conditions. [2]

(b) Assuming that the mobilised friction at the interface between the sheet pile wall and the sand on the passive side is $\delta = \varphi'/3$, compute the active and passive lateral earth pressure coefficients using Rankine's and Lancellotta's static solutions, respectively:

$$K_A = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}$$

$$K_P = \frac{\cos\delta}{1 - \sin\varphi'}\left[\cos\delta + \sqrt{\sin^2\varphi' - \sin^2\delta}\right] e^{2\Theta\tan\varphi'}$$

where

$$2\Theta = \sin^{-1}\left[\frac{\sin\delta}{\sin\varphi'}\right] + \delta$$

[3]

(c) Compute the depth of embedment required to obtain $K_P/K_{mob} = 1.7$, and the prop force and maximum bending moment in the wall for this embedment depth. [10]

(d) If the water table rises to dredge level (Fig. 2b), compute the depth of embedment required to obtain the same ratio K_P/K_{mob} as in (c) and the new values of the prop force and maximum bending moment in the wall. [10]

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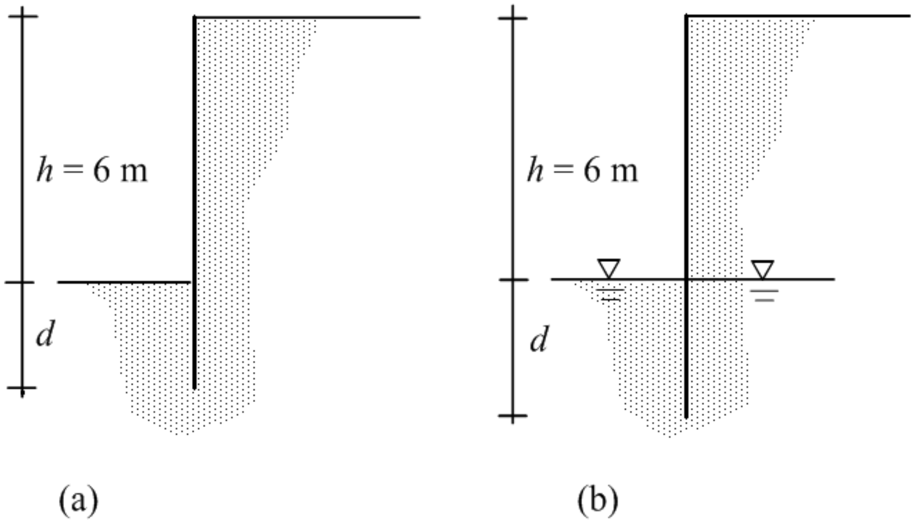

Fig. 2

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13 A fully saturated clayey silt deposit has specific gravity $G_S = 2.75$ and void ratio $e = 1.3$. The friction angle of the silt is $\varphi' = 26°$ and its undrained shear strength is $s_u = 60$ kPa.

(a) Compute the unit weight of the clay and its water content. [1]

(b) Assuming that the water table is 1 m below ground surface and that the clay is fully saturated to ground surface, compute the total vertical and effective stress at a depth of 15 m below ground level, point A in Fig. 3a. [3]

(c) For the soil at 15 m depth, if the vertical total stress is constant but the horizontal total stress decreases, estimate the total horizontal stress and the excess pore water pressure at failure in undrained conditions. [7]

(d) For the same conditions given in (c), estimate the total horizontal stress at failure in drained conditions. [1]

(e) A tunnel has to be constructed in the clayey silt deposit.

(i) Define the stability ratio in terms of vertical total stress, temporary support pressure and undrained shear strength, and explain its significance in the context of tunnel construction. [2]

(ii) Assuming that tunnel construction is an undrained process, would it be safe to excavate the tunnel at a depth of 20 m in open face mode? Justify your answer. [2]

(iii) Settlements are likely to be significant because the tunnel crosses the historical centre of a city. Why might masonry buildings be damaged? What protective measure(s) could be implemented to mitigate tunnelling induced damage? Describe briefly the way they work. [2]

(f) If 5 m of clayey silt are eroded, what are the total and effective vertical and horizontal stresses at the point A in Fig. 3(b), which is the same point as in section (b), now 10 m below ground surface? Draw qualitatively the variation of OCR and K_0 with depth. Assume that after erosion the ground water table is at the surface. [7]

