Version KSD/PAD/3

EGT1
ENGINEERING TRIPOS PART IB

Tuesday 8 June 2021 13.30 to 15.40

13.30 to 14.40 Foreign Language Option

Paper 8

SELECTED TOPICS

*This is an **open-book** exam.*

*If you are not taking the Foreign Language option, answer **four** questions, taken from only two of Sections B–H. Not more than **two** questions from each section may be answered.*

*If you are taking the Foreign Language option, answer **two** questions from one of Sections B–H.*

All questions in Sections B–H carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the top sheet.*

STATIONERY REQUIREMENTS

Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

Attachments: Data Sheet for Section E (3 pages)

You have access to the Engineering Data Book, online or as your hard copy.

10 minutes reading time is allowed at the start of this exam.

The time allowed for scanning/uploading answers is 20 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

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SECTION B: ***Civil and Structural Engineering***

Answer not more than ***two*** *questions from this section.*

1 A sheet pile wall, with one level of temporary props at its top, supports an excavation in sand with a depth of h = 6 m, as shown in Fig. B.1(a). The specific gravity of the sand is G_s = 2.65 and it has a void ratio e = 0.85. The critical state friction angle of the sand is φ' = 32°. It is assumed that the wall moves sufficiently to mobilise full active pressure on the retained side, while the lateral earth pressure coefficient K_{mob} on the excavation side is assumed to be a constant value.

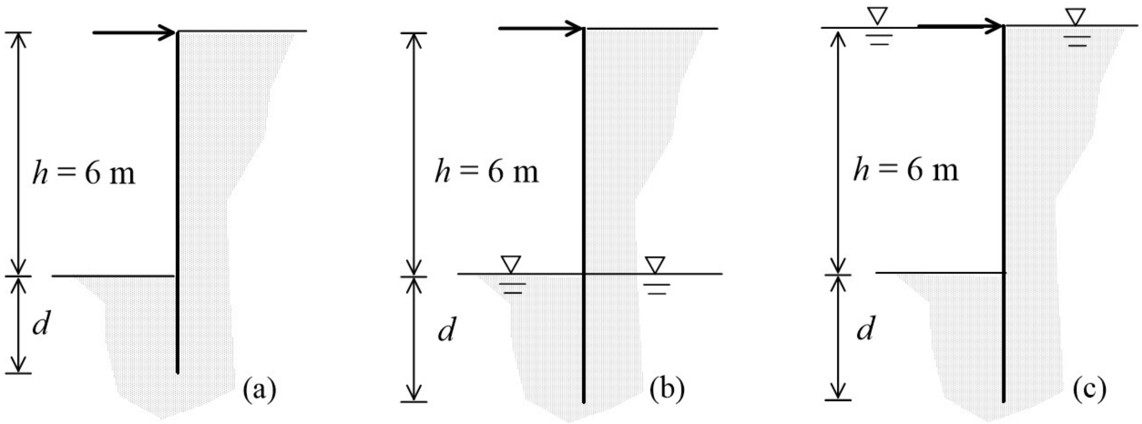

Fig. B.1

(a) Compute the unit weight of the sand in dry and saturated conditions. [2]

(b) Assuming that the mobilised friction at the interface between the sheet pile wall and the sand on the passive side is $\delta = \varphi'/3$, compute the active and passive lateral earth pressure coefficients using Rankine's and Lancellotta's static solutions, respectively:

$$K_A = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}$$

$$K_P = \frac{\cos\delta}{1 - \sin\varphi'}\left[\cos\delta + \sqrt{(\sin\varphi')^2 - (\sin\delta)^2}\right] e^{2\Theta\tan\varphi'}$$

where:

$$2\Theta = \sin^{-1}\frac{\sin\delta}{\sin\varphi'} + \delta$$

[3]

(c) Compute the depth of embedment, d, required to obtain $K_P/K_{mob} = 1.6$, and the prop force and maximum bending moment in the wall for this embedment depth. [7]

(d) If the water table rises to dredge level, as in Fig. B.1(b), compute the depth of embedment required to obtain the same ratio K_P/K_{mob} as in (c) and the new values of the prop force and maximum bending moment in the wall. [7]

(e) If the water table is at ground level, and the excavation is carried out submerged, as in Fig. B.1(c) compute K_P/K_{mob}, prop force and bending moment for the same depth of embedment as in (d). [6]

2 A reinforced concrete bridge pier, made up of a concrete beam and a concrete column as shown in Fig. B.2, supports three structural steel girders for a highway bridge. Each steel girder transmits a factored dead load P_D = 235 kN to the concrete pier. In addition, depending on traffic conditions on the bridge, each of the steel girders may also transmit a maximum factored live load P_L = 265 kN.

