Version wnd/2

EGT2
ENGINEERING TRIPOS PART IIA

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Monday 29 April 2019        9.30 to 11.10

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**Module 3A6**

**HEAT AND MASS TRANSFER**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 Consider the incompressible flow past a flat plate with zero streamwise pressure gradient and with the plate maintained at constant wall temperature $T_s$ .

(a) Sketch the development of the flow and thermal boundary layers from the leading edge of the plate. Explain the physical significance of the Prandtl number (Pr) and comment on the relative thicknesses of the flow and thermal layers, $\delta$ and $\delta_T$ .

[20%]

(b) Define the Stanton number (St) and write down the relationship between the Stanton, Nusselt (Nu), Prandtl and Reynolds (Re) numbers and explain, briefly, the physical significance of each number.

[20%]

(c) By control volume analysis, or otherwise, derive the integral equation relating the thermal thickness $\delta_\theta$ to the Stanton number using the definition:

$$\delta_\theta = \int_0^\infty \left(\frac{T_\infty - T}{T_\infty - T_s}\right)\frac{u}{U_\infty}\,dY$$

where $Y$ is the distance normal to the plate.

[20%]

(d) Assuming cubic profiles for both the flow and thermal boundary layers,

$$\frac{u}{U_\infty} = \frac{3}{2}\left(\frac{Y}{\delta}\right) - \frac{1}{2}\left(\frac{Y}{\delta}\right)^3 \qquad \frac{T - T_s}{T_\infty - T_s} = \theta = \frac{3}{2}\left(\frac{Y}{\delta_T}\right) - \frac{1}{2}\left(\frac{Y}{\delta_T}\right)^3$$

and that the Prandtl number is unity, derive a relationship between $\delta_\theta$ and $\delta$.

[20%]

(e) Given the Blasius result $\delta/X = 5.0/\mathrm{Re}^{1/2}$ derive a relationship between the Nusselt, Prandtl and Reynolds numbers and comment on the result. $X$ is the streamwise distance.

[20%]

2 (a) By considering a small control volume $\Delta X$ by $\Delta Y$ derive the following transport equation for mass fraction $Y_i$

$$\frac{\partial}{\partial t}(\rho Y_i) + \nabla.(\rho\bar{u}Y_i) = -\nabla.(G_{di}) + \dot{\omega}_i$$

Explain the physical significance of each of the terms.

[25%]

(b) Show that summing this transport equation over all species present leads to the standard bulk mass conservation equation.

[20%]

(c) Now, consider the flow over a flat plate and sketch the development of the flow and species concentration boundary layers. State Fick's law and, hence, define the mass transfer coefficient, $h_m$ . Compare and discuss this definition relative to the heat transfer coefficient $h$ and friction coefficient $c_f$ .

[20%]

(d) Consider now the flow in a two-dimensional channel with constant width $H$. After a long, straight section the channel turns a $90^0$ bend before continuing. Assume the flow and species concentration boundary layers are thin, that the boundary layers remain attached and that the bulk flow in the bend can be approximated by a free vortex velocity distribution. Where will the peak mass transfer from the channel to the flow take place?

Given that the Sherwood number is related to the Schmidt and Reynolds numbers by the following,

$$\mathrm{Sh}_x \sim \mathrm{Sc}^{1/3}\,\mathrm{Re}_x^{\,4/5}$$

estimate the relative peak mass transfer between two cases when the inner radius of the bend is equal to $H$ and to $2H$ respectively and comment on the result.

[35%]

**(TURN OVER**

3 (a) Two large plates are held at temperatures $T_1$ and $T_2$ . A thermocouple, modelled as a sphere of diameter $D_{TC}$ with emissivity $\varepsilon_{TC}$ , is placed between the plates as shown in Fig.1. Radiative heat exchange brings the thermocouple and plates into radiative equilibrium. You may assume that the instantaneous temperature within the thermocouple, $T_{TC}$, is spatially uniform and ignore the effect of its support structure.