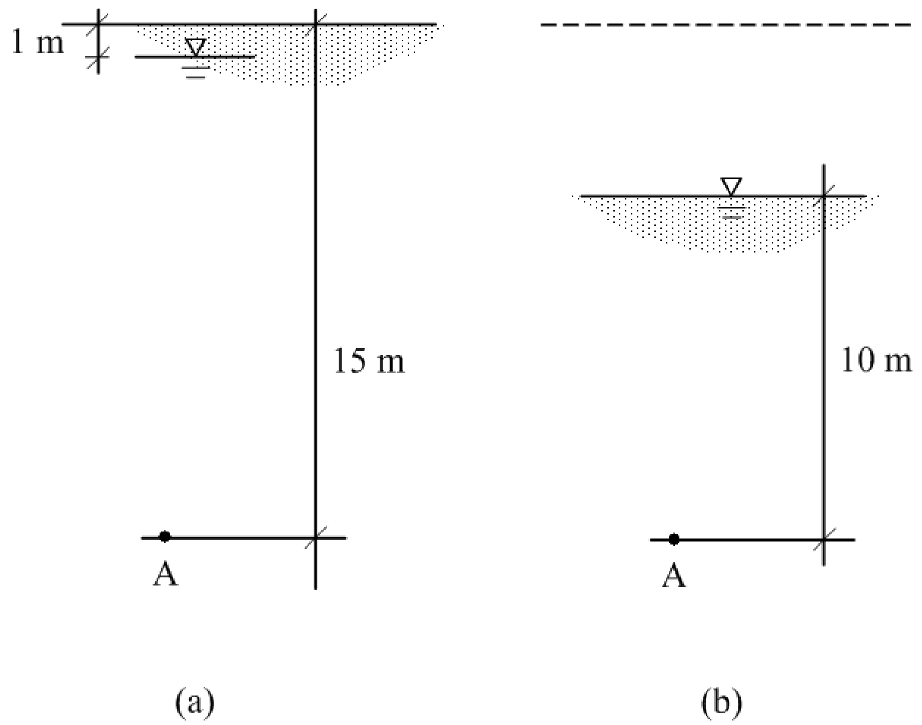

Fig. 3

SECTION C: *Mechanics, Materials and Design*

Answer not more than ***two*** *questions from this section.*

14 (a) Discuss design factors influencing the variation in geometry along the length of the blades of horizontal axis wind turbines. [4]

(b) Show that the incident wind speed V_0, the relative wind speed V_{rel}, the axial flow factor a and the flow angle ϕ for a wind turbine blade are related by

$$V_{rel} = \frac{V_0(1-a)}{\sin\phi}$$

[4]

(c) Consider a horizontal axis wind turbine with an upwind rotor and three blades. Each blade is modelled by three sections of uniform geometry, with aerodynamic and structural properties given in Table 1, connected to a rigid hub of radius $r = 2$ m. The blades rotate at an angular speed of 30 rpm at a wind speed $V_0 = 10$ m s^{-1}. The density of air is 1.225 kg m^{-3}. The flow angle, induction factors and coefficients of lift and drag for the three blade sections given in Table 1 have been calculated using blade element momentum theory.

(i) For the middle section of the blade ($r = 8$ to 14 m), derive the values given in Table 1 for the normal force F_N and tangential force F_T using the above data and other data given in Table 1. [7]

(ii) Calculate the power generated by the wind turbine. [3]

(d) The only contribution to the structural strength and rigidity of the blade is assumed to be from flat spars of breadth b and thickness t separated by a depth d, as illustrated in Fig. 4. The spar dimensions for the three sections are given in Table 1. Estimate the maximum stress in the spars associated with the aerodynamic loading. Detail any assumptions made. [7]

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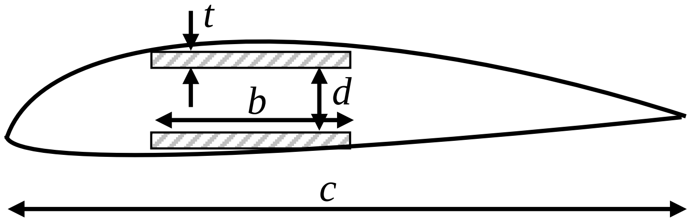

Fig. 4

Radial sections	$r = 2$ to 8 m	$r = 8$ to 14 m	$r = 14$ to 20 m
Chord c (m)	1.5	1.2	0.6
Twist θ (°)	20	5	0.5
Spar depth d (m)	0.2	0.15	0.07
Spar width b (m)	0.5	0.35	0.2
Spar thickness t (m)	0.02	0.01	0.005
Axial induction factor a	0.115	0.180	0.135
Angular induction factor a'	0.036	0.011	0.004
Flow angle ϕ (°)	28.5	13.2	9.16
Coefficient of lift C_L	0.936	0.900	0.950
Coefficient of drag C_D	0.0074	0.0075	0.0080
Normal force F_N (N m^{-1})	260	831	1018
Tangential force F_T (N m^{-1})	139	188	155

Table 1

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15 (a) For a certain wind turbine, the power coefficient C_p is given as a function of tip speed ratio λ as:

$$C_p = -0.005\lambda^2 + 0.08\lambda + 0.08 \quad \text{for} \quad \lambda < 16; \qquad C_p = 0 \ \text{otherwise.}$$

Determine the maximum power coefficient, $C_{p,max}$ and the tip-speed ratio at which it occurs, λ_{opt}. Hence sketch a graph of C_p versus λ. Use your graph to explain why modern wind turbines are usually variable speed devices. [4]