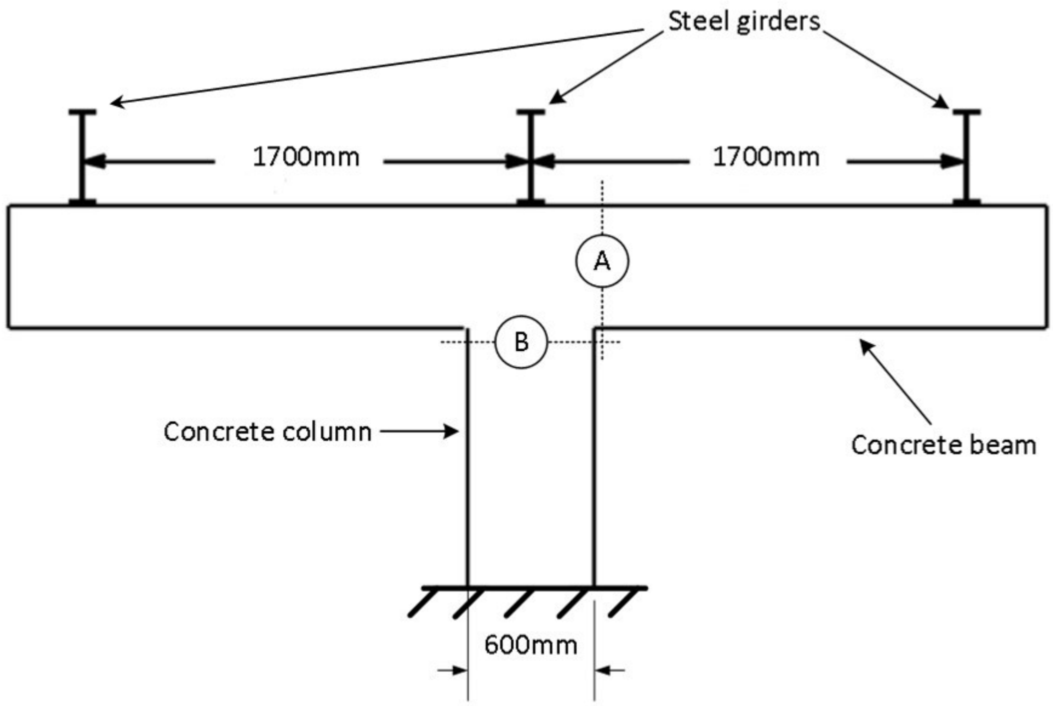

Fig. B.2

(a) Considering Cross Section A, suggest a loading scenario to calculate the maximum bending moment in the cross section. For the loading scenario you have suggested, draw the shear force and bending moment diagram for the concrete beam. [4]

(b) Design the required longitudinal reinforcement for Cross Section A. Assume a width of 400 mm (into the page) and an effective depth of 600 mm (take f_{cu} = 40 Nmm^{-2} and f_y = 460 Nmm^{-2}). Choose specific bar sizes for a singly-reinforced situation. Is shear reinforcement required? Justify your answer. [8]

(c) Considering Cross Section B, suggest a loading scenario to calculate the maximum bending moment in the cross section. For the loading scenario you have suggested, draw the shear force and bending moment diagram for the concrete column. [4]

 (TURN OVER

(d) Cross Section B has a width (into the page) of 400 mm and is singly reinforced with three Φ32 mm bars. The distance from the centre of all longitudinal reinforcing bars to the closest concrete surface is 70 mm. Where is the neutral axis located within the cross section? What is the section's moment capacity? [5]

(e) Without doing any calculations, what other consideration(s) will you need to take into account in designing Cross Section B? [4]

3 A high speed railway connection is planned between two large historical cities A and B. The urban stretches of the line will run in bored tunnels. The soil profile at city A consists of about 5 m of sand and gravel underlain by a stiff clay with an undrained shear strength $s_u = 200\ \mathrm{kNm^{-2}}$, which is approximately constant with depth and has a unit weight $\gamma = 20\ \mathrm{kNm^{-3}}$. The ground beneath city B is a relatively soft clay with an undrained shear strength $s_u = 60\ \mathrm{kNm^{-2}}$, also constant with depth, and a unit weight $\gamma = 20\ \mathrm{kNm^{-3}}$. You are involved in the preliminary design of the tunnels beneath the cities, which run at a typical depth of 20 m below ground level.

(a) Give answers to the following questions.

(i) Why is it safe to construct an open face tunnel beneath city A but not beneath city B? Use the concept of stability ratio to illustrate your answer. What will be needed to construct the tunnels beneath city B? [4]

(ii) Explain the significance of differential settlement in the context of potential damage to masonry buildings caused by tunnelling. Why are hogging deformations potentially more significant than sagging deformations? What general deformation shape can be expected? [3]

(iii) How can compensation grouting be used to control damage to buildings in a tunnelling project, and what is the role of instrumentation in the process? What should be the principal aim of the process when it is used to protect a masonry building? [3]

(b) Part of the proposed tunnel beneath city B will also encounter sand below the water table. Describe the significance of soil permeability in the context of tunnelling below the water table, and give two examples of techniques that can be used to overcome potential problems. [5]

(c) In city B, a structure for a short stretch of twin connecting tunnels will have to be constructed in a shallow river estuary, as shown in Fig. B.3. The structure is sufficiently heavy to resist uplift.

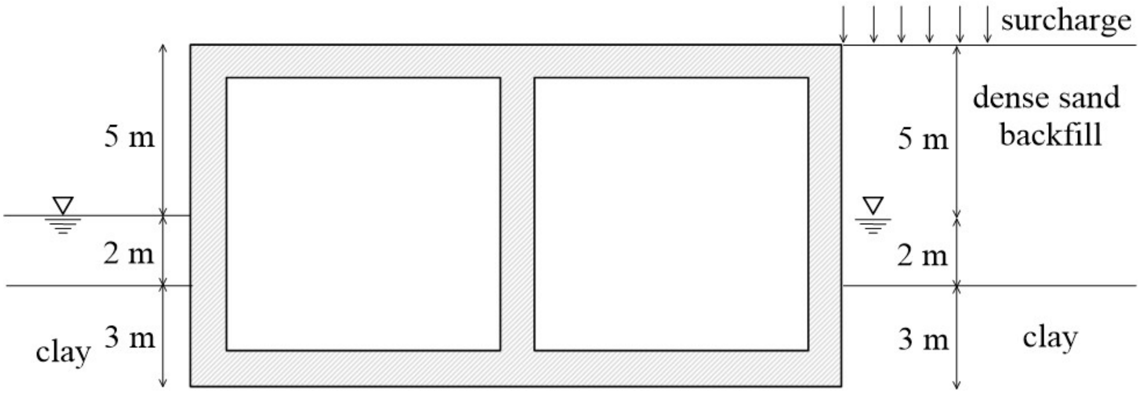

Fig. B.3

The lowest 3 m of the structure are in the base clay. The depth of water in the river estuary is 2 m. Immediately after construction, one side of the tunnel will be backfilled with dense sand to the top of the structure. The sand has unit weight $\gamma = 19\ \text{kNm}^{-3}$, below the water and $\gamma = 17\ \text{kNm}^{-3}$ above the water. The critical state angle of friction of the sand is $\varphi' = 35°$. A surcharge of $50\ \text{kNm}^{-2}$ will be applied at the surface of the backfill. It should be assumed that the tunnel can move sufficiently to mobilize full active pressure on the backfilled side and full passive pressure on the opposite side, and that its walls are smooth.

