EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 23 April 2019 2.00 to 3.40

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number not your name on the cover sheet.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 The Z-section shown in Fig. 1(a) is mounted as a horizontal cantilever and is loaded by its own self-weight as illustrated in Fig. 1(b). The section is made from sheet steel of uniform thickness t, length L, weight per metre w, Poisson's ratio $\nu = 0.3$, elastic modulus E, and shear modulus G. For a Z-section, the restrained warping torsion constant $\Gamma = \frac{d^2}{4} I_{yy}$ (where d is the depth of the section) and the characteristic length $\lambda = \sqrt{\frac{E\Gamma}{GJ}}$.

(a) Determine the second moments of area of the cross-section about its centroid. [20%]

(b) Determine the principal second moments of area of the cross-section and illustrate these clearly on a sketch of the section. [20%]

(c) Determine the deflection of the centroid of the cantilever at the tip. [20%]

(d) Determine the St. Venant's torsion constant J for the cross-section. [5%]

(e) Determine the ratio by which the twist at the tip is reduced by accounting for restrained warping torsion assuming $L = 100t$. [35%]

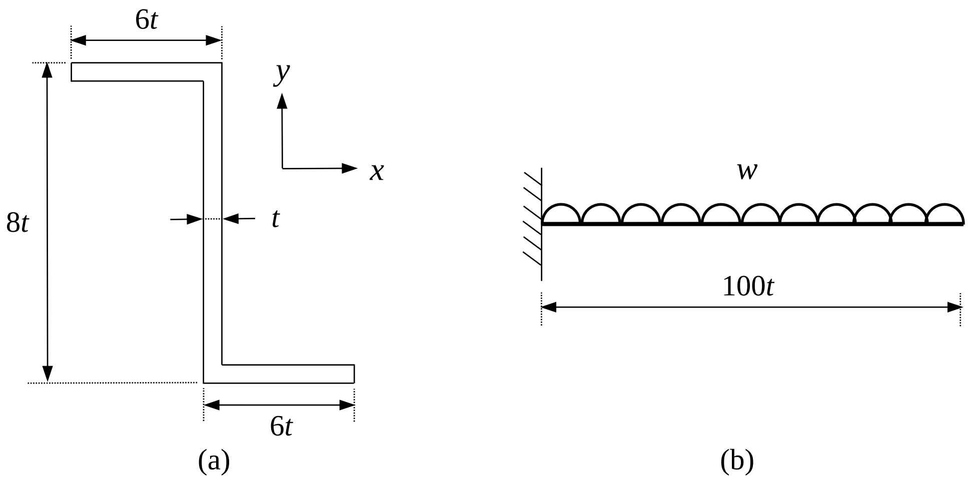

Fig. 1

2 The continuous beam shown in Fig. 2 has flexural rigidity $EI = 2 \times 10^4$ kN m^2, and is loaded on the right-hand span with two point loads, each of 6 kN magnitude.

(a) Determine the reaction forces at each of the supports. [40%]

(b) Develop an expression for the deflection at any point along the beam. [20%]

(c) Calculate the location and magnitude of the maximum deflection. [40%]

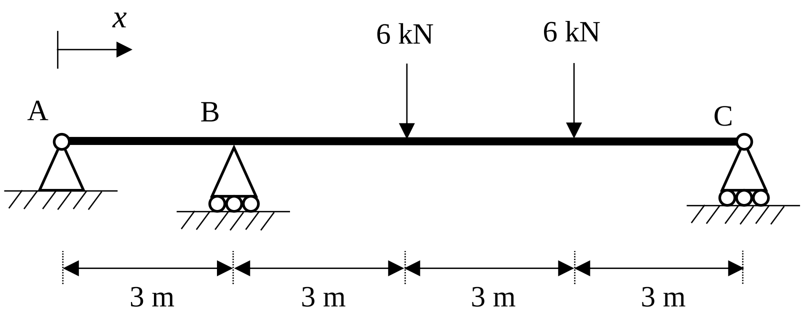

Fig. 2

3 A cantilever column of height L is unstressed when straight, and has uniform flexural rigidity EI. It is fixed at end A where $x = 0$ and a compressive force P is applied axially at the free end B where $x = L$.