(b) The wind turbine of part (a) has a blade radius of 40 m, and the most common wind speed at the turbine site is 7 m s^{-1}. The turbine is coupled via a gearbox to an 18 pole, three-phase, star-connected induction generator. The generator is of wound rotor construction, with the slip-rings short-circuited. The generator stator windings are connected directly to the 3.3 kV, 50 Hz grid. The system is to operate at optimum tip-speed ratio at the most common wind speed. At this wind speed, find:

(i) the turbine output power, torque and angular speed; [3]

(ii) the gearbox ratio required. [2]

(c) The induction generator has the following equivalent circuit parameters:
$R_1 = 0.6\ \Omega$; $R_2' = 0.5\ \Omega$; $X_1 = X_2' = 2.2\ \Omega$; X_m and R_0 are both large enough to ignore. Find, using the simplified torque equation, the generator input torque, slip, phase current and power factor under the conditions of part (b). [4]

The following may be quoted without proof:

$P = 0.5 C_p \rho A v^3$, $\lambda = \dfrac{\omega R}{v}$, $T = \dfrac{3V^2 s}{\omega_s R_2'}$, and take ρ to be 1.23 kg m^{-3}.

(d) The government of a European country declares a target for the growth of its installed wind power capacity over the next decade. A consultant seeks to determine whether this is a sustainable development, using the Ashby methodology outlined in lectures.

(i) Briefly explain what is meant by the Triple Bottom Line, and define the associated Capitals. [4]

(ii) For the proposed expansion in wind power capacity, identify three potential positive consequences and three potential negative consequences that could be considered, stating which Capital they fall under. [4]

(iii) Choosing one of these consequences that can be readily quantified, outline suitable metrics for the analysis and the data that would be needed. [4]

16 (a) Discuss the following factors relating to use of composites in wind turbine blades:

(i) multi-directional composite fabrics are used for the shell of large blades; [3]

(ii) a range of different composite manufacturing processes are used for composite blades; [3]

(iii) unidirectional carbon fibre material is used for the spars in large wind turbine blades; [3]

(iv) steel is preferred to composite material in wind turbine towers. [3]

(b) Figure 5 illustrates a model of a wind turbine tower. The tower is split into two rigid sections, each of length L and with a point mass m at the centre. The two sections are connected by a rotational spring of stiffness k and the bottom section is connected to a rigid foundation by a rotational spring of stiffness $2k$. A tower head mass $2m$ is attached to the top of the upper section of the tower.

(i) Comment on the importance of including the tower head mass in the model. [2]

(ii) Estimate, in terms of m, k and L, the lowest resonant frequency of the system. [11]

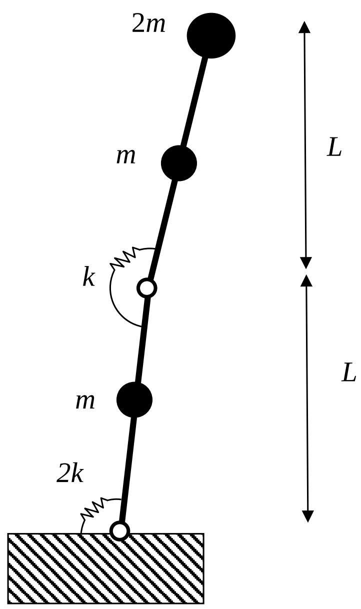

Fig. 5

SECTION D: *Aerothermal Engineering*

Answer not more than ***two*** *questions from this section.*

17 (a) Consider the design of a 2-shaft turbofan engine. The core has fixed mass flow, overall pressure ratio and turbine entry temperature. Assume the bypass and core jet velocities are equal and fixed. Assume there is no pressure drop across the combustor. Sketch a temperature-entropy diagram of the core flow through the engine. Describe how improving the compressor and turbine efficiencies would alter the bypass ratio of the turbofan engine and the diameter of the fan. Explain any other engine parameters that would change. [8]

(b) The core flow in a 2-shaft turbofan engine enters the core compressor with a stagnation temperature of 290 K and stagnation pressure of 0.70 bar. The compressor has an isentropic efficiency of 90% with 8 stages and a stagnation pressure ratio of 25. The mean blade radius is constant through the compressor and is 0.3 m. If the design work coefficient of all compressor stages is $\psi = 0.4$, calculate the rotational speed of the compressor in rotations per minute. [6]

(c) The Mach number at the compressor inlet plane is 0.7. The flow is in the axial direction at the inlet plane. The compressor has a mass flow rate of 40 kg s^{-1}. Calculate the height of the first blade row in the compressor. [6]

(d) The HP turbine that drives the compressor has an isentropic efficiency of 90%, an inlet temperature of 1900 K and a constant mean blade radius of 0.3 m. How many stages of HP turbine would you advise a designer to select? What practicalities should the designer consider when choosing the number of HP turbine stages? [5]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $c_p = 1005$ J kg^{-1} K^{-1}.

