Version EM/3

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 2 May 2019   14.00 to 15.40

Module 3A5

THERMODYNAMICS AND POWER GENERATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Starting from the definition of the specific Helmholtz function $f = u - Ts$, derive the Maxwell relation

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

where the symbols have their usual meanings. Derive also a second Maxwell relation starting from the definition of the specific Gibbs function. [15%]

(b) A particular gas obeys the relation

$$p(v-b) = RT$$

where b is a constant, R is the specific gas constant and the other symbols have their usual meanings. By considering the partial derivative $(\partial u/\partial v)_T$, determine the most general form for the variation of specific internal energy u in terms of temperature T and specific volume v. Briefly explain your result from a molecular perspective. [35%]

(c) Figure 1 shows a sketch of the isotherms for a van der Waal's fluid, for which the equation of state is

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

(i) Determine expressions for the constants a and b in terms of the specific gas constant R, the critical temperature T_c and the critical specific volume v_c. Hence calculate a value for the compressibility of a van der Waal's fluid at the critical point

$$Z_c = \frac{p_c v_c}{RT_c}$$

where p_c is the critical pressure. [30%]

(ii) Figure 1 also shows the saturation line and the 'Maxwell construction' f-m-g for an isotherm in the two-phase region, where f and g here denote wet and dry saturated conditions respectively. Show that the two shaded areas f-k-m and m-l-g for this construction must be equal. [20%]

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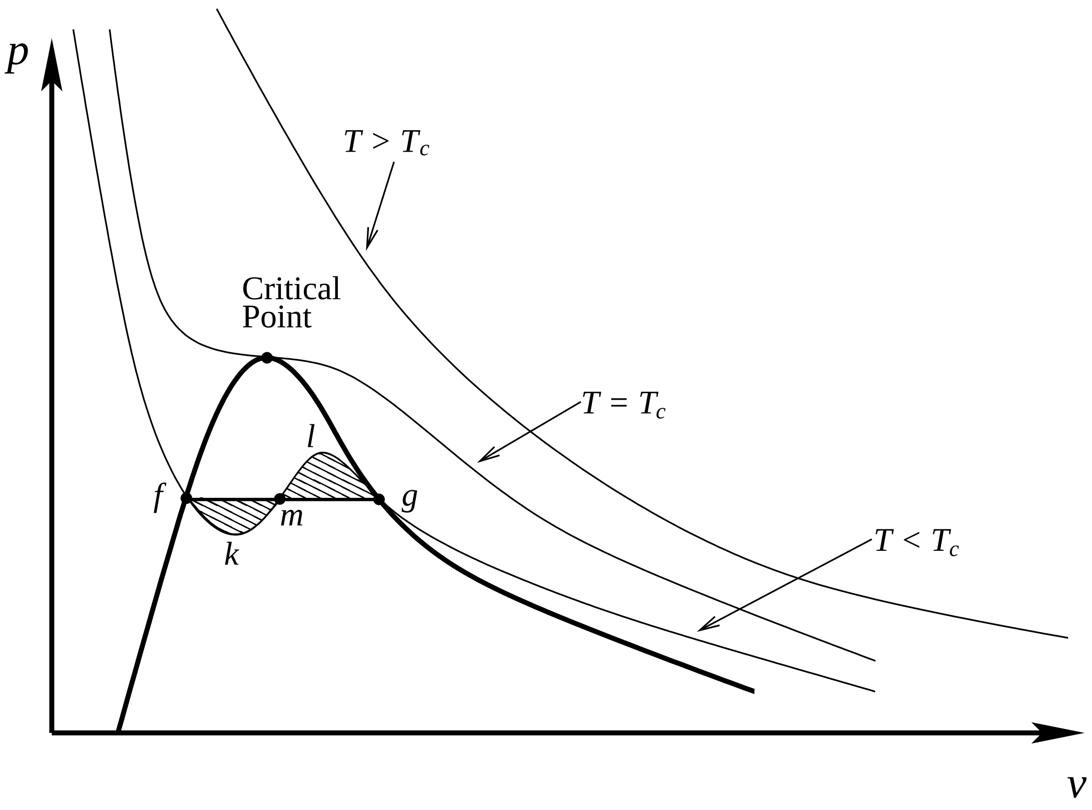

Fig. 1

2 Tests are being carried out on an adiabatic combustion chamber as part of a rocket propulsion system. The chamber is supplied with pure O_2 and pure H_2, each at 1 bar and 25 ^{o}C. The equivalence ratio (defined as the actual fuel to oxygen ratio divided by the stoichiometric fuel to oxygen ratio) is $\phi > 1$. The combustion gases leave the chamber at 1 bar, and the design exhaust temperature is 2200 K.

(a) If the combustion gases contain only H_2 and H_2O, determine the values of ϕ required to achieve the design temperature, assuming all species :

(i) have a constant isobaric molar heat capacity of $30\,\mathrm{kJ\,kmol^{-1}\,K^{-1}}$; [20%]

(ii) behave as semi-perfect gases. [10%]

Comment on the validity of each of these two assumptions. [10%]

(b) In practice, a small amount of dissociation occurs at this temperature such that four additional species are present in the combustion gases in low concentrations.

(i) By considering the possible reactions in the 'Thermochemical Data for Equilibrium Reactions' section of the Data Book, identify the four additional species. [5%]

(ii) Stating your assumptions, estimate the mole fractions of each of these four species. Comment on the validity of your assumptions. [45%]

(c) The combustion chamber is now operated at higher pressure but ϕ remains unchanged. Explain fully whether you would expect the combustion gas temperature to increase or decrease. [10%]

3 (a) For an ideal air-standard Joule cycle, show that the efficiency η is given by $\eta = 1 - 1/r_p^{(\gamma-1)/\gamma}$, where r_p is the pressure ratio. [10%]

(b) Consider an air-standard ideal recuperated Joule cycle. The temperature at the inlet of the compressor is T_1 and the temperature at the inlet of the turbine is T_3. Derive an expression for the efficiency of the cycle in terms of r_p and θ, where $\theta = T_3/T_1$. [40%]

(c) Consider now a realistic air-standard recuperated Joule cycle, where the compressor and turbine polytropic efficiencies are 0.9, the recuperator effectiveness is 0.8, $r_p = 6$, T_1=300 K, and T_3=1800 K. Ignore the fuel flow rate. Calculate the cycle thermal efficiency. [40%]

(d) Discuss the advantages and disadvantages of recuperation in gas turbines. [10%]

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4 Consider an ideal Rankine cycle with reheat and a single direct-contact feedheater, where the exit of the feedheater is saturated liquid. The steam for the feedheater is bled from the exhaust of the high-pressure turbine. The mass fraction of steam bled from the high-pressure turbine is m.

(a) Sketch the cycle in a $T-s$ diagram, carefully showing the saturation line and the conditions at the inlet and exit of each component. [20%]

(b) Derive an expression for the efficiency of the cycle in terms of m and the specific enthalpies at various points in the cycle. Discuss why the efficiency of the cycle with feedheating is greater than that without feedheating. [40%]

(c) The condenser operates at 0.06 bar and the steam leaves the boiler at 150 bar and 550 °C. After expanding at the high-pressure turbine to a pressure of 40 bar, the steam is reheated to 550 °C. Assume the turbines to be isentropic and neglect feedpump work. Calculate the fraction m, the power output for each turbine and the rates of heat transfer in the main boiler, reheater, and condenser per kg of steam flowing through the boiler. Calculate also the cycle thermal efficiency. [40%]

END OF PAPER