Let any small general lateral deflection $w(x)$ be approximated using a polynomial expansion:

$$w(x) = L \sum_{j=2}^{N} \alpha_j \left(\frac{x}{L}\right)^j$$

and let the total potential energy $\Pi(w)$ of the loaded system be written as:

$$\Pi(w) = \frac{1}{2} \sum_{i=2}^{N} \sum_{j=2}^{N} k_{ij} \alpha_i \alpha_j$$

(a) Explain why it is sensible to start the polynomial expansion at $j = 2$. [10%]

(b) Determine an expression for a general coefficient k_{ij} in the total potential energy function. [50%]

(c) Taking a single term approximation with $N = 2$, estimate the critical load P_{cr} for elastic flexural buckling of the column. [10%]

(d) Compare your estimate of P_{cr} with the true critical load, and give reasons for any difference. [10%]

(e) Without further calculation, explain how your estimate could be improved. [10%]

(f) Without further calculation, describe how you could estimate the buckling load of a tapered column. [10%]

4 A simply-supported Universal Beam is uniformly loaded along its span such that it undergoes lateral-torsional buckling.

(a) Provide three sketches to show the patterns of shear stress on a general cross-section that would be associated with resisting the directly applied load, with the St Venant torsion and with the restrained warping torsion. [20%]

(b) The formula for the critical value of equal and opposite end moments that cause lateral-torsional buckling of a beam includes a warping correction factor $\left(1+\frac{\pi^2 E\Gamma}{L^2 GJ}\right)^{1/2}$. Here, all symbols have their usual meanings, and in particular the warping constant $\Gamma = I_{minor} D_f^2/4$ where D_f is the distance between flange centroids.
Determine the critical value of equal and opposite major-axis end-moments that will cause lateral-torsional buckling of a simply-supported steel Universal Beam of length 8 m and of $406 \times 140 \times 46$ section. [20%]

(c) Fig. 3 shows a rigidly-jointed frame ABCDE with two columns and two beams. The supports are pinned at A and fixed at D and E. All members have length L and flexural rigidity EI with respect to bending within the plane of the diagram. Equal vertical loads P are applied at B and C as shown.

(i) Determine a 2×2 tangent stiffness matrix relating the moments and rotations at B and C using s and c stability functions. [20%]

(ii) Using the graph of s and c functions provided in Fig. 4, where P is the load and P_E is the Euler buckling load, estimate the value of P that will cause flexural buckling of the frame. [20%]

(iii) Sketch the buckling mode, and indicate the magnitudes of the rotations of joints A, B, D and E as a proportion of the rotation at C. [20%]

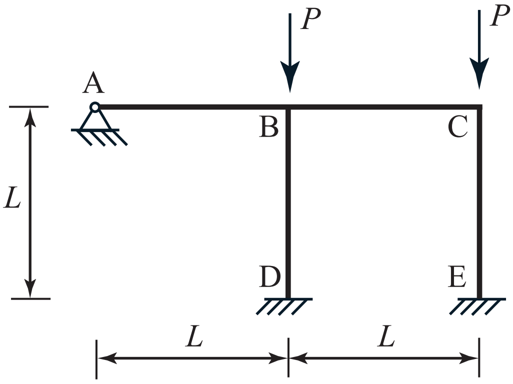

Fig. 3

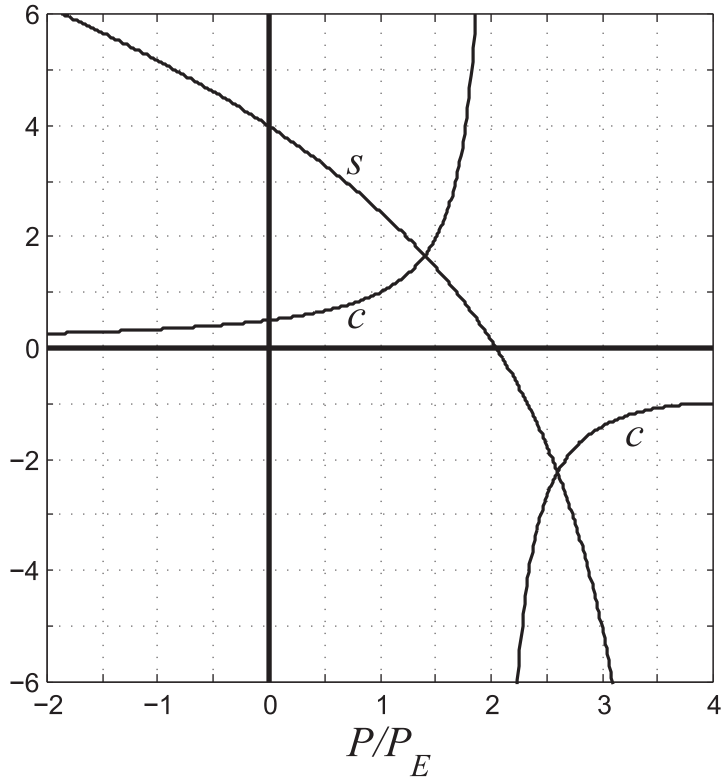

Fig. 4

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