18 (a) An aircraft at cruise flies at Mach 0.8 at an altitude of 10,000 m. Calculate the stagnation pressure p_0 and the stagnation temperature T_0 at the inlet of the engines of the aircraft. What are the assumptions that are made in deriving the relationships between static and stagnation pressures, and static and stagnation temperatures? Determine the flight velocity of the aircraft. [6]

(b) The engines on the aircraft in part (a) have a combined total mass flow of 600 kg s^{-1} and a jet velocity of 380 m s^{-1}. Determine the net thrust and the gross thrust of the engines. Assume no loss in the inlet duct. [4]

(c) The non-dimensional mass flow $\tilde{m}$ and the non-dimensional thrust $\tilde{F}$ of an engine are defined as

$$\tilde{m} = \frac{\dot{m}\sqrt{c_p T_0}}{p_0 A_N} \quad \text{and} \quad \tilde{F} = \frac{F_G + p_a A_N}{p_0 A_N}$$

where A_N is the nozzle exit area, p_a is the ambient pressure and F_G is the engine gross thrust. Explain the significance of the term $F_G + p_a A_N$ in the expression for $\tilde{F}$. If the combined nozzle area of the two engines in part (b) is 3 m^2, calculate $\tilde{m}$ and $\tilde{F}$ for the engines when the aircraft is in cruise. [5]

(d) Derive a relationship for the non-dimensional net thrust $F_N/(p_a A_N)$ in terms of the non-dimensional thrust $\tilde{F}$, the non-dimensional mass flow $\tilde{m}$, the cruise Mach number M, the ratio of specific heats γ, the stagnation to static pressure ratio at the inlet of the engine p_0/p_a and the stagnation to static temperature ratio at the inlet of the engine T_0/T_a. [5]

(e) The aircraft increases its altitude. At the new altitude the stagnation pressure and stagnation temperature at the inlet of the engines are 0.216 bar and 244.3 K respectively. The flight Mach number is held constant. Assuming that the non-dimensional operating point is unchanged, calculate the total mass flow through the engine and the net thrust at the new cruise altitude. [5]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $c_p = 1005$ J kg^{-1} K^{-1}.

19 (a) Explain why passenger aircraft cruise at high altitude. Why do aircraft often increase their cruise altitude during a long flight? [5]

(b) The Breguet range equation for the distance, s, between the start and end of cruise is given by

$$s = \frac{V}{g}\frac{L/D}{sfc}\ln\left(\frac{W_{\text{start}}}{W_{\text{end}}}\right)$$

where g is the acceleration due to gravity, V is the flight speed, sfc is the thrust specific fuel consumption, L/D is the aircraft lift-to-drag ratio and W_{start} and W_{end} are the total aircraft weights at the start and end of cruise respectively. Use this to show that F', the mass of fuel needed to move a unit mass of payload by a unit distance, can be expressed as

$$F' = \frac{1}{s}\left(1 + \frac{W_e}{W_p}\right) \times \left[\exp\left(\frac{s\,g}{V}\frac{sfc}{L/D}\right) - 1\right]$$

where W_e is the empty weight of the aircraft and W_p is the weight of the payload. [5]

(c) State two ways in which airframe and engine design could be used to minimise F'. Discuss how the operation of the aircraft could be used to minimise F'. [5]

(d) An aircraft cruises at 10,000 m, where the ambient temperature is 223 K, at a Mach number of 0.85. The $L/D = 20$ and the $sfc = 1.7 \times 10^{-5}$ kg s^{-1} N^{-1}. The empty weight $W_e = 400$ tonne, the payload weight $W_p = 70$ tonne and the fuel weight $W_f = 250$ tonne at take-off and negligible when landing. Taking the weight of a typical passenger with baggage to be 100 kg, and assuming a combustion reaction of the form

$$C_8H_{18} + 12.5O_2 \rightarrow 8CO_2 + 9H_2O$$

calculate the amount of CO_2 produced per passenger kilometre flown. How does this figure compare with other types of transport? [10]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $c_p = 1005$ J kg^{-1} K^{-1}, and $g = 9.81$ m s^{-2}.

SECTION E: *Electrical Engineering*

Answer not more than ***two*** *questions from this section.*

20 (a) (i) How are the momentum and wavelength of a wave-particle related?

(ii) An electron microscope uses a 40 keV electron beam. What is the approximate theoretical resolution of the microscope? Ignore relativistic effects.

[5]

(b) (i) Consider a particle existing in the one-dimensional potential energy profile shown in Fig. 6. There exist three regions: region I defined as $-\infty < x \leq L_1$, region II defined as $L_1 < x \leq L_2$ and region III defined as $L_2 < x < \infty$. The particle has total energy E and mass m with $0 < E < U_1$. Write Schrödinger's equation (do not solve it) for the particle in region I, region II and region III.

(ii) If this particle has wavefunction ψ in region III, can the wavefunction have the form $\psi(x) = Ae^{\beta x}$? Here A and β are positive real constants. Explain your answer.

[10]

(c) (i) Explain what is meant by 'tunneling' in the context of quantum mechanics.

(ii) Name and explain one application of the phenomenon of tunneling.