(i) Calculate the horizontal total stresses acting in the short term on each side of the tunnel structure and sketch the stress distribution with depth. [5]

(ii) Ignoring any friction on the base of the tunnel, calculate the factor of safety against sliding in the short term. Comment on its value. [5]

END OF SECTION

SECTION C: *Mechanics, Materials and Design*

Answer not more than ***two*** *questions from this section.*

1 (a) A wind turbine utilises a 3-phase, star-connected induction generator connected directly to the 3.3 kV, 50 Hz grid. It has the following equivalent circuit parameters: $R_1 = 1.1\ \Omega$; $R_2' = 0.8\ \Omega$; $X_1 = 1.4\ \Omega$; $X_2' = 1.1\ \Omega$. The magnetising reactance and iron loss resistance are large enough to be ignored.

(i) At a slip of −0.03, find the generator phase current and hence its output real and reactive power. [4]

(ii) Derive an expression for the real part of the generator phase current in terms of the equivalent circuit parameters, the slip and the phase voltage. [2]

(iii) By considering that the generator output power is related only to the real part of the generator phase current, find the generator slip if the output power is 1 MW. [6]

(b) Sketch a typical overall gear arrangement for large commercial wind turbines and explain why such an arrangement is used. [3]

(c) What is meant by a gearbox reaction torque and how does it arise? [3]

(d) The definition of the tooth module m and the tooth bending strength formula, along with a table of relevant data, are given below. Use this to find the number of teeth required on the final spur-gear stage of a gear box used to drive the generator of a wind turbine. The required speed ratio through the stage is four, with a transmitted power of 2 MW and a generator speed of 1800 rpm. [7]

$$m = \frac{2r}{N}, \qquad \sigma_b = \frac{2.08F}{wm}$$

Tooth module m	8 mm
Tooth width w	60 mm
Failure stress σ_b	400 MPa

Table C.1

2 (a) Briefly discuss the following observations in relation to wind turbine design and use:

(i) there has been significant growth in the offshore wind energy generation capacity of the UK in the past few years; [3]

(ii) technological progress does not necessarily contribute to sustainable development; [3]

(iii) light-weight composite materials are used in blades but not generally in the gearbox or tower of wind turbines; [3]

(iv) modelling vibration is critical in wind turbine design. [3]

(b) A horizontal-axis wind turbine is 20 m in diameter and has a hub of diameter 2 m. The chord c of the blade follows the following function:

$$c = c_0 \left(1 - \frac{r}{D}\right)$$

where r is the radial position, D is the diameter of the turbine and $c_0 = 1$ m. The density of air is $\rho = 1.2$ kg m^{-3}.

(i) Calculate the maximum bending moment in the blade due to storm loading with a wind velocity of 50 m s^{-1}. You may assume the drag coefficient $C_d = 1.5$. [6]

(ii) The structural component of the wind turbine blade, assumed to carry all the bending loads, is composed of a hollow cylinder with constant radius $r_c = 0.1$ m and wall thickness $t_c = 10$ mm. Calculate the minimum yield strength of the material needed for the cylinder for it to withstand the storm loading. [3]

(iii) By how much does the yield strength need to change, if the self-weight loading of the blade is also considered? The mass per unit length of the blade is 50 kg m^{-1}. [4]

3 Figure C.1 shows a simplified model of a wind turbine blade of length L, built in at the hub. The hollow rectangular cross section is uniform along the length of the blade with side lengths d and $4d$. The blade has a uniform wall thickness t which can be varied and can be assumed to be small compared with d. The blade material is isotropic with Young's modulus E, failure strength σ_f and density ρ. The blade is subject to self-weight loading due to gravity g acting vertically, and storm wind loading modelled as a uniform pressure p acting on the front vertical face of the blade. The tip deflection due to wind loading is δ. The design aim is to minimise the mass of the blade.

(a) Considering deflection-limited design due to wind loading alone, derive a material property performance index to minimise mass. [7]

(b) Considering strength-limited design due to self-weight loading alone, with the aim of minimising mass, derive:

(i) a constraint on material properties;

(ii) a material property performance index. [7]

(c) Now consider the case with parameter values as listed in Table C.2. Using the materials data book, identify appropriate materials which minimise the mass of the blade, while meeting both the deflection and strength constraints considered in parts (a) and (b). [7]

(d) Discuss, without doing detailed calculations, how the blade material choice will be affected by changes in the blade dimensions. [4]

L	d	δ	g	p
20 m	0.3 m	0.2 m	9.81 m s^{-2}	1500 Pa

Table C.2

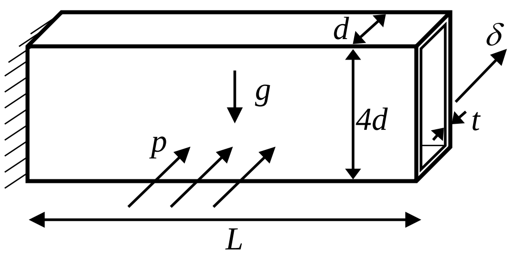

Fig. C.1

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Engineering Tripos Part IB 2021
Paper 8 Selected Topics

