Version DC/3

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 1 May 2019 9.30 to 11.10

Module 3C6

VIBRATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C5 Dynamics and 3C6 Vibration data sheet (6 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A shaft of length L, cross-sectional area A and polar second moment of area J is made of a material with density ρ and shear modulus G, as illustrated in Fig. 1. One end of the shaft is prevented from rotating (at $x = 0$) and the other end is free (at $x = L$). The shaft can undergo small amplitude torsional oscillations.

(a) Write down the appropriate boundary conditions at each end and find expressions for the mode shapes $u_n(x)$ and natural frequencies ω_n. [20%]

(b) A torque T is applied at the free end, $x = L$.

(i) Derive an expression for the transfer function $H_1(L, x, \omega)$ from T to the output angular displacement θ at an arbitrary position x. Your answer should be expressed as a summation and should be in terms of the properties of the shaft. [20%]

(ii) By appropriately differentiating your answer to (b)(i) or otherwise, derive an expression for the transfer function $H_2(L, 0, \omega)$ from T to the reaction torque at the boundary $x = 0$. [20%]

(iii) Find the transfer function $H_3(L, 0, \omega)$ from input angular displacement at $x = L$ to reaction torque at $x = 0$ in terms of $H_1(L, L, \omega)$ and $H_2(L, 0, \omega)$, noting that H_1 is evaluated at the output position $x = L$. [10%]

(iv) Sketch the transfer functions $H_1(L, L, \omega)$ and $H_3(L, 0, \omega)$, labelling both resonant and anti-resonant frequencies. [30%]

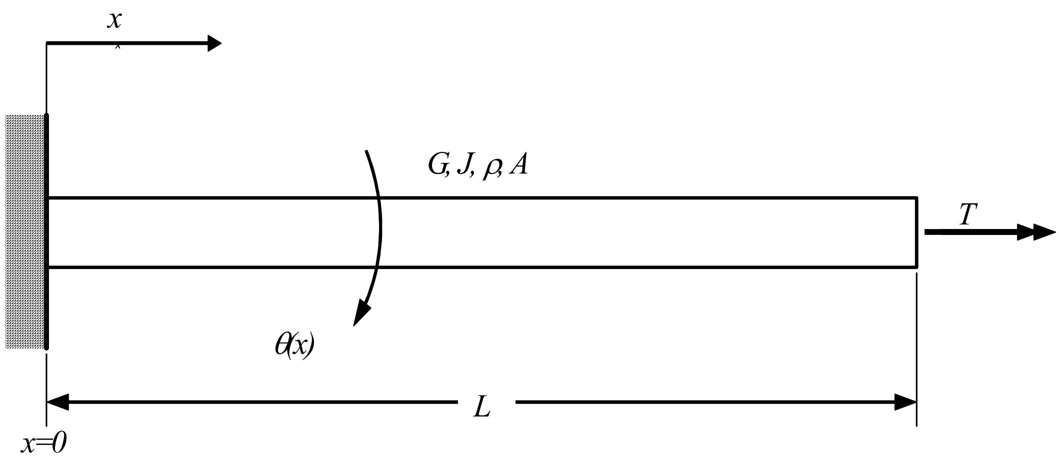

Fig. 1

2 A beam of length L and uniform cross-section with second moment of area I, is made of material with density ρ and Young's modulus E. The beam is clamped at both ends and undergoes small-amplitude transverse vibration.

(a) Starting from the governing equation for transverse vibration of a beam, derive an expression whose solutions give the wavenumbers k_n for the modes of the beam. [20%]

(b) Using a graphical construction, estimate the first six natural frequency ratios of the beam, i.e. estimate ω_n/ω_1 for $1 \le n \le 6$. [20%]

(c) A measurement is carried out to find the driving point transfer function at the centre of the beam G_a. An instrumented hammer is used to strike the beam at the centre, and the response is measured using an accelerometer mounted at the centre. The accelerometer has a small but non-zero mass m, resulting in a measured transfer function G_m.

(i) Sketch the driving point transfer functions G_m and G_a, i.e. both with and without including the effect of the added mass of the accelerometer. Include in your sketch frequencies up to and including ω_6. [20%]

(ii) By considering the coupled driving point transfer function at the accelerometer, derive an expression to calculate the actual transfer function G_a (without the mass) from the measured transfer function G_m (with the mass). [30%]

(iii) Under what conditions will the mass-compensated estimate be best and why? [10%]

3 The gearbox system shown in Fig. 2 has two discs (A and B), both with polar moments of inertia $4J$, connected by a light shaft of torsional stiffness $2k$ and two discs (C and D), both with polar moments of inertia J, connected by a light shaft of torsional stiffness k. The shafts are supported by frictionless bearings. Discs B and C are gears with pitch circle radii of $2R$ and R, as shown. The gears mesh perfectly, with no backlash. Small torsional rotations from equilibrium are defined by the coordinate vector $\begin{bmatrix}\theta_1 & \theta_2 & \theta_4\end{bmatrix}^{\mathrm{T}}$, noting that θ_3 is determined by θ_2.

