

# EGT1

## ENGINEERING TRIPPOS PART 1B

## Paper 8

## SELECTED TOPICS

*Answer **all** questions from Section A. In addition:*

*If you are taking the Civil Engineering option **AND** the Foreign Language option, no further questions should be answered.*

If you are taking **EITHER** the Civil Engineering option **OR** the Foreign language option, answer **two** questions from one of the Sections C-H.

If you are taking **NEITHER** the Civil Engineering option nor the Foreign language option, answer **four** questions from two of the Sections C-H. Not more than two questions from each Section may be answered.

*All questions in Sections C–H carry the same number of marks.*

*The approximate number of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

Section A: <i>The Engineer in Business</i> .....	A.1
Section B: <i>Civil and Structural Engineering has been assessed as coursework</i>	
Section C: <i>Mechanics, Materials and Design</i> .....	C.1
Section D: <i>Aerothermal Engineering</i> .....	D.1
Section E: <i>Electrical Engineering</i> .....	E.1
Section F: <i>Information Engineering</i> .....	F.1
Section G: <i>Bioengineering</i> .....	G.1
Section H: <i>Manufacturing and Management</i> .....	H.1

## **STATIONERY REQUIREMENTS**

Single-sided script paper.

## **SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**

CUED approved calculator allowed

Engineering Data Book

**10 minutes reading time is allowed for the paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationary from the Examination room.**

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## **SECTION A: *The Engineer in Business***

*Answer all questions from this section, on the special answer sheet.*

1 Vodafone is a UK mobile network operator which does not act as a provider to virtual mobile operators who retail mobile services using another retailer's mobile network. Three is a UK mobile network operator which does act as a network provider to virtual mobile network operators. If Vodafone acquires Three, which of the following best describes its growth strategy:

- (a) Growth by merger.
- (b) Growth by backward vertical integration.
- (c) Growth by forward vertical integration.
- (d) Growth by diversification.
- (e) None of the above.

[2.5]

2 Four equally sized companies make undersea cables for the European electricity industry. They make high rates of return. When one firm puts down its prices, the other three immediately follow. This is likely to be an example of what type of pricing behaviour?

- (a) Predatory Pricing.
- (b) Collusive Pricing.
- (c) Price Discrimination.
- (d) Competitive Pricing.
- (e) None of the above.

[2.5]

3 Which of these statements is consistent with the Marris theory of the firm?

- (a) Firms in the financial services sector pay the employees a salary equal to the value of their output.
- (b) Firms in the financial services sector have a sector specific experience.
- (c) Firms in the financial services sector favour growth of sales over profit maximisation.
- (d) Firms in the financial services sector rotate employees between clients to reduce their client specific knowledge.
- (e) None of the above. [2.5]

4 Looking across all firms, which of the following situations is most favourable for firm growth?

- (a) GDP rises by 1%, technological opportunity increases by 2%, managerial experience increases by 2%.
- (b) GDP rises by 1%, technological opportunity increases by 1%, managerial experience increases by 2%.
- (c) GDP rises by 1%, technological opportunity increases by 2%, managerial experience decreases by 3%.
- (d) GDP rises by 1%, technological opportunity increases by 2%, managerial experience increases by 3%.
- (e) None of the above. [2.5]

5 Amazon tracks its performance against approximately 500 measurable goals, with nearly 80% focused on customer objectives. For example, Amazon has metrics showing that a 0.1 second delay in page rendering can lead to a 1% drop in customer activity. Which marketing philosophy does this example best describe?

- (a) Product-oriented.
- (b) Selling-oriented.
- (c) Promotion-oriented.
- (d) Customer-oriented.
- (e) None of the above.

[2.5]

6 Coca-Cola's "Share a Coke" campaign replaced its iconic logo on bottles with popular names and phrases, allowing customers to find bottles with their own name or the names of friends and family. The campaign encouraged people to share these personalized bottles, creating memorable moments and strengthening emotional connections with the brand. Which stage of brand management does this campaign best exemplify?

- (a) Brand identity.
- (b) Brand meaning.
- (c) Brand relationship.
- (d) Brand response.
- (e) None of the above.

[2.5]

7 Spotify offers various subscription plans tailored to different customer needs, such as a free ad-supported tier, a premium individual plan, and a family plan. Which strategy does this example best describe?

