EGT2
ENGINEERING TRIPOS PART IIA

Monday 29 April 20192 to 3.40

## Module 3C8

## MACHINE DESIGN

Answer not more than three questions.
All questions carry the same number of marks.
The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

Engineering Data Book
CUED approved calculator allowed
Attachment: Module 3C8 data sheet (9 pages).

## 10 minutes reading time is allowed for this paper at the start of the exam. <br> You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## Version DJC/3

1 Figure 1 shows a continuously variable transmission consisting of two rotating cones contacting a wheel that is attached to a layshaft (intermediate shaft). One of the contacts is labelled A. Normal force in the contacts is provided by a mechanism that is not shown. Both cones have half-cone angle $\alpha$, minimum radius $R_{1}$ and maximum radius $R_{2}$. The wheel has radius $r_{a}$, thickness $t$, and a barrelled surface with radius of curvature $r_{b}$. Bearings, indicated by hatched areas, support the cones on their respective shafts, allowing angular velocities $\omega_{\text {in }}$ and $\omega_{\text {out }}$. Further bearings allow the layshaft and wheel to rotate, with angular velocity $\omega_{\text {lay }}$, and also to be positioned axially (by a mechanism not shown), with the resulting axial position $x$ determining the ratio of input to output speeds. The axes of the cones are parallel to one another, and the layshaft and edges of the cones indicated by // are parallel to one another.
(a) Explain why it is advantageous that the wheel is barrelled, rather than being cylindrical in shape.
(b) Find an expression for the input to output speed ratio.
(c) Assuming no slip, find an expression for the spin speed at contact A.
(d) For contact A , find expressions for the effective radii of curvature of the contact:
(i) in the plane of the shafts; and
(ii) in the plane of the wheel.
(e) Determine the efficiency of power transfer from the input cone to the wheel when the input torque to the cone is half of its limiting value. Assume the contact is circular and of radius 0.25 mm . Assume that the spin speed divided by the rolling speed of the contact is $10 \mathrm{~m}^{-1}$.


Fig. 1

## Version DJC/3

2 Figure 2 shows schematically a compound epicyclic gearbox. The primary input shaft, which rotates at speed $\omega_{i}$, is connected to the sun wheels $S_{1}$ and $S_{2}$ of the first and second epicyclics, respectively. The planet carrier $\mathrm{C}_{1}$ is connected to the annulus $\mathrm{A}_{2}$. The planet carrier $\mathrm{C}_{2}$ is connected to the output shaft. The annulus $\mathrm{A}_{1}$ is connected to a secondary input shaft which rotates at a speed $\alpha \omega_{i}$. The ratio of annulus tooth number to sun tooth number for both the first and second epicyclics equals $R$.
(a) Show that the speed $\omega_{o}$ of the output shaft is given by

$$
\omega_{o}=\frac{1+2 R+\alpha R^{2}}{(1+R)^{2}} \omega_{i}
$$

(b) For the case of $R=4, \alpha=0.5$ :
(i) find expressions in terms of the torque $T_{i}$ on the primary input shaft for the torques on the output shaft and the secondary input shaft, identifying the directions of these torques;
(ii) show that the power transmitted through $\mathrm{S}_{2}$ equals $\frac{5}{9} \omega_{i} T_{i}$.
(c) The two epicyclic gears are geometrically self-similar, with the same tooth numbers and with modules $m_{1}$ and $m_{2}$ for the first and second epicyclic gears, respectively. The face widths of the two epicyclics are proportional to their modules. Assuming that failure in both epicyclics is due to surface failure with the same contact stresses, find the ratio $m_{1} / m_{2}$ which would correspond to simultaneous failure of the two epicyclics.


Fig. 2

## Version DJC/3

3 (a) In the form of a table, compare the performance of deep groove ball bearings, cylindrical roller bearings, taper roller bearings and spherical roller bearings in terms of their ability to accommodate radial load, axial load, angular misalignment, and axial displacement.
(b) Calculate the life (in revolutions) of a cylindrical roller bearing with designation NU1010 when the equivalent radial load is 20 kN , the oil viscosity is $8 \mathrm{~mm}^{2} \mathrm{~s}^{-1}$ and the rotation speed is 500 revolutions per minute. The reliability is required to be $95 \%$.
(c) Figure 3 shows a rotating shaft supported by two taper roller bearings. Bearing A has axial stiffness $100 \mathrm{MN} \mathrm{m}{ }^{-1}$. Bearing B has axial stiffness $50 \mathrm{MN} \mathrm{m}{ }^{-1}$. External axial and radial forces $P_{a}$ and $P_{r}$ can be applied to the shaft. The shaft and bearings are initially assembled with zero preload and zero axial clearance.
(i) State the disadvantages of having zero preload when external axial force $P_{a}$ is applied to the rotating shaft. What additional problem can arise when external radial force $P_{r}$ is applied to the shaft?
(ii) For the case of zero external radial force, determine the minimum preload force and the corresponding preload displacement necessary to ensure that there is no axial clearance in either bearing when an external axial force $P_{a}$ of 10 kN is applied to the shaft in the direction shown in the figure.
(iii) Explain, with the aid of a diagram, how the application of a radial force $P_{r}$ would affect the minimum preload force required in part (ii).


Fig. 3

## Version DJC/3

$4 \quad$ An electric vehicle of mass $m$ has a motor driving a wheel of radius $R$ through a speed ratio $n=\omega / \Omega$, where $\omega$ is the motor speed and $\Omega$ is the wheel speed. The electric motor's maximum output torque is 100 N m at zero speed and 0 N m at $\omega=1000 \mathrm{rad} \mathrm{s}^{-1}$, with a linear variation of torque between these two speeds. The force resisting motion of the vehicle travelling up a gradient can be taken as $m g \alpha$ where $g$ is the force per unit mass due to gravity and $\alpha$ is the gradient. Speed-dependent resistance can be neglected.
(a) Determine the motor speeds between which the motor is able to generate at least $90 \%$ of its maximum power output.
(b) Sketch a graph of the maximum output torque of the motor against speed of the motor. On the same graph, sketch the load characteristic of the vehicle as seen by the motor.
(c) For $R=0.3 \mathrm{~m}$ and $m=1000 \mathrm{~kg}$ determine the minimum number of speed ratios necessary to allow the motor to generate at least $90 \%$ of its maximum power output when ascending gradients $\alpha$ in the range 0.05 to 0.3 , and determine suitable numerical values for the speed ratios.

## END OF PAPER