(a) By calculating the potential energy or otherwise, show that the stiffness matrix for small vibration of this system is

$$\begin{bmatrix} 2k & -2k & 0 \\ -2k & 6k & 2k \\ 0 & 2k & k \end{bmatrix}$$ [20%]

(b) Find the natural frequencies and corresponding mode shapes. Illustrate these mode shapes with sketches, including θ_3 and the approximate position of any nodes. [60%]

(c) Sketch a log-amplitude plot of the transfer function describing the rotation of disc D due to a sinusoidal torque applied to disc A. Comment on the presence or absence of anti-resonances in the transfer function. [20%]

 (cont...

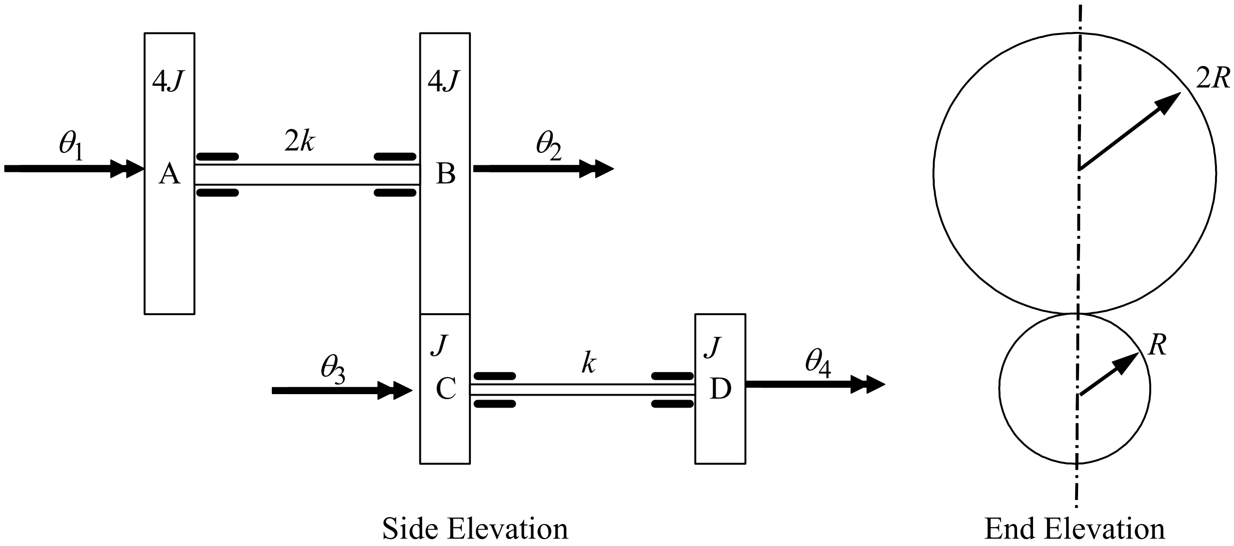

Fig. 2

(TURN OVER

4 A uniform straight rigid rod of length L and mass M is supported at its ends by two equal springs of stiffness k, whose other ends are attached to horizontal ground. A mass m is attached to the rod at position x as shown in Fig. 3. The rod undergoes small vibration confined to the vertical plane. The vertical displacement of the centre of the rod is y and its angle to the horizontal is θ.

(a) For the case where $m = 0$, write down the mode shapes of the system. Calculate the corresponding natural frequencies. [30%]

(b) For the case where $m = \varepsilon M$ ($\varepsilon \ll 1$), use Rayleigh's principle to show that the lower natural frequency is given by:

$$\omega^2 = \left(\frac{2}{1+\varepsilon}\right)\frac{k}{M}$$

and find a corresponding expression for the higher natural frequency. [40%]

(c) What position of the additional mass m makes the difference between the two natural frequencies:

(i) largest

(ii) smallest?

Justify your answers. [30%]

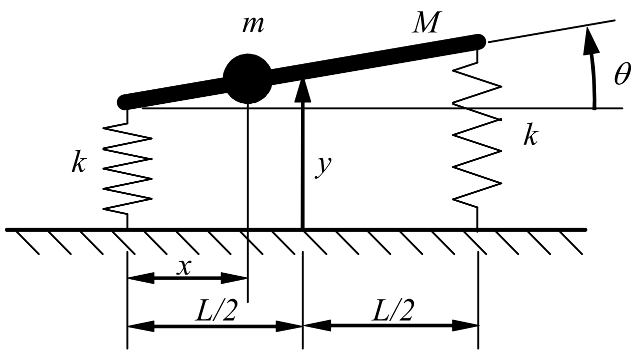

Fig. 3

END OF PAPER