- (a) Market segmentation.
- (b) Positioning.
- (c) Targeting.
- (d) Branding.
- (e) None of the above. [2.5]

8 When Deepseek released its chatbot for download in January 2025, this was an example of managers following:

- (a) Porter's ideas about differentiation.
- (b) Barney's ideas about leveraging core resources.
- (c) McGrath's ideas about exiting opportunities at the right time.
- (d) Prahalad and Hamel's ideas about engaging the workforce in innovation.
- (e) Adner's ideas about ecosystem keystones. [2.5]

9 When managers consider adopting one of Porter's generic strategies, their implicit assumption is:

- (a) They can identify opportunities to grow.
- (b) They can increase willingness to pay among customers.
- (c) They can achieve cost leadership.
- (d) They can identify core competences.
- (e) They must identify ways to differentiate from competitors. [2.5]

10 Which of the following is a product of VRIO analysis?

- (a) Sources of temporary advantage.
- (b) A BHAG.
- (c) An ecosystem map.
- (d) An analysis of the firm's activity system.
- (e) A financial projection of future profits.

[2.5]

**END OF SECTION**

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**SECTION C: Mechanics, Materials and Design**

*Answer not more than two questions from this section.*

1 (a) Discuss how variations in materials and shape provide a range of different design solutions for the tower and blades in a wind turbine. Include comments on design requirements, manufacturing routes and sustainability, along with other relevant issues. [6]

(b) Sketch a typical gearbox arrangement for a large commercial wind turbine. What factors affect the arrangement? [3]

(c) Consider the first epicyclic stage of a gearbox for a wind turbine which is generating power  $P$ . The hub and blades of the turbine are attached to the planet carrier which rotates at angular speed  $\Omega_i$ . The annulus is held fixed and the output is from the sun gear which rotates at angular speed  $\Omega_o$ . The sun and the four planet wheels all have the same radius  $r$ , with involute gear teeth of module  $m$ . Power losses in the gearbox can be neglected.

(i) Show that the ratio of output to input speeds  $\Omega_o/\Omega_i = 4$ . Are the planet carrier and sun rotating in the same or opposite directions? [4]

(ii) Find expressions for the torques  $T_i$  and  $T_o$  on the input and output shafts, respectively, and the contact force  $F$  between the sun and each of the planet wheels in terms of  $P$ ,  $\Omega_i$  and  $r$ . [6]

(iii) Find the required sun gear radius  $r$  to avoid bending failure of the sun teeth. The sun gear has  $N = 60$  teeth and a tooth width  $w = 100$  mm. The input speed  $\Omega_i = 20$  rpm and the power  $P = 2$  MW. Equations for the module and allowable tooth bending failure stress  $\sigma_b$ , which is equal to 300 MPa, are given below.

$$m = \frac{2r}{N}, \quad \sigma_b = \frac{2.5F}{wm}$$

[6]

2 A three-bladed wind turbine is located at a site with a free wind speed  $U = 10 \text{ m s}^{-1}$ . Consider the ring swept by a blade element between radii  $r$  and  $r + \delta r$ , as shown in Fig. C.1(a), where  $r = 50 \text{ m}$  and  $\delta r = 1 \text{ m}$ . The blades have chord  $c = 10 \text{ m}$  at that radius. You may take density of air to be  $\rho = 1 \text{ kg m}^{-3}$ .

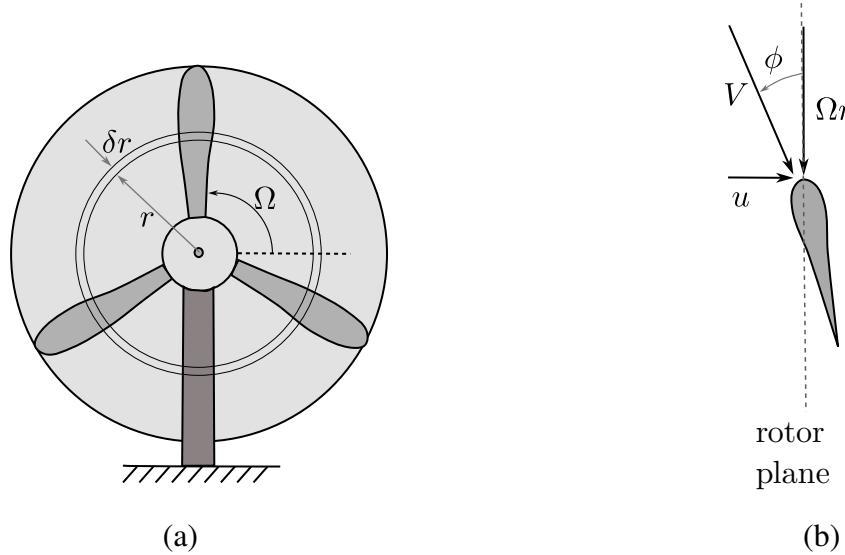


Fig. C.1

(a) Assume that the axial induction factor is to be designed equal to  $a = 1/3$ . Determine the induced wind speed  $u$ , shown in Fig. C.1(b). [5]