(iii) A particle only exists in a one-dimensional region of space $0 \leq x \leq L$ where its wavefunction is $\psi(x) = Ax(L-x)$ with A being a real constant. Find the value of A.

[10]

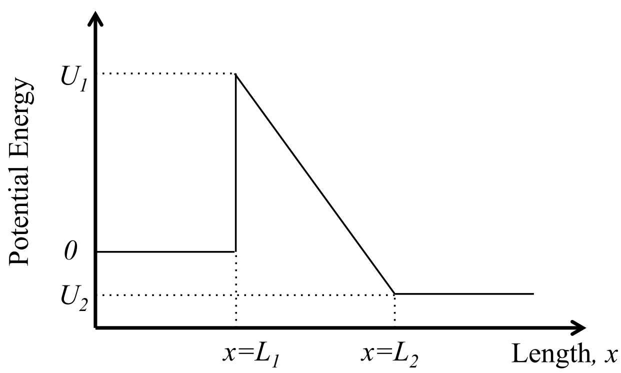

Fig. 6

21 (a) (i) A semiconductor has an intrinsic carrier concentration of 1×10^{10} cm^{-3}. Find the electron and hole concentration in the semiconductor when it is doped with 1×10^{15} cm^{-3} acceptor dopants. Also find the electron and hole concentration if the acceptor dopant concentration is 1×10^{10} cm^{-3}.

(ii) An n-type semiconductor of thickness 1×10^{-6} m is connected in series with a capacitor of capacitance 100 pF forming a series resistor-capacitor (RC) circuit. The film has equal length and width of 5×10^{-6} m. The semiconductor has an intrinsic carrier concentration of 1×10^{10} cm^{-3} and is doped with 2×10^{15} cm^{-3} of donor dopants. The electrons in the semiconductor have an effective mass of 1.6×10^{-31} kg and a mean collision time of 1×10^{-13} s. Find the time constant of the RC circuit.

[10]

(b) A metal-semiconductor junction, shown in Fig. 7, consists of a semiconductor with permittivity ε_s that is doped with acceptor dopants having a concentration $N(x) = N_0 e^{-x/\lambda}$. Here N_0 is the dopant concentration at the metal-semiconductor interface and λ is a constant defining an effective length. The semiconductor near the interface is depleted up to a width $x = d$. Assuming the electric field beyond this depletion region (i.e. $x > d$) is zero, find the electric field at $x = d/2$. Also find the potential difference between the points $x = d/2$ and $x = 0$. [10]

(c) For a MOSFET scaling factor k, how does the power consumption scale with

(i) constant voltage scaling?

(ii) constant field scaling?

[5]

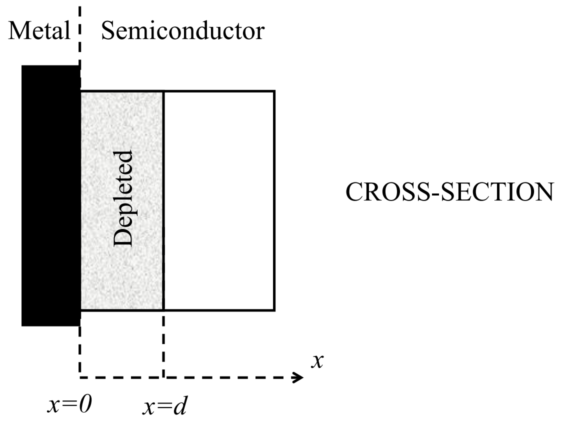

Fig. 7

22 (a) (i) What is the difference between the deposition techniques of 'sputtering' and 'e-beam evaporation'? Which technique is preferred to ensure better step coverage and why?

(ii) Describe the photolithography assisted fabrication process flow required to fabricate the structure shown in Fig. 8. The structure consists of two 50×10^{-6} m by 50×10^{-6} m pads of chrome film on glass substrate. The pads are separated by a distance of 20×10^{-6} m.

[10]

(b) Metal oxides are a versatile class of materials that are used in integrated circuits. Answer the following.

(i) Describe an application of indium tin oxide.

(ii) One of the high-k dielectrics compatible with silicon is hafnium oxide. Why is a high k dielectric desirable?

(iii) In a light emitting diode (LED) display, the pixel current driver is a gallium indium zinc oxide thin film transistor (TFT). For full brightness, the TFT is expected to drive 40 μA of current. The data voltage supplied to the pixel to achieve full brightness is 7 V. The device parameters of the TFT are as follows: mobility of 20 cm^2/Vs, insulator capacitance per unit area of 20 nF/cm^2, threshold voltage of 2 V. Find the aspect ratio required for the TFT to achieve these specifications during full brightness operation.

[10]

(c) (i) Magnetic memory disks can either be read 'inductively' or by using the concept of 'magnetoresistance'. Briefly explain the difference between the two approaches of reading.

(ii) X-rays of wavelength 1.5×10^{-10} m undergo a first order reflection at a glancing angle of 10° from a crystal. Find the spacing of the atomic planes in the crystal.