SECTION C – Mechanics, Materials and Design

Answers

1 (a) (i) 74.2 A -174.4 º , -422 kW, 41.4 kVA, (ii) -0.071

(d) 29 and 116 teeth

2. (b) (i) 59.2 kNm (ii) 188.5 MPa, (iii) 5% increase

SECTION D: Aerothermal Engineering

*Answer not more than **two** questions from this section*

1 (a) An aircraft cruises at an altitude where the ambient temperature and pressure are 220 K and 29 kPa. At the inlet of the turbojet engine the stagnation pressure is 45 kPa. Calculate the flight Mach number and the stagnation temperature at the engine inlet. [3]

(b) The mass flow of air entering the compressor of the turbojet engine is 50 kg s^{-1}. The air flow at inlet of the compressor is axial with a Mach number of 0.6. The stagnation pressure at the inlet of the compressor is 45 kPa. The height of the first compressor blade at inlet is 30 cm. Calculate the mean radius of the compressor. [4]

(c) The compressor of the turbojet has a pressure ratio of 35 and is driven by a turbine, which has an inlet stagnation temperature of 1600 K. If the isentropic efficiency of both the compressor and turbine is 90%, find, stating any assumptions made:

(i) the stagnation temperature at exit to the turbine; [4]

(ii) the fuel flow rate, if the fuel has a lower calorific value of 43 MJ kg^{-1}; [2]

(iii) the jet velocity and the propulsive efficiency of the engine. [5]

(d) Instead of using the mass flow from the turbojet for propulsion directly, it is suggested that an additional turbine be added to drive a generator that would power one or more electrically driven fans.

(i) Assuming the additional turbine has an isentropic efficiency of 90%, how much shaft power could be produced? [2]

(ii) Discuss the potential advantages and challenges of this hybrid-electric approach. [5]

Assume that the working fluid is air throughout with γ = 1.4 and R = 287 J kg^{-1} K^{-1}.

2 (a) The dimensionless thrust $\tilde{F}$, the dimensionless air mass flow rate $\tilde{m}_a$ and the dimensionless fuel mass flow rate $\tilde{m}_f$ are given by

$$\tilde{F} = \frac{F_G + p_a A_N}{p_{02} A_N} \quad , \quad \tilde{m}_a = \frac{\dot{m}_a \sqrt{c_p T_{02}}}{p_{02} A_N} \quad , \quad \tilde{m}_f = \frac{\dot{m}_f LCV}{p_{02} A_N \sqrt{c_p T_{02}}}$$

where all the symbols have their usual meaning. Explain carefully the significance of using the term $F_G + p_a A_N$ in the expression for $\tilde{F}$. Explain the physical reasoning for the different forms of the dimensionless mass flow rates for air and fuel. [2]

(b) An aircraft cruises at a Mach number of 0.8 at an altitude where the ambient pressure is 29 kPa and the ambient temperature is 220 K. The turbofan engines that power the aircraft have a propulsive efficiency of 80%. Each engine produces a net thrust of 50 kN. The bypass and the core of the engine have the same jet velocity. Calculate the stagnation pressure and stagnation temperature at the inlet of the engine. Calculate the mass flow rate through each engine and the gross thrust of each engine. [7]

(c) The same turbofan engine is tested in a static test bed at an ambient pressure of 100 kPa and ambient temperature of 288 K. It is run at the same dimensionless operating point as the cruise condition. The bypass and core propelling nozzles are choked in both the static test bed and cruise condition. If the nozzle area A_N is 1 m^2, determine the engine air mass flow rate during testing. Also calculate the gross thrust during testing and comment on the value obtained. [8]

(d) At cruise, the thermal efficiency of the engine is 42%. The fuel has a lower calorific value of 43 MJ kg^{-1}. Calculate the engine thrust specific fuel consumption during cruise and during testing. [4]

(e) State how the dimensionless fuel flow rate of the engine changes between the top of climb and cruise. What consequences does this have for engine design? [4]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $R = 287$ J kg^{-1} K^{-1}.

3 (a) The Breguet range equation for the distance travelled during cruise, s , is

$$s = \frac{V}{g}\frac{L/D}{\text{sfc}}\ln\left(\frac{W_1}{W_2}\right)$$

where W_1 and W_2 are the total aircraft weights at the start and end of cruise, V is the flight speed, sfc is the thrust specific fuel consumption, L/D is the aircraft lift-to-drag ratio, and g is the acceleration due to gravity. With reference to this equation, discuss ways in which aerothermal engineering influences aircraft range. [4]

(b) The aircraft cruises at a constant lift coefficient C_L = 0.5. At the start of cruise the ambient pressure is 0.2 bar and the mass of the aircraft is 100 tonnes. The total wing area is 250 m^2. Calculate the Mach number at the start of cruise. [4]

(c) The aircraft cruises with an sfc = 0.014 kg s^{-1} kN^{-1} and L/D = 20 for a distance of 4000 km. During cruise, the ambient temperature remains constant at 220 K.

(i) Assuming the Mach number is constant during cruise, calculate the ambient pressure at the end of cruise. [4]

(ii) If the altitude is instead constrained to be constant during cruise, explain the effect on the aircraft's Mach number and range. [4]

(d) Find, stating any assumptions, an expression for the rate of change of range s with respect to the overall efficiency of the engine. For the aircraft described above, evaluate the increase in range obtained by an increase in engine overall efficiency of 0.01. The fuel has a lower calorific value of 43 MJ kg^{-1} . [4]

(e) If the propulsive efficiency of the engine is fixed, describe, highlighting any challenges, ways in which the overall efficiency of the engine could be increased. [5]

Assume that the working fluid is air throughout with γ = 1.4 and R = 287 J kg^{-1} K^{-1} .