(b) In the idealized analysis, the thrust coefficient of a blade element is related to  $a$  by  $C_T = 4a(1 - a)$ . Based on the induction factor in part (a) and the idealized analysis, determine the thrust  $\delta T$  experienced by this blade element. [5]

(c) The magnitude  $V$  and direction  $\phi$  of the relative velocity of the wind with respect to the blade element are shown schematically in Fig. C.1(b). Assume that the lift coefficient  $C_L = 1.5$  for the airfoil cross-section at radius  $r$ , and that the drag coefficient is zero. Assume also that the lift force  $\delta L$  from the three blades between  $r$  and  $r + \delta r$  aligns with the thrust  $\delta T$  calculated in part (b); i.e. assume the angle  $\phi$  shown in Fig. C.1(b) is small. Determine the magnitude  $V$  which leads to the thrust calculated in part (b). [10]

(d) Combine your result in parts (a) and (c) to find  $\phi$  and the rotation rate  $\Omega$ . Is  $\phi$  small as assumed in part (c)? [5]

3 (a) The UK Government recently committed to doubling onshore wind energy by 2030 and removed the de facto ban on onshore wind in England, in place since 2015. The five-step methodology outlined in the lecture course is used to assess the sustainability of the decision.

(i) Rephrase, in your own words, the United Nations Brundtland Commission definition of sustainable development. [2]

(ii) List three possible stakeholders who might be impacted by the decision and describe briefly their interest and influence. [3]

(iii) Create a synthesis table, consisting of the six fact-finding areas versus the three capitals. Populate the table with positive and negative impacts of the decision.

[7]

(b) (i) Variable speed offshore wind turbines commonly use permanent magnet generators. Explain why such generators require a fully-rated power electronic converter in order to connect to the grid. What is the main advantage of the permanent magnet generator compared to the doubly-fed induction generator in offshore wind applications? [3]

(ii) A wind turbine has a power coefficient ( $C_p$ ) vs tip speed ratio ( $\lambda$ ) characteristic given by:

$$C_p = 0.15\lambda \exp\left(-\frac{\lambda}{8}\right)$$

Determine the optimum tip-speed ratio and the corresponding power coefficient, and sketch a graph of  $C_p$  vs  $\lambda$  for  $0 < \lambda < 16$ . [4]

(iii) The turbine of part(ii) is mechanically coupled to a three-phase, 20 pole, star-connected cage rotor induction machine via a gearbox. The induction machine is connected directly to the 6.6 kV, 50 Hz grid, and its equivalent circuit parameters are  $R_1 = 0.2 \Omega$ ,  $R'_2 = 0.15 \Omega$ ,  $X_1 = X'_2 = 0.8 \Omega$ . It has been designed to produce 1 MW at the most common wind speed of  $8 \text{ m s}^{-1}$ , at which it operates at its optimum tip-speed ratio. Stating any assumptions find:

- the turbine blade radius and its angular speed at the wind speed of  $8 \text{ m s}^{-1}$ ; [2]

- the gearbox ratio required, and the turbine power to the generator at the rated wind speed of  $12 \text{ m s}^{-1}$ ; [2]

- the induction generator output real and reactive power if the slip is -0.01. [2]

The following may be quoted:  $\lambda = \frac{\omega R}{v}$ ,  $P = 0.5C_p\rho Av^3$  and take  $\rho$  to be  $1.23 \text{ kg m}^{-3}$ .

## END OF SECTION

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## SECTION D: Aerothermal Engineering

*Answer not more than two questions from this section.*

1 In this question we will explore propulsive efficiency and related concepts.

(a) (i) Show that the distance,  $s$ , between the start and end of an aircraft flight is given by the Breguet range equation

$$s = -\frac{VL/D}{g \text{ sfc}} \ln \frac{w_{\text{end}}}{w_{\text{start}}}$$

where  $V$  is the aircraft speed, sfc is the thrust specific fuel consumption,  $L/D$  is the lift-to-drag ratio,  $g$  is the gravitational acceleration and  $w$  is the weight of the aircraft. State any assumptions made in your derivation. [4]

(ii) Express the Breguet range equation in terms of overall efficiency. Hence show that aircraft range is proportional to propulsive efficiency. [2]

(b) For a turbojet fitted to an aircraft flying at velocity  $V$ , show that the propulsive efficiency  $\eta_p = 2V/(V+V_j)$ . Assume that the engine produces a single jet with relative leaving velocity  $V_j$ . [3]

(c) Now consider an unmixed turbofan engine with a core jet velocity  $V_{jc}$  and a bypass jet velocity  $V_{jb}$ . The bypass ratio is 1 and the ratio of the exhaust stream velocities  $V_{jc}/V_{jb} = \alpha$ , which can be varied. The total mass flow rate of both streams together is  $\dot{m}$ .

