EGT2
ENGINEERING TRIPOS PART IIA

---

Tuesday 23 April 2019 9.30 to 11.10

---

**Module 3B5**

## SEMICONDUCTOR ENGINEERING

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number* ***not*** *your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 A GaAs crystal of thickness $t$, length $l$ and width $w$ is measured in a Hall effect experiment as illustrated in Fig. 1. A current of density $J$ flows in the positive $x$-direction and the magnetic field $B$ is applied in the positive $z$-direction. The Hall voltage $V_H$ is defined as illustrated in Fig. 1.

(a) In GaAs at room temperature, the intrinsic carrier density is $2.1\times10^{12}$ m$^{-3}$ and the effective densities of states in the valence band and in the conduction band are $9.0\times10^{24}$ m$^{-3}$ and $4.5\times10^{23}$ m$^{-3}$, respectively. Calculate the band gap of GaAs. [10%]

(b) Show that the magnitude of the measured voltage $V_H$ is given by

$$|V_\mathrm{H}| = \frac{JB}{Ne} w,$$

where $e$ is the electronic charge and $N$ is the density of majority carriers in the GaAs crystal. State any assumptions made. [25%]

(c) A positive Hall voltage $V_H$ of 25 μV is established across the GaAs crystal when the applied current density $J$ is 100 Am$^{-2}$ and the applied magnetic field $B$ is 0.1 T.

(i) Explain whether the majority carriers are electrons or holes. [10%]

(ii) Given that the width $w$ is 1 cm, calculate the concentration of dopants and the concentration of minority carriers. State any assumptions made. [20%]

(iii) Calculate the position of the Fermi level above the valence band edge. [10%]

(iv) A laser pulse of wavelength 800 nm and pulse energy $P$ illuminates the GaAs crystal uniformly and generates a hole density of $1\times10^{23}$ m$^{-3}$. If the mobilities of electrons and holes are 0.9 m$^2$V$^{-1}$s$^{-1}$ and 0.04 m$^2$V$^{-1}$s$^{-1}$, respectively, calculate the conductivity of the sample immediately after the laser pulse is incident. [15%]

(v) A laser pulse of wavelength 1600 nm and the same pulse energy $P$ illuminates the GaAs crystal uniformly. Calculate the conductivity of the sample immediately after the laser pulse is incident. [10%]

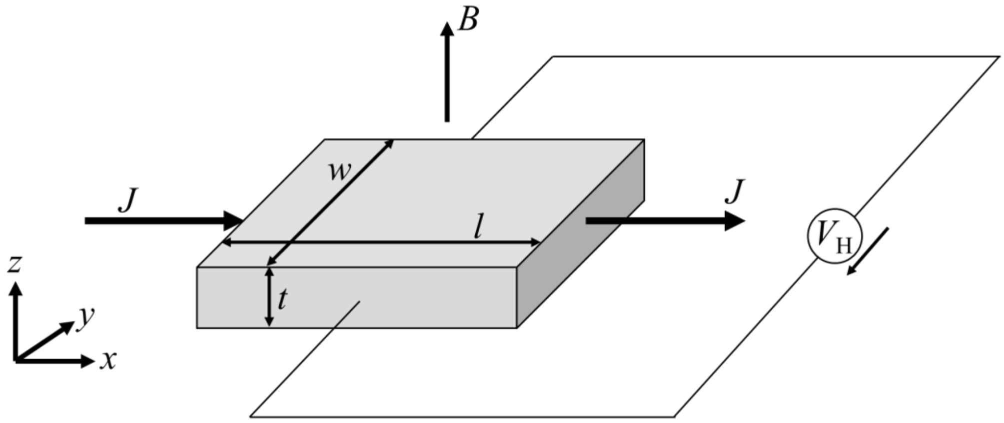

Fig. 1

2 (a) (i) Starting from expressions for the electron and hole carrier densities, show that the contact potential $V_0$ of an abrupt p–n junction can be given by

$$V_0 = \frac{kT}{e}\ln\left(\frac{N_\mathrm{A}N_\mathrm{D}}{n_\mathrm{i}^2}\right),$$

where $N_A$ and $N_D$ are the doping densities in the p- and n-regions, respectively, and $n_i$ is the intrinsic carrier concentration. State all assumptions made. Sketch an energy band diagram of the junction indicating $V_0$. [20%]

(ii) Outline how $V_0$ could be measured. [10%]

(b) A p–n diode is fabricated where the n-region has a resistivity of $2\times10^{-4}$ Ωm and the p-region a resistivity of $4\times10^{-3}$ Ωm. Calculate the percentage change in contact potential if the doping concentration of the n-side were to be increased by a factor of 100. Assume electron and hole mobilities of 0.13 and 0.05 $\mathrm{m^2V^{-1}s^{-1}}$, respectively, and an intrinsic carrier density of $1.5\times10^{16}$ $\mathrm{m^{-3}}$. [15%]

(c) A p–n junction is formed from Si with a concentration of acceptors in the p-region of $N_\mathrm{A}=10^{24}$ $\mathrm{m^{-3}}$, and a concentration of donors in the n-region of $N_\mathrm{D}=10^{21}$ $\mathrm{m^{-3}}$.

