EGT2
ENGINEERING TRIPOS PART IIA

Thursday 25 April 2019 9.30 to 12.40

Module 3A3

FLUID MECHANICS II

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Compressible Flow Data Book (38 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Draw a diagram to show the main components of a fixed-geometry, closed-circuit supersonic wind tunnel. Explain why such a wind tunnel cannot be operated isentropically. Describe the development of the flow field during the process of starting the tunnel. [30%]

(b) The test section of a closed-circuit supersonic wind tunnel is 20 cm square. The design Mach number in the working section is 2.5. Upstream of the inlet nozzle the stagnation pressure is 100 kPa and the stagnation temperature is 320 K. Calculate the area of the inlet nozzle throat. [15%]

(c) Stating any assumptions, calculate the minimum area of the downstream throat. [15%]

(d) What is the minimum power required to start the tunnel? [20%]

(e) During continuous operation at the design Mach number, how much power must be removed by the cooler in order to maintain the original inlet conditions? [20%]

2 (a) Air is flowing steadily through a pipe of constant cross-sectional area. The velocity is found to decrease with distance downstream. Explain the variation along the pipe of the mass flow rate, the impulse function, the stagnation enthalpy and the entropy for the case of:

(i) adiabatic flow with friction, [20%]

(ii) frictionless flow with heat addition. [20%]

Illustrate each case on an $h - s$ diagram.

(b) The pipe is well-insulated and the inside surface has a skin friction coefficient of 0.0025. The air flow is decelerating throughout and the exit is found to be choked. The pipe has inside diameter 20 cm and is 3.3 m long. What is the Mach number at the upstream end of the pipe? [15%]

(c) A new pipe section of length 1 m and a skin friction coefficient of 0.004 is added to the downstream end of the pipe. The diameter of the new section is unchanged at 20 cm. The exit remains choked, and a shock wave is formed in the pipe at a location 1.7 m upstream of the new exit location. What is the Mach number immediately upstream of the shock wave? [30%]

(d) What is the new Mach number at the upstream end of the pipe? [15%]

3 (a) Show that the Froude number upstream of a hydraulic jump is given by the expression

$$Fr_1^2 = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1}+1\right)$$

where h is the depth of water and subscripts 1 and 2 denote conditions upstream and downstream of the jump respectively. [25%]

(b) Find an expression for the Froude number downstream of the hydraulic jump, and show that the flow changes from supercritical to subcritical in passing through the jump. [15%]

(c) A long water channel of uniform rectangular cross section is filled with water to a depth of 1.3 m. At the upstream end the channel is separated from a large reservoir by a sluice gate. Suddenly the sluice gate is opened and the water level at the upstream end of the channel rises by 0.3 m. Calculate the speed of the flow into the channel. [20%]

(d) After a time interval of 40 s the sluice gate is closed suddenly and fully. Calculate the speed of the head and the tail of the resulting wave system. How long does it take for the wave head to catch up with the hydraulic jump? [40%]

4 Two-dimensional, isentropic, steady flow with a free stream Mach number of M_∞ that is slightly disturbed from uniform flow by a thin airfoil may be described by a flow potential ϕ in Cartesian coordinates as:

$$\left(1-M_\infty^2\right)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

(a) Explain briefly how the behaviour of this equation correctly describes important changes in the flow physics when the free stream Mach number M_∞ changes from subsonic to supersonic. [20%]

(b) Show how the potential equation above may be reduced to Laplace's Equation for the subsonic case of flow over an aerofoil of chord c and maximum thickness τ. Carefully indicate any substitutions you have made. [25%]

(c) Show that it is possible to express the lift coefficient c_L of the airfoil at an arbitrary subsonic Mach number M_∞ in terms of the equivalent incompressible value c_{L0} by

$$c_L=\frac{c_{L0}}{\sqrt{1-M_\infty^2}}$$ [25%]

(d) Explain why it would be incorrect to write down a similar expression for the drag coefficient c_D of the airfoil. [15%]

(e) Explain the challenges of computing steady transonic flow (i.e. a flow containing both subsonic and supersonic regions) and suggest approaches that may permit analyses of such flows. [15%]

5 A new supersonic single seat aircraft is to be built in two different models, A and B. While model A is conventional, model B incorporates a lift fan behind the cockpit to enable vertical take-off and landing. The forward fuselage of model A, shown in Fig 1(a), has a conventional cockpit canopy. Model B has a shortened canopy that is fared into the dorsal spine of the aircraft which is raised over the lift fan as shown in Fig. 1(b). Linear dimensions are given for surfaces that are not parallel to the aircraft datum. The rest of the airframe may be assumed to be identical in both models.