(i) Find an expression for the gross thrust  $F_G$  in terms of the average jet velocity  $\bar{V}_j$  and total mass flow rate  $\dot{m}$ , where  $\bar{V}_j = (V_{jc}+V_{jb})/2$  is the average jet velocity and the mass flow rate through each of the core and bypass is  $\dot{m}/2$ . Show that the total kinetic energy flow in the exhaust stream  $\dot{KE} = \dot{m}\bar{V}_j^2 \frac{1+\alpha^2}{(1+\alpha)^2}$ . Find an expression for propulsive efficiency,  $\eta_p$ , in terms of  $\alpha$  and  $\varphi = \bar{V}_j/V$ . [9]

(ii) Assuming all components are 100% efficient, **state** the optimal value of  $\alpha$  from an efficiency perspective. Briefly explain why this is the case. Sketch a plot of  $\dot{KE}/(\dot{m}\bar{V}_j^2)$  versus  $\alpha$ , labeling values at salient points. [4]

(iii) In reality, extracting power from the core flow and delivering it to the bypass stream occurs with some losses. Qualitatively describe how you expect this to change the optimal value of  $\alpha$  and why. [3]

2 An aircraft fitted with a turbojet engine cruises at  $400 \text{ ms}^{-1}$  at an altitude where the atmospheric pressure is 26.5 kPa and the atmospheric temperature is 225 K.

(a) Draw a schematic of the turbojet with station numbering. [2]

(b) If the compressor pressure ratio is 10 and it has an isentropic efficiency of 85%, show that the stagnation temperature at compressor exit is approximately 639 K. [4]

(c) The compressor blade speed is  $280 \text{ ms}^{-1}$  at the mean radius. If design stage loading factor  $\Delta h_0/U^2$  must be less than 0.4, calculate the minimum number of compressor stages required. [2]

(d) Given that the stagnation temperature at turbine entry is 1300 K and the turbine isentropic efficiency is 90%, calculate the stagnation temperature and stagnation pressure at the exit from the turbine. State any assumptions made. [4]

(e) If the turbine blade speed is  $300 \text{ ms}^{-1}$  and the turbine stage loading factor must be less than 2.0, calculate the number of turbine stages. Why are fewer stages needed compared to the compressor? [2]

(f) Calculate the jet velocity at the exit from the propelling nozzle assuming that the flow is isentropic. Comment on the type of nozzle geometry that would be needed. [5]

(g) Now the engine is to be modified to have reheat (afterburner) added after the turbine and before the propelling nozzle. This has the effect of raising the stagnation temperature between the turbine and the propelling nozzle by 500 K. The nozzle is redesigned such that the stagnation pressure in the propelling nozzle and the mass flow rate through the engine is unchanged.

- (i) Calculate the new jet velocity and the percentage increase in thrust. [3]
- (ii) Calculate the percentage increase in fuel used. [2]
- (iii) Comment on the type of aircraft for which this engine might be suitable. [1]

Assume that the combustion products behave as a perfect gas with the same properties as air. Take  $\gamma = 1.4$ ,  $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ , and  $c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$  for air.

3 A new electric ducted fan (EDF) engine with ultra-efficient components has been designed. It consists of a fan taken from an existing large turbofan engine, but the shaft driving the fan is now powered by an electric motor rather than a turbine. It has a thrust of 445 kN when the engine is stationary on the ground. The fan and propelling nozzle can be assumed to be perfectly efficient and all processes are adiabatic.

(a) Explain why, if losses are neglected, the static temperature at the exit of the nozzle is equal to the atmospheric temperature. [1]

(b) Derive an expression for the thrust-per-unit power from the motor,  $F_N/P$ , in terms of the jet velocity  $V_J$  and flight velocity  $V$ . [4]

(c) Consider a stationary ground test. The ambient temperature for the ground test is 288 K and the ambient pressure is 101.3 kPa. Assume that the propelling nozzle is choked and is designed to perfectly expand the flow to ambient pressure at Mach 1.0. Calculate the thrust-per-unit power for the engine, the propelling nozzle area  $A_N$ , and the mass flow rate through the engine. [5]

(d) The same EDF is intended to operate on a new electric aircraft that flies at Mach 0.7, at an altitude with ambient pressure 26.5 kPa and temperature 223 K.

(i) Assuming the engine is operating at the same non-dimensional operating point both at ground test and flight conditions, show that the group

$$\frac{F_G + p_a A_N}{p_{02} A_N}$$

can be taken as constant. The symbols have their usual meaning, which should be defined. [5]

(ii) Hence calculate the gross thrust that this engine will have at cruise conditions by using the ground test measurement and assuming the engine will operate in the same non-dimensional condition. [5]

(iii) Calculate the net thrust at cruise. [5]

Take  $\gamma = 1.4$  and  $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$  for air.