Numerical answers:
1 (a) 0.818; 249.4 K
(b) 0.274 m
(c)(i) 1111.8 K
(ii) 0.99 kg s^{-1}
(iii) 1074 m s^{-1}; 0.37
(d)(i) 26.0 MW

2 (b) 44.2 kPa; 248.2 K; 420.4 kg s^{-1}; 150 kN
(c) 882.8 kg s^{-1}; 305 kN
(d) 16.5 g kN^{-1} s^{-1}; 6.58 g kN^{-1} s^{-1}

3 (b) 0.749
(c)(i) 17.7 kPa
(d) 108 km

SECTION E: *Electrical Engineering*

Answer not more than ***two*** *questions from this section*

1 Figure E.1 shows the structure of a pure element with one crystal plane shaded.

(a) (i) What is this structure called? [1]

(ii) Name three elements that naturally display this structure. [2]

(iii) What are the Miller Indices of the plane shown in grey in Fig. E.1? [2]

(b) (i) Sketch a similar figure to Fig. E.1 showing the (1,1,3) crystal plane. [3]

(ii) If the dimension L in Fig. E.1 is 0.543 nm, what is the spacing between (1,1,3) planes in this element? [4]

(iii) At what angle would a peak be expected for the (1,1,3) plane of this element in an X-ray diffraction spectrum if X-rays with a photon energy of 8.04 keV are used? [4]

(iv) What is the minimum X-ray photon energy that could be used to produce a diffraction peak from the (1,1,3) plane of this element? [4]

(c) The element in Fig. E.1 is a semiconductor with an electron mobility of 1400 $cm^2\ V^{-1}\ s^{-1}$, a hole mobility of 450 $cm^2\ V^{-1}\ s^{-1}$ and a conductivity of $5\times10^4\ \Omega^{-1}\ cm^{-1}$ at room temperature (298 K). Estimate the probability that any one atom has been ionised through the thermal excitation of an electron into the conduction band at room temperature. [5]

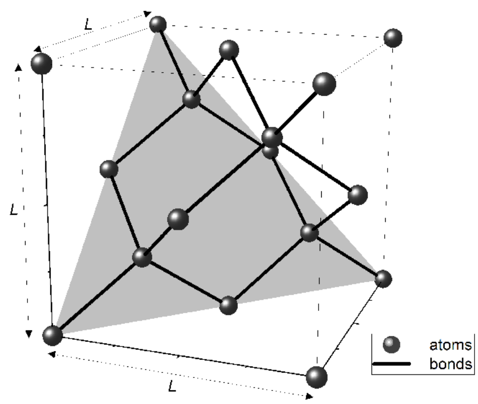

Fig. E.1

2 (a) Figure E.2 shows the pixel circuit on a display backplane.

(i) In what type of display would you find this pixel circuit? [2]

(ii) How is the Thin Film Transistor (TFT) used in this circuit different from a common MOSFET in both design and operation? [4]

(b) An electronic engineer needs to specify a TFT to control the current flow through an Organic Light Emitting Diode (OLED) pixel in a display. The OLED needs to draw a current of 1 μA when the voltages on the TFT are V_{ds} = 1 V and V_{gs} = 2 V. The threshold voltage V_t = 0.5 V. The engineer has fabrication facilities which allow them to choose from the materials shown in Table E.1 (over page) and the minimum channel length which can be fabricated is 1 μm. They have been asked to ensure that the area occupied by the TFT in the pixel is minimised.

(i) Design a TFT to meet these requirements. Your answer should specify the channel semiconductor material, the gate dielectric material, the channel length *L*, the channel width *W* and the thickness of the gate dielectric *t*. [14]

(ii) Estimate the current I_{ds} when V_{ds} = 1 V and V_{gs} = 0 V for the TFT you have designed if the channel semiconductor in the TFT has a thickness of 100 nm. You may assume that the TFT is 'off' under these bias conditions (i.e. there is no accumulation and that conduction due to holes is negligible). [5]

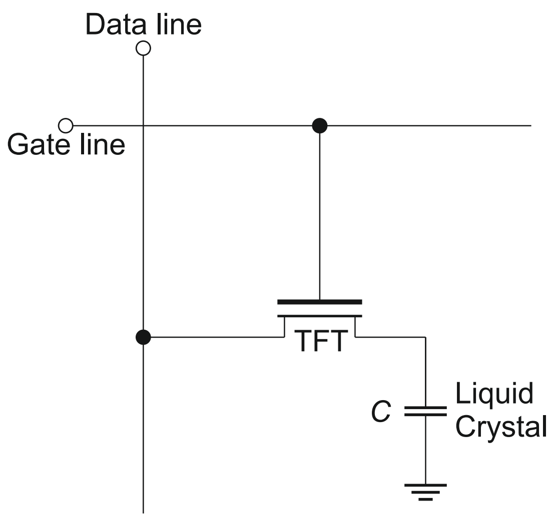

Fig E.2

Material	**Type**	**Properties**
Silicon Nitride	Dielectric	ε_r = 7.5
Aluminium Oxide	Dielectric	ε_r = 9.1
Amorphous Silicon	Semiconductor	μ_e = 1 cm^2 V^{-1} s^{-1} n = 10^{16} cm^{-3}
Amorphous Indium Gallium Zinc Oxide (a-IGZO)	Semiconductor	μ_e = 10 cm^2 V^{-1} s^{-1} n = 10^{13} cm^{-3}

Table E.1

NOTE: For the TFT the equation governing current flow is

$$I_{ds} = \mu \frac{W}{L} C_{ox}\left[\left(V_{gs} - V_t\right)V_{ds} - (V_{ds}^2/2)\right]$$

3 (a) A silicon wafer of diameter L is to be coated with chromium via thermal evaporation. In order to obtain a gradient in the coated film, the substrate is placed perpendicular to the plane of the chromium source to be evaporated in the manner shown in Fig. E.3.