(a) Considering the forward fuselage as a series of flat surfaces and the flow around it to be two-dimensional, draw a carefully labelled sketch of the flowfield when the aircraft is flying horizontally at a Mach number $M = 2.0$. [25%]

(b) Calculate the ratio of the pressure drag of model A to that of model B at $M = 2.0$. [35%]

(c) The air inlet to the lift fan is sealed by an airtight door during supersonic flight. Assuming that the pressure inside the airframe is the ambient static air pressure, and given that the door measures 2 m by 1.5 m, calculate the force that the door latches must withstand at $M = 2.0$ at an altitude of 10,000 m. [25%]

(d) Briefly comment on the relative benefits of the two canopy designs. [15%]

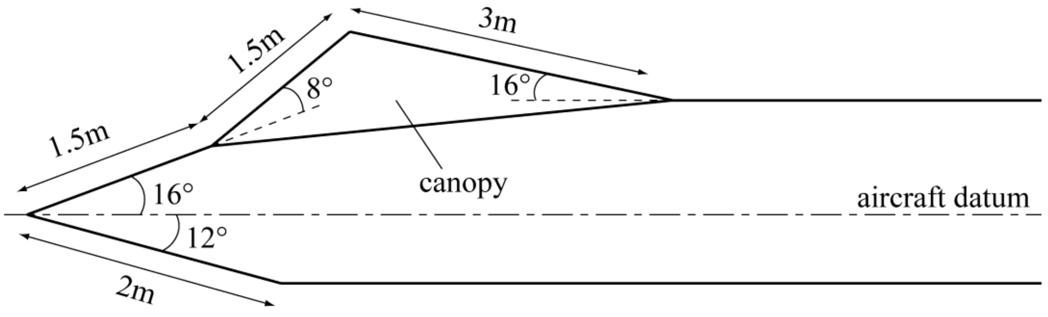

Fig. 1(a) (not to scale)

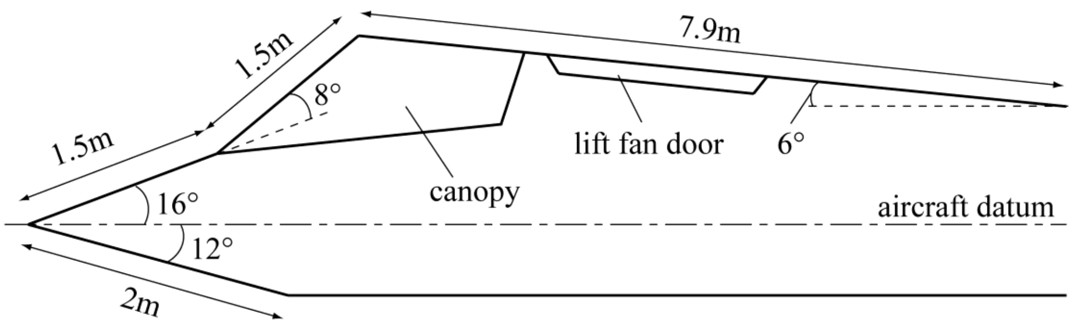

Fig. 1(b) (not to scale)

6 (a) Explain what is meant by *upwind differences* in the context of the numerical solution of hyperbolic partial differential equations. [15%]

(b) The one-dimensional scalar advection equation

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0$$

where A is a positive constant, is to be modelled using the following difference equation

$$u_i^{n+1} = u_i^n - c\left(u_i^n - u_{i-1}^n\right)$$

where time $t = n\Delta t$, distance $x = i\Delta x$ and $c = A\Delta t/\Delta x$. Show that the equivalent differential equation of the scheme takes the form

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = \alpha\frac{\partial^2 u}{\partial x^2}$$

and find an expression for the constant α. Hence determine the accuracy of the numerical scheme. [45%]