**END OF SECTION**

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## SECTION E: Electrical Engineering

*Answer not more than two questions from this section*

1 (a) A common use of thin film transistors (TFTs) is in pixel circuits for active matrix liquid crystal displays (AMLCDs). Sketch the AMLCD pixel circuit and explain the function of the TFT. [6]

(b) The current-voltage characteristics of a TFT is given by

$$I_{ds} = \mu \frac{W}{L} C_{ox} \left[ (V_{gs} - V_t) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

Hence derive an expression for the small-signal resistance of the TFT operating in the linear regime, where  $V_{ds} \ll (V_{gs} - V_t)$ . [4]

(c) A full colour (RGB) AMLCD for a laptop computer has a  $28.8 \text{ cm} \times 18.0 \text{ cm}$  display area, a resolution of  $2560 \times 1600$  and refresh rate of 60 Hz. The liquid crystal material has a relative permittivity of 8 and is  $10 \mu\text{m}$  thick. The light-emitting part of the display (i.e. where there is switched liquid crystal) makes up 90% of the total area.

(i) At the time of manufacture, the display is designed so that the liquid crystal is charged to within 5% of the data voltage in no more than 3  $\mu\text{s}$ . Calculate the maximum small-signal resistance of the pixel TFTs. [7]

(ii) The TFT uses hydrogenated amorphous silicon as the channel semiconductor with a field effect mobility of  $1 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$  and a 100 nm thick layer of silicon nitride as the gate dielectric with a relative permittivity of 7.5. When turned on, the TFT has operating voltages of  $V_{gs} = 3 \text{ V}$ ,  $V_t = 0.5 \text{ V}$  and  $V_{ds} = 1 \text{ V}$ . What ratio of channel width and length ( $W/L$ ) is required for the TFT? [4]

(iii) During operation, charge is trapped in the gate dielectric of the TFTs resulting in an increase in  $V_t$  with time. Estimate the maximum value of  $V_t$  that can be tolerated before it is no longer possible for the liquid crystal to be charged to within 5% of the data voltage during the time that an individual row of pixels is being addressed. [4]

2 (a) What is the quantum mechanical *wavefunction* of a particle? [2]

(b) A fundamental postulate of quantum mechanics is that the wavefunction  $\psi$  of a particle must be a valid solution to the *Schrödinger equation*, whose time-independent form in one dimension  $x$  is

$$\frac{-\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi = E\psi$$

What are the terms in the time-independent Schrödinger equation (TISE) and what is the overall physical significance of this equation? [6]

(c) Figure E.1 shows a one-dimensional situation where an electron of energy  $E$  is incident on a potential step. The solution to the TISE for the electron wavefunction in each of the three regions is

$$\psi_I(x) = A_1 e^{jk_1 x} + B_1 e^{-jk_1 x}$$

$$\psi_{II}(x) = A_2 e^{k_2 x} + B_2 e^{-k_2 x}$$

$$\psi_{III}(x) = A_3 e^{jk_1 x}$$

where  $k_1 = \sqrt{2mE}/\hbar$  and  $k_2 = \sqrt{2m(V-E)}/\hbar$ .

(i) Derive four equations using boundary conditions that together relate the constants  $A_1, A_2, A_3, B_1$  and  $B_2$ . [8]

(ii) Sketch the wavefunction for the electron across the three regions shown in Fig. E1 for the situation where  $E < V$ . [4]

(iii) Explain the significance of your sketch for the scaling of MOSFETs to smaller feature sizes and give an example of how the design of MOSFETs has been engineered to mitigate this. [5]

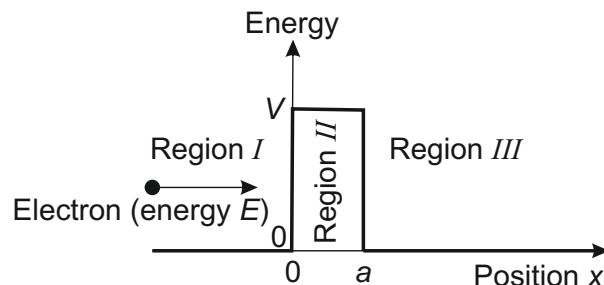


Fig. E.1

3 (a) Since the early 2010s, a FinFET structure has been preferred to a planar structure for MOSFETs in microprocessors. Explain the physical architecture of the FinFET with the aid of a schematic diagram and explain its advantages over the planar structure. [6]

(b) It is proposed to fabricate a lateral gate-all-around (GAA) MOSFET by growing a silicon nanowire directly across a gap between two metal electrodes as shown in Fig. E.2. This is achieved by having a thin layer of polycrystalline silicon (poly-Si) underneath one of the metal electrodes. Poly-Si is a thin film form of silicon which comprises crystals of silicon with grain boundaries between the crystals and it is commonly fabricated by crystallising hydrogenated amorphous silicon at elevated temperatures. The poly-Si seeds the subsequent growth of the silicon nanowires by a chemical vapour deposition process. Critically, the edge of the poly-Si is recessed from the edge of the metal electrode so that the silicon nanowires grow in a lateral direction only.