(i) What is the ratio R_A / R_B if R_A and R_B are the rates of film deposition at points A and B respectively? [4]

(ii) Explain how electron beam evaporation differs from thermal evaporation. [2]

(iii) What are the advantages and disadvantages of using electron beam evaporation instead of thermal evaporation for coating a plastic substrate? [3]

(b) (i) What are the advantages of hard disk drives over other data storage technologies and what are their main applications? [3]

(ii) What are the three conditions that must be satisfied in a system that is to be used for binary data storage and in each case explain how ferromagnetic materials can be made to fulfil these conditions? [3]

(iii) Draw a diagram for a longitudinal scheme showing the magnetisation direction in a series of domains storing the following string of binary data: 11101001 [4]

(iv) Why does a disk designed for perpendicular recording require an additional layer of magnetic material compared to a disk designed for longitudinal recording? With the aid of a sketch of a magnetisation curve, describe the important properties that this material should possess. [6]

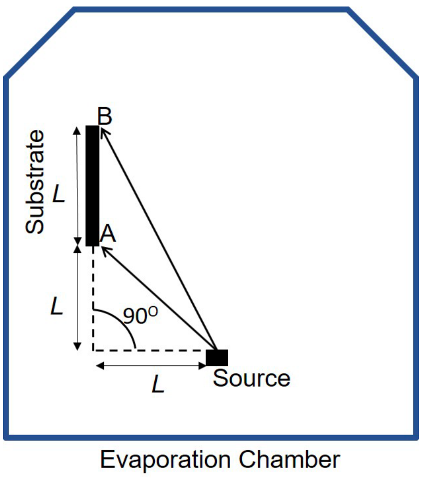

Fig. E.3

Numerical Solutions

1. (a) (iii) (1 1 1)

(b) (ii) 0.164 nm

(iii) $\theta = 28.0^{\circ}$

(iv) 3.78 keV

(c) 3.4×10^{-3}

2 (b) (i) Amorphous indium gallium zinc oxide, aluminium oxide, $L = 1$ µm with other parameters depending on design and good values being $W = 1$ µm and $t = 80.6$ nm.

(ii) 160 pA (based on above design)

3 (a) (i) 25/8

SECTION F: *Information Engineering*

Answer not more than **two** *questions from this section.*

1 (a) List five factors which influence the intensity, $I(x, y)$, of a monochrome CCD image. Explain why edge detection is commonly used in computer vision applications. [3]

(b) A greyscale image, $I(x, y)$, is to be smoothed by low-pass filtering with a Gaussian filter, $G(x, y)$, and differentiated as part of the edge detection process.

(i) Explain why smoothing is required and why the Gaussian is a suitable low-pass filter. [3]

(ii) How can smoothing followed by differentiation be implemented efficiently? Compute the number of operations (multiply and add) needed per pixel. Take the 2-D filter to be of size $(2n + 1) \times (2n + 1)$ pixels, where n is a positive integer. [3]

(c) The Laplacian of the Gaussian, $\nabla^2 G(x, y)$, is a useful operator for both edge detection and blob detection.

(i) How is its use different in edge detection and in blob detection? [2]

(ii) Derive a discrete 1-D Laplacian filter which approximates the second-order spatial derivative of a 1-D image. [2]

(iii) Using the filter derived above calculate a 3×3 filter that can be used to compute a discrete approximation to the Laplacian of a 2-D image. Why are integer valued filters preferred to float valued filters in practice? [2]

(iv) Show that the Laplacian of the Gaussian, $\nabla^2 G(x, y)$, is a suitable band-pass filter for localising *blob-like* features in images. How are image positions and scales of the features computed efficiently without the need for differentiation? [4]

(d) The SIFT descriptor is often used to describe interest points or image features in order to match them in different images and over different viewpoints.

(i) Describe the main steps in computing the SIFT descriptor and how it achieves its invariance to lighting, image and viewpoint changes. What are its limitations? [4]

(ii) How are these descriptors used to find correspondences in images from different viewpoints? [2]

2 A company that makes autonomous vehicles is in the process of designing a Convolutional Neural Network (CNN) for classifying road conditions. The specification of the CNN is only partly complete. Currently it takes an image Z as input and returns an output $\mathbf{a}$ which is a 100 dimensional vector with real-valued elements a_d that lie in the range $-\infty < a_d < \infty$.

(a) Explain why a CNN is more suitable for this task than a single neuron or a Multi-Layer Perceptron. [5]

(b) The company has collected a dataset comprising images Z taken by front facing cameras with associated labels y that indicate whether the road is dry ($y = 1$), wet ($y = 2$), or icy ($y = 3$).

(i) Describe how to complete the design of the CNN by transforming the output vector $\mathbf{a}$ into a form that is suitable for classifying unseen images into the three categories of road condition. [4]

(ii) Mathematically define an objective function that measures the quality of the complete CNN's predictions on training data $\{Z_n, y_n\}_{n=1}^{N}$. Explain the rationale behind the form of the objective function. [4]

(iii) Briefly describe how the objective function can be used to train the CNN. Your answer should outline the general approach to performing learning, but detailed derivations are not required. [4]

(c) The company's autonomous vehicle fleet includes cars with different numbers of front facing cameras. This means that, in practice, several images are simultaneously collected of the same road from different positions, but the number of images K depends on the car. It is important to use all available cameras to classify the road conditions as patches of ice or water are often visible to only one camera. The company would therefore like to build a single system for all vehicles that can take a set of K images as input $\{Z_{n,k}\}_{k=1}^{K}$ (where K can vary) and return a single classification of the road condition.

Design such a system using the trained CNN developed in part (b). Your answer should explain how aspects of your design reflect the need to handle different numbers of input images K and hard-to-spot road conditions that are visible to only one camera. [8]

3 Figure F.1 depicts a map of locations with the road distance between each pair of locations, in km, indicated on the line joining them. The (x, y) location, in km relative to node C, is also given for each node.