[5]

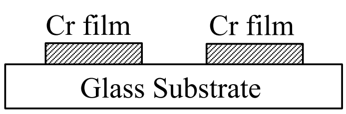

Fig. 8

SECTION F: *Information Engineering*

Answer not more than **two** *questions from this section.*

23 (a) A greyscale image, $I(x,y)$, is to be smoothed by *low-pass* filtering with a Gaussian filter, $G(x,y)$, and differentiated as part of the edge detection process.

(i) Explain why smoothing is necessary before differentiation. [2]

(ii) Give an expression for computing the intensity of a smoothed pixel, $S(x,y)$, in terms of two discrete 1-D convolutions. [3]

(iii) By first considering the Taylor series expansion of $S(x,y)$, show that approximations for the first-order derivatives, $\partial S/\partial x$ and $\partial S/\partial y$, can also be computed by convolving $S(x,y)$ with discrete 1-D convolutions. Identify the filter coefficients needed for each derivative. [3]

(b) The Laplacian of the smoothed image, $\nabla^2 S(x,y)$, can be used to localise *blob-like* features in both image position and scale.

(i) Show that this is equivalent to *band-pass filtering*. [2]

(ii) Explain how this can be implemented efficiently using an *image pyramid* in this application. [4]

(iii) How is the feature localised in image position and scale? [2]

(c) Image features are to be extracted in a window at each *keypoint* (blob centre). These are then to be matched in different images and over different viewpoints.

(i) The neighbourhood of each image feature is first geometrically normalised to a 16×16 patch of pixels by sampling the image at an appropriate scale and orientation. How is this achieved in practice? [3]

(ii) The SIFT (Scale-Invariant Feature Transform) descriptor is used to describe the 16×16 patch of pixels. Describe the main steps in computing this descriptor and how it achieves its invariance to lighting, image and viewpoint changes. [4]

(iii) How are these descriptors used to find correspondences in images from different viewpoints? [2]

24 In binary classification tasks, binary output labels y_n must be predicted from vectors of real-valued inputs $\mathbf{z}_n$ using N training data points $\{\mathbf{z}_n, y_n\}_{n=1}^{N}$. A machine learner uses the following model for this task

$$p(y_n = 1|\mathbf{z}_n, \mathbf{w}, b) = \frac{1}{1+\exp(-\mathbf{w}^\top \mathbf{z}_n - b)} = \sigma_n(\mathbf{w}, b)$$

where the parameters are a vector of weights $\mathbf{w}$ and a bias b. Here $\sigma_n(\mathbf{w}, b)$ is a shorthand for these class probabilities.

(a) The machine learner would like to train the model using maximum likelihood estimation.

(i) Compute the log-likelihood of the parameters $\log p(\{y_n\}_{n=1}^{N}|\{\mathbf{z}_n\}_{n=1}^{N}, \mathbf{w}, b)$ in terms of $\sigma_1(\mathbf{w}, b), \ldots, \sigma_N(\mathbf{w}, b)$. [4]

(ii) Compute the derivative of the log-likelihood with respect to the weights $\mathbf{w}$. [5]

(iii) Explain how the derivative of the log-likelihood can be used to find a (local) maximum of the likelihood. [4]

(b) The machine learner trains the parameters of the model (the weights and bias) using maximum likelihood estimation on the three datasets shown in Fig. 9. These datasets have two dimensional inputs. Solid circles indicate $y_n = 0$ and crosses indicate $y_n = 1$.

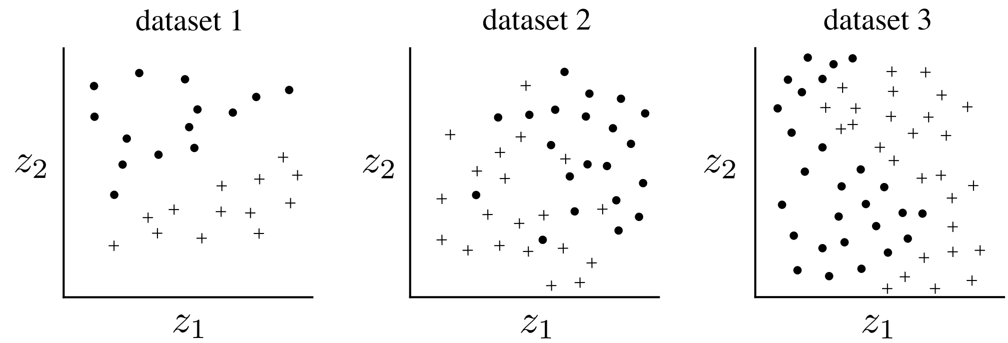

Fig. 9

(i) For each dataset, make a rough sketch showing the *decision boundary* you would expect to be obtained from the maximum likelihood fit. Assess how useful the trained model will be for prediction, noting any limitations. Your answer should include a discussion about the *confidence of the predictions*. [6]

(ii) Discuss how the model and the parameter estimation might be improved. Your answer should explain how these improvements might address any limitations you identified in the previous model fits. [6]

25 (a) What is meant by *reinforcement learning*? Define the terms *state*, *value function*, *policy iteration*, *value iteration* and the *action-value function*, Q. [8]

(b) Consider the following two problems (both of which are to be solved offline).