(i) Suggest a method for depositing the silicon nitride layer shown in Fig. E.2. Justify your choice and explain which process conditions it will be important to control during deposition and why. [6]

(ii) Write a short process flow to follow on from the deposition of the silicon nitride in part (b)(i) to create the structure of the electrodes that would then allow the growth of the silicon nanowires. Your process flow should end before the growth of the silicon nanowires (which would be the final step in the whole production process). [9]

(c) The growth of the silicon nanowires across the gap between the electrodes is an example of a *bottom-up* process. What are the key differences between bottom-up and top-down processes and why is there a growing interest in using bottom-up processes in the fabrication of electronic devices? [4]

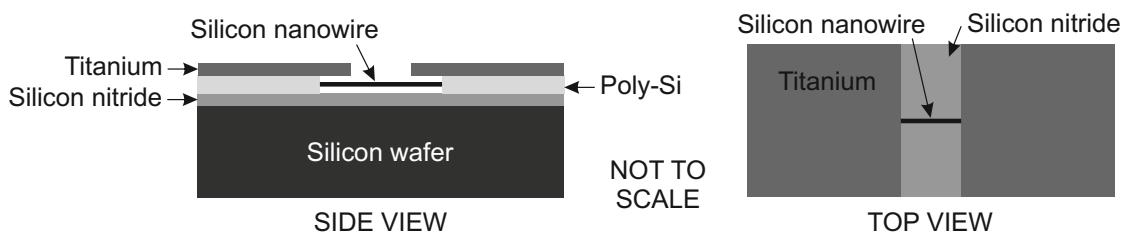


Fig. E.2

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## SECTION F: *Information Engineering*

*Answer not more than two questions from this section*

1 In computer vision, point correspondences over different viewpoints are often used to recover an object's pose and 3-D shape. Point features are first detected in each image and then matched to features in the other viewpoints.

(a) The detection of features first requires the smoothing of a greyscale image,  $I(x, y)$  by *low-pass* filtering with a Gaussian filter,  $G_\sigma(x, y)$ , at different scales  $\sigma$ .

- (i) Why is smoothing required? [1]
- (ii) Give an expression for computing the intensity of a smoothed pixel,  $S_\sigma(x, y)$ , using two discrete 1-D convolutions. [2]
- (iii) How is low-pass filtering at multiple scales implemented efficiently using an *image pyramid*? [3]
- (iv) Describe the convolutions required to construct an image pyramid of low-pass filtered images,  $S_{\sigma_0} \dots S_{8\sigma_0}$ , with  $s = 3$  distinct images in each octave. [4]

(b) The image feature is usually localised in position and scale by filtering the image with a *band-pass* filter over different scales.

- (i) Show that convolution with a Difference of Gaussians filter,  $G_{k\sigma}(x, y) - G_\sigma(x, y)$ , produces a band-pass filtered image. [2]
- (ii) Describe the type of feature that can be localised by this filter and show how to detect it and how to compute its apparent size. [3]

(c) The SIFT (Scale-Invariant Feature Transform) descriptor is used to describe each feature. It is computed from a  $16 \times 16$  patch of pixels around each feature centre.

- (i) How is the  $16 \times 16$  patch of pixels sampled at an appropriate scale and orientation from the image pyramid? [3]
- (ii) How does the SIFT descriptor achieve its invariance to lighting, image and viewpoint changes? [3]
- (iii) What are its limitations? [2]
- (iv) How are these descriptors used to find correspondences in images from different viewpoints? [2]

2 (a) (i) Let  $\mathcal{G}$  be a graph with vertices  $\mathcal{V} = \{v_j\}$  and positive edge lengths  $\mathcal{W} = \{w_i\}$ . Suppose  $P = (v_s, \dots, v_e)$  is the shortest path between  $v_s$  and  $v_e$ . Prove that any sub-path in  $P$  is the shortest path between its endpoints. [4]

(ii) Write down the Bellman-Ford algorithm and explain how it relies on the proposition in part (i). [6]

(b) Consider the undirected graph in Figure F.1. We wish to compute the shortest path from vertex  $A$  to vertex  $F$ .