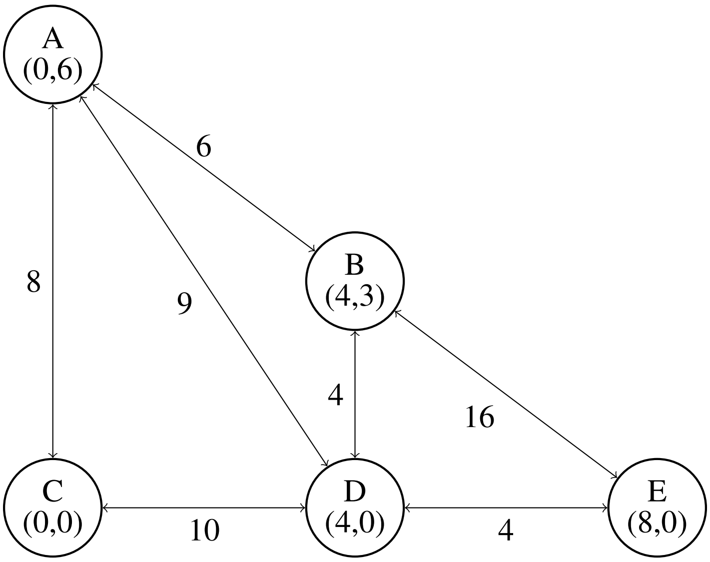

Fig. F.1

(a) Use *Dynamic Programming* to label each node with the shortest path (i.e. distance along roads) to node E. Explain how you know this is the optimal solution. [6]

(b) Use *Dijkstra's algorithm* to find the shortest path between nodes C and E, starting at C (i.e. labelling E with a distance of 0). Is it more efficient than Dynamic Programming for this example? [6]

(c) Now use the A^* *algorithm* to find the shortest path between nodes C and E, starting at C. Does it help in this case? [6]

(d) Now consider an agent trying to discover the shortest path from C to E using *Q-learning* with an ϵ-greedy action selection method. Set the initial value of the state-action function, Q, for node E to 0, for all actions. Give a possible sequence of actions, starting at C and restarting at C whenever E is reached, and the corresponding estimate of the values of Q at each stage, whenever it is changed for each state-action pair. [7]

END OF SECTION

SECTION G: ***Bioengineering***

Answer not more than ***two*** *questions from this section.*

1 (a) Explain what the term *tomography* means in Optical Coherence Tomography (OCT) of the eye. Describe how tomographic imaging is achieved in OCT, including how the resolution and the location of the image data are controlled. [6]

(b) An OCT system illuminates the fundus with a short optical pulse E, of duration a (in s) and centre frequency ω_0 (in rad/s):

$$E = \begin{cases} e^{j\,\omega_0\,t} & -\frac{a}{2} < t < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

What is the bandwidth B of this pulse (in Hz) ignoring all but the main lobe of the frequency response? [6]

(c) A spectral OCT system uses a linear array of N photodiodes to detect the spectrum of the reflected pulse. These diodes are positioned such that they cover a frequency range Z (in Hz), which is always centred around ω_0. The speed of light in the fundus is c.

(i) Explain why the depth spacing between image samples, d, is given by $d = \frac{c}{Z}$. [3]

(ii) If this OCT system uses the pulse E given in part (b), how does the spacing d relate to the pulse duration a? Explain any assumptions you make about the system design. [2]

(iii) The pulse E and number of photodiodes N are fixed, but the frequency range Z is allowed to decrease, such that the total imaging depth increases. What limits the maximum imaging depth in this scenario, and what are the other consequences of increasing the imaging depth in this way? [4]

(iv) The pulse E and number of photodiodes N are again fixed, but now the frequency range Z is allowed to increase in order to improve the axial (depth) resolution. To what extent is this possible, and what would be the limiting factor in improving the resolution? What would eventually happen to the image data if Z was allowed to increase substantially? [4]

2 (a) (i) Describe the functional roles of the cornea, sclera, and aqueous humour in the eye. [2]

(ii) Explain how the organisation of collagen and extracellular matrix fibres relates to the different functions of these eye components. [4]

(iii) Based on the microstructure of the cornea and sclera, explain why it is more convenient to deliver liquid drug formulation to the retina through the sclera than through the cornea. [2]

(b) Explain how ageing could alter the mechanical properties of soft biological tissues. [2]

(c) (i) Explain what the term 'non-linear elasticity' means when referring to the mechanical properties of materials. Describe the origin of non-linear elasticity of soft biological tissues, and use appropriate sketches to aid your answer. [4]

(ii) Sequential recruitment of linear springs is a useful model for simulating the non-linear elasticity of biological tissues. For example, the stress-strain data shown in Fig. G.1 can be modelled using a four-spring sequential recruitment model. Sketch an appropriate configuration for such a model, and estimate the appropriate parameter values for the model. [6]

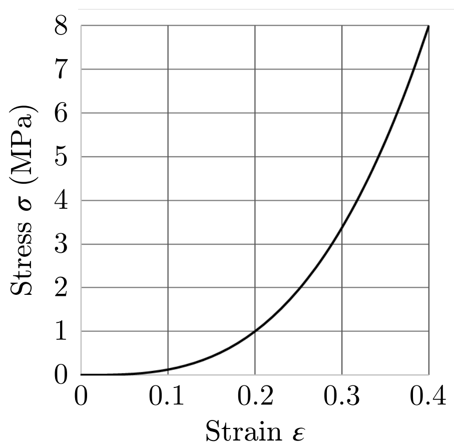

Fig. G.1

(d) A simplified elastic plate model can be used to model the deformation of *lamina cribrosa* resulting from the intraocular pressure of the eye. Describe such a model, explain the assumptions made, and justify the validity of the model. You may use sketch(es) to aid your answer. [5]