(i) Show that the view factors from the thermocouple to each of the two plates $F_{TC,1} = F_{TC,2} = 0.5$ . [10%]

(ii) Draw an equivalent resistance network for the arrangement whilst the thermocouple is reaching thermal equilibrium and derive expressions for the various surface and space resistances. [20%]

(iii) Using the network, or otherwise, derive an expression for the net heat transfer to the thermocouple in terms of $T_{TC}$, $\varepsilon_{TC}$, $T_1$, $T_2$, and $D_{TC}$ . [25%]

(iv) Derive an expression for the equilibrium temperature of the thermocouple. [15%]

(b) Once in equilibrium, the thermocouple arrangement in Part (a) is then exposed to a flow of air at $T_{air}$ with average heat transfer coefficient $h$.

(i) Using your result from Part (a) (iii) derive an expression for the steady state difference (i.e. error) in temperature between the flow and the thermocouple. [10%]

(ii) For the case when $T_{air}$ = 400 K, $T_1$ = 500 K, $T_2$ = 300 K, $h$ = 50 W m$^{-2}$ K$^{-1}$ and $\varepsilon_{TC}$ = 0.5 evaluate the error (you will need to use an iterative method). [10%]

(iii) Suggest how this error could be reduced. [10%]

**(cont**

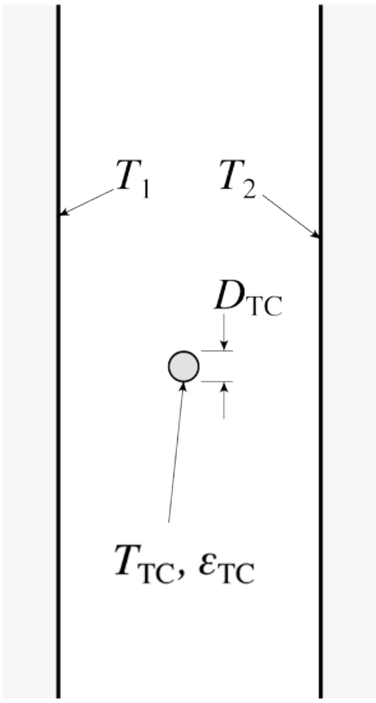


**Fig.1**

**(TURN OVER**

4 A two-dimensional model of a circuit element is shown in Fig.2. It has uniform thermal conductivity $\lambda$. The lower edge, at $y = 0$, is held at uniform temperature $T_b$. A portion of the upper edge, $y = H$, from $x = 0$ to $x = a$ is subject to a heat flux of $\dot{q}$ per unit width. All other edges are perfectly insulated.

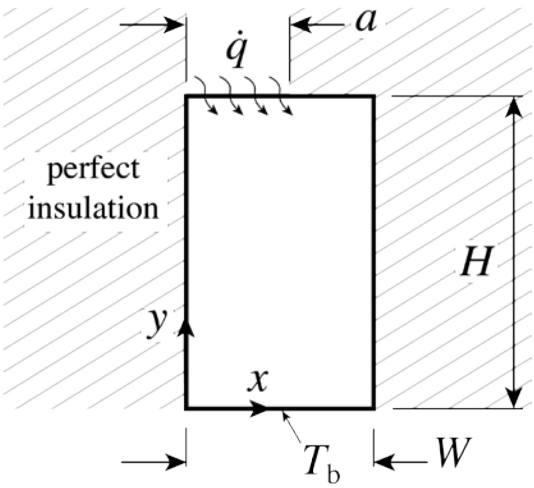


**Fig.2**

(a) Derive an approximation for the temperature field, $\theta = T(x, y) - T_b$ , when $a \to W$. Give your answer in terms of $\dot{q}$, $a$, $W$ and $\lambda$. Is this an upper or lower bound estimate for the temperature in the general case? [10%]

(b) Sketch isotherms and adiabats within the element for $a = W$ and $a << W$. [10%]

(c) Considering a solution of the form $\theta = X(x)Y(y)$, use separation of variables to show that the temperature field in the element can be satisfied by the expression

$$\theta = \sum_{n=0}^{\infty} \cos m_n x \, (A_n \sinh m_n y + B_n \cosh m_n y)$$

where $m_n = n\pi/W$ and $A_n$ and $B_n$ are undetermined constants. [40%]

(d) Considering the $y$ = constant boundaries, derive expressions for the constants $A_n$ and $B_n$ in order to find a solution for the temperature field. [40%]

**END OF PAPER**