(i) Finding the optimal trajectory for reversing a car into a parking space while minimising the distance travelled (see Fig. 10). An accurate model of the kinematics of the car is available. [7]

(ii) Determining the ability of a car to avoid obstacles when driven at speed on wet surfaces. A highly detailed simulation environment is available, which incorporates important nonlinear and stochastic characteristics from a low-level model of the interaction between the tyre and the road surface. [10]

For each problem choose a suitable algorithm using reinforcement learning or optimal control. In each case describe the algorithm and its application to the problem in detail, explaining why it is appropriate to the problem. A different algorithm should be given for each problem. Your answer should also explain how the terms defined above in (a) apply to these two applications. You may make reasonable assumptions.

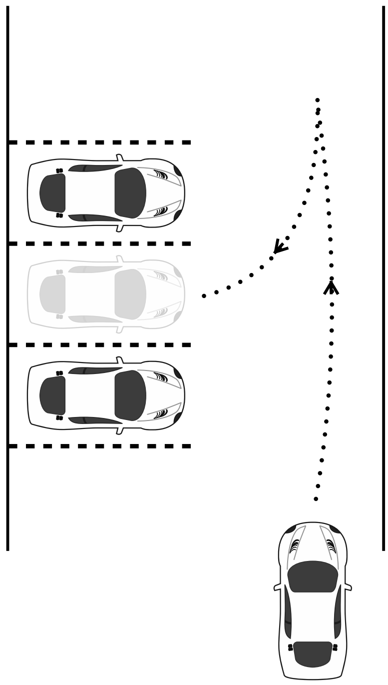

Fig. 10

SECTION G: *Bioengineering*

*Answer not more than **two** questions from this section.*

26 (a) Both ultrasound (US) and optical coherence tomography (OCT) make use of *pulse-echo* signals to image the fundus. Compare and contrast US and OCT in the following areas:

(i) lateral focusing of the signal (i.e. across the direction of travel); [3]

(ii) attenuation of the signal and maximum imaging depth; [3]

(iii) detection of the echo and subsequent determination of the signal depth. [4]

(b) For an ideal OCT system, with no dispersion, the intensity I at the output of the interferometer, as a function of frequency ω, is given by:

$$I(\omega) = S(\omega) + S(\omega)\left|\int_{-\infty}^{\infty} r_{\mathrm{s}}(l_{\mathrm{s}})\, e^{j\frac{\omega}{c}l_{\mathrm{s}}}\, \mathrm{d}l_{\mathrm{s}}\right|^2 + 2\Re\left\{S(\omega)\int_{-\infty}^{\infty} r_{\mathrm{s}}(l_{\mathrm{s}})\, e^{j\frac{\omega}{c}(l_{\mathrm{r}}-l_{\mathrm{s}})}\, \mathrm{d}l_{\mathrm{s}}\right\}$$

(i) Define the terms S, r_{s}, l_{r}, l_{s} and c in this equation. [3]

(ii) Explain how r_{s} can be recovered from $I(\omega)$ using Spectral OCT. Include in your answer the mathematical derivation of the equation used to calculate r_{s}, and the experimental measurements required in order to use this equation. [8]

(iii) What are the consequences of the $S(\omega)$ term in your answer to (ii) for the design of a spectral OCT system? [4]

27 (a) A patient no longer sees anything in their left visual field when closing the right eye, and similarly in their right visual field when closing the left eye. Comment on the plausibility of each of the following potential causes:

(i) A stroke damaged the left part of the visual cortex

(ii) The right optic nerve was damaged before the optic chiasm

(iii) An injury caused damage to the optic chiasm

[3]

(b) This question is about the contrast sensitivity function in vision, and hybrid images.

(i) In a few sentences, describe the so-called ‘contrast sensitivity function’ in the context of sinusoidal gratings, including a brief account of how contrast thresholds can be measured experimentally. [4]

(ii) Look at the large and small versions of the same hybrid image below in Figure 11. In the large version, you see one British prime minister. In the small version, you see another one. Explain how this image might have been constructed, and why it is perceived differently when viewed from up-close and from far away. [5]

Fig. 11

(c) For each of the following terms, name one species or group of animals whose eyes are characterised by that term: negative lens, apposition eye, lens capable of protruding through the iris, multiple lenses, adjustable corneal curvature, inverse eye, flat cornea, reflective mirror, pin hole camera, and scanning telescope. [5]

(d) The following questions are about the information theoretic analysis of two neurons' responses about two stimuli. Table 2 below shows the (joint) probabilities with which these neurons give each of three possible responses for the two stimuli. Please provide your answers using bits as the unit of information theoretic quantities.