3 (a) (i) A dim star is more easily seen in peripheral vision than when the gaze is directed at it. Explain this phenomenon by referring to differences between the two types of photoreceptors in the retina. [2]

(ii) Explain how V1 cells can develop orientation selectivity despite receiving visual input from LGN cells that have circular (i.e. not orientation selective) receptive fields. [3]

(iii) How can a point-like retina achieve two-dimensional vision? Provide an example for this from biology, including the name of the animal that achieves this, the components of its eye, and the principles of vision in it. [4]

(b) In Fig. G.2, which cues allow the viewer to perceive depth? Provide a very brief explanation for each cue that applies. [3]

(c) This question is about efficient coding theory for color vision. The reflectance spectra of 20 different materials, found on recently discovered planet 2P8, are shown in Fig. G.3. What types of retinal ganglion cells would an inhabitant of this planet need in order to efficiently encode these reflectance spectra (in the sense of Principal Components Analysis, as shown in lectures)? Sketch their receptive fields and explain your reasoning. Note that the required ganglion cells need not be identical to those found in mammalian retinas. [5]

(d) This question is about efficient coding in the presence of sensitivity-dependent noise. A cell encodes a scalar stimulus variable, s, in its response, r. The encoding is linear with sensitivity $a \geq 0$, such that $r = a\,s + \epsilon$, where ϵ is noise. Let us assume that both s and ϵ are normally distributed (and, for simplicity, all quantities are dimensionless). Without loss of generality, the mean of both s and ϵ is zero. The variance of s is $\sigma_{\mathrm{s}}^2 > 0$. Critically, the noise in the response grows supralinearly with the sensitivity of the cell, such that the variance of ϵ is $\left(a^4 + b\right)\,\sigma_{\mathrm{r}}^2$ with $b > 0$ and $\sigma_{\mathrm{r}}^2 > 0$.

(i) What quantity needs to be maximised for efficient coding? [1]

(ii) For $\sigma_{\mathrm{s}}^2 = 1$, $\sigma_{\mathrm{r}}^2 = 2$, and $b = 1$ what should be the sensitivity of the cell to achieve maximal coding efficiency? Ensure that you define all relevant intermediate quantities necessary for deriving the answer. [7]

Fig. G.2

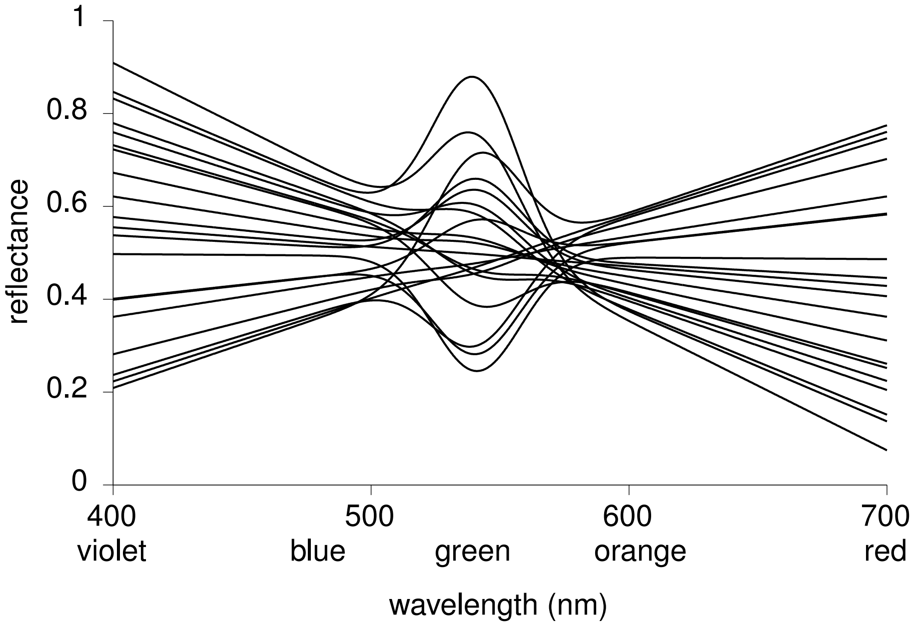
1
0.8
0.6
0.4
0.2
0
reflectance
400
violet
500
blue
green
600
orange
700
red
wavelength (nm)

Fig. G.3

Answers

1(b)

$B = \frac{2}{a}$

1(c)(ii)

$d = \frac{ca}{2}$

3(d)(ii)

$a = 1$

SECTION H: *Manufacturing and Management*

Answer not more than ***two*** *questions from this section.*

1 (a) Sketch and describe the *Kano* model for televisions. [5]

(b) Explain the link between the Kano and the diffusion of innovation models. [12]

(c) Discuss, providing examples, how the diffusion of innovation model can be used by those who are trying to take their innovation to market. [8]

2 A group of university students has developed an innovative augmented reality (AR) technology, combining new sensors and cameras, and new processing software, and is considering setting up a start-up.

(a) What different types of Intellectual Property (IP) could apply to the AR technology and how could they be protected? [8]

(b) Based on your understanding of their pros and cons, and by providing suitable examples of possible business models, advise the start-up on the business model choice. [10]

(c) Based on your answers to (a) and (b), advise the students on how to pitch their idea to Venture Capital investors. [7]

3 (a) Using suitable examples, explain what is meant by *radical* and *incremental* innovation. Discuss how companies pursue different types of innovation. [5]

(b) Discuss the principles and challenges for balancing supply and demand for a company manufacturing a domestic robot to be used to assist with various tasks in a kitchen. [12]

(c) Demonstrate how you would define and segment the market for personal computers. [8]

END OF PAPER