(i) For what values of  $w$  can we use Bellman-Ford and Dijkstra's algorithms? [3]

(ii) Let  $w = 3$ . Compute the shortest path from  $A$  to  $F$  using Dijkstra's algorithm, showing your working. [7]

(c) Suppose the graph in Figure F.1 is a schematic of a subway network, and that the GPS coordinates of all stations  $A, \dots, F$  are known.

(i) Describe a modification to Dijkstra's algorithm that might improve its efficiency on this network. [2]

(ii) A colleague claims that all of the subway tracks between each station are completely straight. Could this claim be true? Justify your answer. [3]

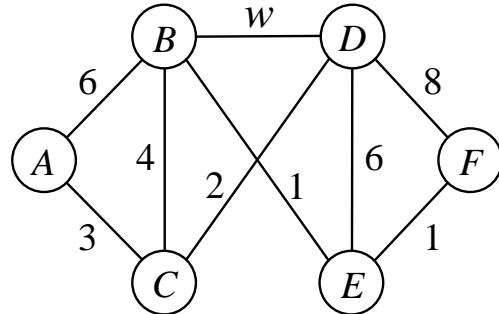


Figure F. 1

3 (a) A machine learner would like to train a neural network to classify natural images.

- (i) Explain why *Multi-Layer Perceptrons* (MLPs) are not suitable for this task. [2]
- (ii) Describe how *Convolutional Neural Networks* (CNNs) overcome the shortcomings in MLPs for modelling images. [3]

(b) The machine learner trains a large CNN from scratch on a small dataset of labelled images to perform binary classification. At the end of training, the network is found to attain a very small loss on the training dataset, but it performs very poorly on a test dataset.

- (i) Why might the network be performing poorly? Justify your answer. [2]
- (ii) What changes to the model and training procedure might improve performance on the test dataset? [6]

(c) The machine learner is applying the CNN to a new dataset. The dataset  $\mathcal{D} = \{x_n, y_n\}_{n=1}^N$  comprises  $N$  images  $x_n$  and class labels  $y_n$ . Each image belongs to one of  $K$  classes  $y_n \in \{1, \dots, K\}$ .

- (i) Explain how to modify the network and the training objective to support multi-class classification. [5]
- (ii) One of the classes is very common and the others are very rare which is causing the trained network to only predict the majority class on the test dataset regardless of the input. What changes to the training procedure could address this limitation? [2]
- (iii) An additional real-valued output  $z_n$  is now observed for each input so the dataset becomes  $\mathcal{D} = \{x_n, y_n, z_n\}_{n=1}^N$ . Describe how the network and training objective can be modified so that the network can predict both outputs at once. [5]

**END OF SECTION**

Version TO/2

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## SECTION G: *Bioengineering*

*Answer not more than two questions from this section*

1 (a) Ultrasound imaging systems and optical scanning laser ophthalmoscopy systems are both able to make measurements of tissue at specific depths ( $z$ -axis) in the eye as well as different lateral ( $x$ -axis) locations. For both of these systems, explain:

- (i) how the measurements are restricted to specific depths and what the approximate depth ( $z$ ) resolution is; [6]
- (ii) what determines the lateral ( $x$ ) resolution and what is its approximate value; [4]
- (iii) what is measured, to what extent this represents a physical property of the tissue, and why this is the case. [4]

(b) The acoustic pulse in an ultrasound imaging system is generated by an electrical pulse with duration  $2t_u$  and shape  $V(t)$  given by:

$$V(t) = \begin{cases} +1 & -t_u < t < 0 \\ -1 & 0 < t < t_u \\ 0 & \text{otherwise} \end{cases}$$

- (i) Calculate and sketch the magnitude of the frequency spectrum of  $V(t)$  for  $t_u = 0.04 \mu\text{s}$ . [6]
- (ii) How would the frequency spectrum change after passing through tissue at the back of the eye and what effect would this have on the pulse shape? [2]
- (iii) How does this pulse differ from optical pulses used in optical coherence tomography? [3]

2 (a) What are the main tissue structures forming the outer and the middle layers of the eye? For each layer, describe the tissue structures' main functions, and how these functions are likely to be affected by intraocular pressure. [7]

(b) Describe how aging can change the microstructure of the crystalline lens, and demonstrate its effects on the lens' regional mechanical property by plotting the dependence of log shear modulus versus age. How is the accommodative ability of the eye affected by these changes? [4]

(c) Discuss the function and material selection of an artificial intraocular lens. [3]