(c) At time $t = 0$, u_i^0 is a square pulse as shown in Fig. 2. Describe, with the aid of sketches, the evolution of u_i^n with time for:

(i) $c = 1$,

(ii) $c = 0.5$ and

(iii) $c = 1.5$ [40%]

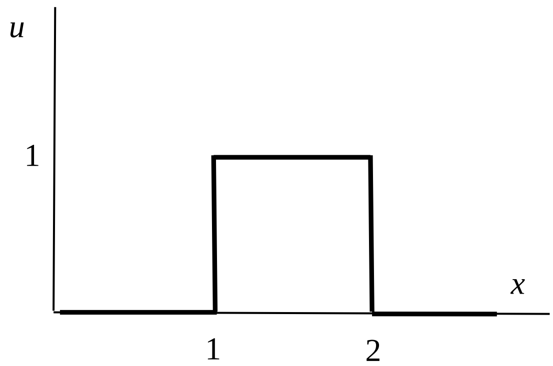

Fig. 2

7 (a) (i) Using a finite volume approximation on a uniform two-dimensional Cartesian mesh, show that Laplace's equation can be discretised as

$$\frac{\phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j}}{\Delta x^2} + \frac{\phi_{i,j+1} - 2\phi_{i,j} + \phi_{i,j-1}}{\Delta y^2} = 0$$ [35%]

(ii) Show that this is second order accurate. [15%]

(b) (i) Starting from Euler's work equation in the absolute frame, show that it may be written in the relative frame as:

$$h_0^{rel} - \tfrac{1}{2} r^2 \Omega^2 = constant$$

where h_0^{rel} is the relative stagnation enthalpy, r is the radius and Ω is the angular velocity. State the conditions under which the above relationships can be applied. [20%]

(ii) An incompressible fluid with density $1000\ \text{kg}\,\text{m}^{-3}$ passes through an adiabatic radial flow rotor which rotates at $700\ \text{rpm}$. At the inner radius, $r_A = 0.1\ \text{m}$, the relative stagnation pressure, p_{0A}^{rel}, is $1\ \text{bar}$ and at the outer radius, $r_B = 0.2\ \text{m}$, the relative stagnation pressure, p_{0B}^{rel}, is $2\ \text{bar}$. Giving your reasons, determine whether this is a radial outflow compressor or radial inflow turbine. [30%]

8 The flow entering a single-stage axial flow turbine has stagnation pressure, p_{01}, of $8\,\text{bar}$ and stagnation temperature, T_{01}, of $1200\,\text{K}$ and the mass flow rate is $50\,\text{kg}\,\text{s}^{-1}$. The turbine rotates at $10{,}000\,\text{rpm}$ and the flow has a constant axial velocity at a mean radius of $0.3\,\text{m}$. The flow coefficient is 0.7, the stage loading coefficient is 1.4, there is no exit swirl and the overall total-to-total isentropic efficiency is 90%. You may assume that the working fluid is a perfect gas with $R = 287\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$, $\gamma = 1.333$ and $c_p = 1149\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$.

(a) Determine the turbine power output and the stagnation pressure, p_{03}, at exit. [20%]

(b) Determine the (absolute) flow angle, α_2, downstream of the stator and the Mach number of the flow at that location. [20%]

(c) If the stagnation pressure downstream of the stator, p_{02}, is $7.85\,\text{bar}$ determine the stagnation pressure loss coefficient for the stator blade row. [20%]

(d) Determine the total-to-total isentropic efficiency of the flow through the rotor. [20%]

(e) Comment on the choice of zero exit swirl at turbine exit if:

(i) the exit flow enters a combustion chamber;

(ii) there is another turbine downstream. [20%]

END OF PAPER

Numerical Answers

1 (b) 0.015 m^2 (c) 0.03 m^2 (d) 242 kW (e) 157 kW

2 (b) 1.58 (c) 1.35 (d) 1.71

3 (b) $Fr_2^2 = \frac{1}{2}\frac{h_1}{h_2}\left(\frac{h_1}{h_2}+1\right)$ (c) .784 ms^{-1} (d) 298 s

5 (b) 1.105 (c) 21.8 kN

8 (a) 6.91 MW, 4.99 bar (b) 63.4° , .760 (c) .062 (d) 93 %