(i) Starting from the Poisson equation, derive an expression for the peak value of the electrical field across the junction. State all assumptions made. [15%]

(ii) Sketch the electric field across the unbiased junction. Will the depletion region be predominantly in the p-region or n-region? Calculate the peak value of the electric field for an assumed depletion region width of 30 µm. Assume a relative permittivity for Si of 12. Comment how this value compares to the breakdown field of Si which is 30 $\mathrm{MVm^{-1}}$. [15%]

(d) Explain the origins of junction capacitance and charge storage capacitance for a p–n junction. Which capacitance dominates under forward bias conditions? [15%]

(e) Explain how band engineering can be used for a $p^+$–n diode to prevent electron injection into the $p^+$-region under forward bias. Draw a band diagram of such an engineered junction and comment which materials can be used for this. [10%]

3 For a hydrogen atom, the electron potential energy $V$ varies with distance $r$ from the proton according to

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r},$$

where $\varepsilon_0$ is the permittivity of free space. The wavefunction of an electron in the 1s state in this potential is given by

$$\psi(r) = \frac{1}{\sqrt{\pi}}\left(\frac{1}{\alpha}\right)^{3/2} \exp\left(-\frac{r}{\alpha}\right),$$

where $\alpha$ is a constant.

(a) Explain the physical significance of this wavefunction and how its form is governed by the basic postulates of quantum mechanics. [10%]

(b) Derive expressions for the constant $\alpha$ and for the electron's total energy $E$ in terms of fundamental physical constants. Do not calculate numerical values. [30%]

(c) Sketch how the probability of finding the particle varies with distance $r$. Determine the distance $r_{\max}$ at which this probability is maximum. [20%]

(d) If the electron is located at a radial distance $r = \alpha$, its kinetic energy is given by $\hbar^2/(2m\alpha^2)$. Derive an expression for the de Broglie wavelength $\lambda$ of this electron. Comment on this result. [20%]

(e) Two isolated hydrogen atoms bond to form $H_2$. Describe how the electron energy levels in $H_2$ differ from those in the isolated hydrogen atoms. [20%]

Note: The time-independent Schrödinger equation in three dimensions is

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi.$$

The Laplace-equation in spherical polar coordinates is

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2\psi}{\partial\phi^2}\right).$$

4 (a) A metal with a work function of 4.6 eV is deposited on Si that is doped with a boron concentration of $10^{24}$ $\mathrm{m}^{-3}$.

(i) Explain if this results in a Schottky or Ohmic contact. Draw an equilibrium band diagram highlighting the Fermi level, band edges and vacuum level across the contact. Assume the Si has a band gap of 1.12 eV, an electron affinity of 4.05 eV, and effective densities of states in the conduction and valence bands of $N_C = 2.8\times10^{25}$ $\mathrm{m}^{-3}$ and $N_V = 1.04\times10^{25}$ $\mathrm{m}^{-3}$, respectively. Assume no interface traps and room temperature (300 K) conditions. State all other assumptions. [15%]

(ii) How much should the metal work function be altered to change the type of contact? Draw an equilibrium band diagram for such an altered contact. [10%]

(b) A metal with an effective work function of 4 eV is now deposited onto Si that is doped with a donor concentration of $10^{23}$ $\mathrm{m}^{-3}$ but has an insulating oxide layer on its surface. The $SiO_2$ thickness is 15 nm and its relative permittivity is 3.9. Assume that the resistivity of the oxide is infinite, i.e. that no carrier transport is possible through that layer.

(i) Draw a band diagram of the resulting structure without any external bias applied. State all assumptions made. [15%]

(ii) Calculate what external voltage needs to be applied to the metal with respect to the Si to achieve the so called flat band condition. [10%]

(iii) Calculate the threshold voltage which has to be applied to the metal to induce strong inversion. Assume room temperature conditions (300 K) and that the width of the depletion region in the n-type Si is 100 nm at strong inversion. The intrinsic carrier concentration of Si is $1.5\times10^{16}$ $\mathrm{m}^{-3}$. State any other assumptions made. [30%]

(c) The Metal-Oxide-Semiconductor (MOS) structure described in (b) is used to form a MOS Field Effect Transistor (MOSFET).

(i) Sketch the output characteristics of such a MOSFET. Sketch how the inversion layer varies across the channel at a drain-source bias beyond the so-called pinch-off point. [10%]

(ii) Sketch how the gate structure can be modified to create a so-called Flash memory cell structure that has non-volatile memory. [10%]

**END OF PAPER**

**Numerical answers**

1a: 1.4 eV

1c(i): holes

1c(ii): $N_A = 2.5\times10^{22}$ m$^{-3}$, n = 176 m$^{-3}$

1c(iii): 150 meV

1c(iv): $1.52\times10^{4}$ $\Omega^{-1}$m$^{-1}$

1c(v): 160 $\Omega^{-1}$m$^{-1}$

2b: 15% increase

2c(ii): 45 MV/m

4a(ii): >+0.51 eV

4b(ii): -0.2 V

4b(iii): -1.7V