(d) The lamina cribrosa is a porous connective tissue filling the optic nerve head through which the nerve passes. A simplified plane membrane model predicts that the local strain  $\varepsilon_l$  experienced at the lamina cribrosa is approximately four times that of the remote strain  $\varepsilon_r$  experienced at the sclera. Describe, including sketches, such a plane membrane model, stating the assumptions and simplifications made. [7]

(e) Figure G.1 shows the time-dependent uniaxial compressive strain  $\varepsilon$  applied to a hydrated tissue with linear poroelastic behaviour. Sketch the stress response versus time, briefly explaining the key features of your plot. [4]

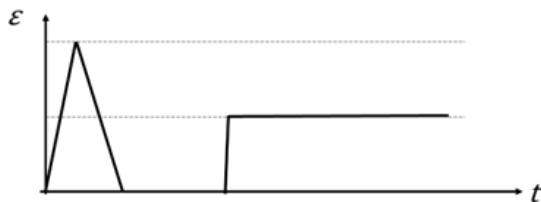


Fig. G.1

3 (a) Write short notes on:

(i) what is meant by “the colour” of an object, and two reasons why it is non-trivial for our visual system to perceive it; [3]

(ii) the principles according to which accommodation and vergence can be used as physiological cues in the perception of depth. [3]

(b) Consider a terrestrial animal species whose eye, shown in Fig. G.2, has a simple, symmetric (convex) lens at a distance of 1 cm from the back of the eye, where the retina is located. The lens has an effective refractive index  $r_{\text{lens}} = 1.6$ .

(i) How can such an effective refractive index be achieved? [2]

(ii) What should be the curvature radius of the lens when focussing on objects at a distance of hundreds of meters? [4]

(iii) What should be the curvature radius of the lens when focussing on objects at a distance of 1 cm? [2]

(iv) Explain if there are any constraints on the curvature radius of the lens if its half-height (shown as  $h$  in Fig. G.2) is fixed at  $h = 1.1$  cm. Support your explanation with calculations. [6]

(v) Given the constraint on the curvature radius that you derived in (b)(iv), explain, with reasons, if the animal can focus on objects at each of the distances discussed in (b)(ii) and (b)(iii), respectively. [2]

(vi) What biological mechanism might allow the animal to focus on objects at different distances, and how does this violate some of the assumptions made about the lens in this question? [3]

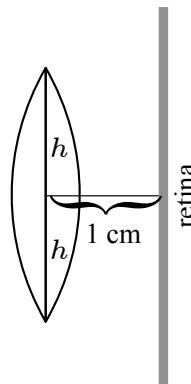


Fig. G.2

**END OF SECTION**

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## Section H: Manufacturing and Management

*Answer not more than two questions from this section*

1 (a) Describe the generic *industry evolution* model. Apply the model to the bicycles industry. [5]

(b) “VitalScan” is a new medical device which can be used to monitor key parameters of patients with chronic conditions both at home and in hospitals.

(i) Describe the role of at least eight key stakeholders in the development of “VitalScan”. [8]

(ii) Discuss why the analysis of the key stakeholders is important for those who are taking “VitalScan” to market. [5]

(iii) Describe which techniques the company could use to capture the perspectives of the key stakeholders at the start of the process of bringing “VitalScan” to market. Describe in detail how the company could combine two different techniques to do this, explaining your reasoning. [7]

2 (a) Discuss the key strategic issues companies face when they grow. Use real examples to illustrate your answers. [8]

(b) Your company is developing a new AI-driven kitchen robot which it wants to market under the name “AI Dente”. It plans to present it publicly for the first time at the CES trade exhibition in Las Vegas next January. The advertisement will feature the slogan “Served AI Dente”, accompanied by a photograph taken by a professional photographer of you preparing to taste a sophisticated dish made by “AI Dente”. You have been asked to advise on the types of *Intellectual Property (IP)* which the company should consider in planning this product’s development, production and marketing. For each type of IP, outline the steps needed to protect the IP under UK law, and comment on any specific points which the company should consider before the exhibition. [9]

(c) Describe the different perspectives of a large and of a small/new company when they are approaching the setup of a partnership to develop a new technology or product. [8]

3 (a) Describe the key types of innovation and explain how companies typically balance their efforts across these different types of innovation. [7]

(b) You are the developer of a new AI-driven kitchen robot, called “AI Dente”, designed to aid in the preparation of sophisticated dishes (described also in question 2(b)). You are applying for backing from a Venture Capital (VC) fund.

(i) Explain what you would include in the business plan for the VC fund, providing examples of details specific to “AI Dente”. [10]

(ii) Which information in the business plan do you think will have the greatest impact on the VC’s decision to back your venture? Justify your answer. [5]

(c) What is the difference between invention and innovation? Give an example from your personal studies of an invention that led to innovation. [3]

**END OF PAPER**