stimulus 1		neuron 1		
		1	2	3
neuron 2	1	0	0.05	0.2
	2	0.2	0.2	0.1
	3	0.1	0.05	0.1

stimulus 2		neuron 1		
		1	2	3
neuron 2	1	0.25	0.25	0
	2	0.25	0.25	0
	3	0	0	0

Table 2

(i) How much surprise do we get when the response of neuron 1 to stimulus 1 is 3? [1]

(ii) How much surprise do we get when the response of neuron 2 to stimulus 1 is 2? [1]

(iii) How much surprise do we get when, for stimulus 1, the response of neuron 1 is 3 and the response of neuron 2 is 2? [2]

(iv) Are the responses of the two neurons to stimulus 1 independent? How is this reflected in your answers to the previous questions? [1]

(v) What is the entropy of the joint responses of these two neurons for stimulus 1? [1]

(vi) When each stimulus is used with an equal probability, what is the conditional entropy of the joint responses of these two neurons? [1]

(vii) When each stimulus is used with an equal probability, what is the mutual information between the stimuli and the responses of these two neurons? [1]

28 (a) Discuss why the mechanical properties of biological materials, such as the cornea of the eye, are difficult to measure compared to conventional engineering materials. [3]

(b) State the role played by intra-ocular pressure (IOP) in the eye. How is IOP in the eye maintained? [3]

(c) Goldmann tonometry is a common testing method to measure IOP. Assisted by a sketch of the testing geometry of Goldman tonometry, describe how the value of IOP is determined and how this can be used to detect a potential disease state of the eye. [5]

(d) To evaluate whether potential damage on the optic nerves can occur in the eye, two methods are used to model the deformation of lamina cribrosa, one being the plane membrane model, and the other being the elastic plate model. (For the following questions, you can take 1 mmHg = 133 Pa and Poisson's ratio = 0.5.)

(i) Describe how the plane membrane model can be used to estimate the exerted strain on lamina cribrosa, given that the radial strain ε_r is 0.5% for IOP = 40 mmHg. State your assumptions. [5]

(ii) The elastic plate model indicated in Figure 12 assumes a uniformly loaded circular plate of radius R and thickness h with pressure p. The central deflection of the lower surface (d) is given by $\frac{d}{h} + 1.85\left(\frac{d}{h}\right)^3 = 0.7\frac{p}{E}\left(\frac{R}{h}\right)^4$ and the central membrane stress (σ_m) and the bending stress (σ_b) are given by $\sigma_m = 0.91E\left(\frac{d}{R}\right)^2$ and $\sigma_b = 1.78E\frac{dh}{R^2}$. Calculate the strain, taking $p = 40$ mmHg, $R = 600$ μm, $h = 110$ μm and $d = 128$ μm. [6]

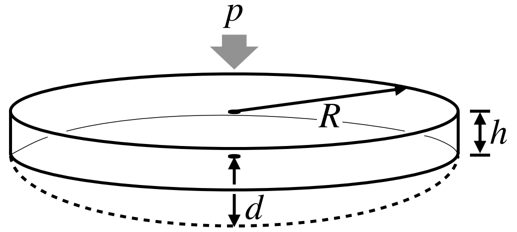

Fig. 12

(iii) It is assumed that a strain of 5–8% in the lamina cribrosa can induce potential damages in the optic nerve. Briefly discuss the applicability of the above two models for modelling optical nerve damage. [3]

SECTION H: *Manufacturing and Management*

*Answer not more than **two** questions from this section.*

29 A group of university researchers have developed a new sensor technology to detect very small concentrations of gaseous molecules in air. The team think that it can be used for a medical device which can collect and analyse people's breath in order to detect various illnesses.

(a) Describe the potential stakeholders for this medical device. [5]

(b) Describe **two** different techniques the researchers could use to understand the needs of customers and users for such a medical device. Explain the strengths and weaknesses of each technique. [10]

(c) Discuss the potential business model choices for commercialising the sensor technology. Explain the advantages and disadvantages of each business model. [10]

30 (a) For a small electric car designed for short distance urban journeys, draw three different perceptual maps and explain your choice of axes in each map. [8]

(b) Describe the main sources of funds available to business during the commercialisation of a technology. Explain at what stage in the commercialisation process each source might be most appropriate. [6]

(c) Explain the difference between sustaining and disruptive innovation. Use examples to illustrate your answer. [3]

(d) Describe the key challenges in managing partnerships between large established firms and small new firms. [8]

 (TURN OVER

31 (a) Describe a possible *product specification* for **one** of the following:

(i) a new high-performance luxury electric car; or

(ii) a new low-cost training shoe; or

(iii) a domestic coffee maker; or

(iv) a product of your choice.

[5]

(b) Sketch and describe the model which explains how industries evolve and provide examples for each phase of the model. [6]

(c) List and describe the key elements of a business plan which should be presented by an entrepreneur to venture capitalists to raise investment. [6]

(d) Explain the key characteristics of:

(i) batch production;

(ii) mass production.

[8]

END OF PAPER