Version PM/2

EGT2
ENGINEERING TRIPOS PART IIA

Friday 3 May 2019 2 - 3.40

Module 3E3

MODELLING RISK

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 3E3 Modelling Risk data sheet (3 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A job shop consists of three machines and two repair workers. The amount of time a machine works before breaking down is exponentially distributed with a mean of 12 time units. The amount of time it takes a single repair worker to fix a machine is exponentially distributed with a mean of 10 time units.

(i) What is the average number of machines not in use? [25%]

(ii) What proportion of time are both repair workers busy? [25%]

(b) A racing bike manufacturer (XMB) is using a special seat in its brand-new bike model. The bike sales for this model show a fairly steady demand of 5,600 bikes per year. Traditionally, XMB purchases these seats from a producer in Germany at a price of £8/unit. It costs XMB £100 to place an order. Inventory holding costs are based on an annual interest rate of 20%. Suppose that the seat supplier is offering a quantity discount applied to all units as follows:

Price per seat	**Order quantity, Q**
£8	$Q \leq 800$
£7	$800 < Q < 1000$
£6	$Q \geq 1000$

(i) What is the optimal order quantity in this case? [30%]

(ii) Using your optimal order quantity, what is the total cost? [10%]

(c) In regression analysis, briefly explain what R^2 is and how it can be used. What is one shortcoming of the R^2 statistic? [10%]

2 (a) A firm is considering introducing a new product. There are initially three possibilities: (i) Introduce the product immediately; (ii) Wait and carry out a test market; (iii) Do not introduce the product. The firm is undecided about which choice to make because it does not know whether the new product will be a 'hit' (H) or 'miss' (M); introducing a 'hit' product will yield £100K, a 'miss' product nothing.

Product introduction costs £30K. Carrying out a test market costs £15K. A test market will tell for certain whether the product is H or M. If it shows H, the firm will proceed with product introduction; if M they will not introduce the product, which at that stage will cost £10K due to reputational issues. The chance of H is now assessed at 0.5.

(i) Build a decision tree for the firm's decision. [15%]

(ii) Based on this decision tree, solve for the firm's optimal decision if the firm wishes to maximize its payoff. [10%]

(iii) Instead of carrying out a test market themselves, the firm can employ a market research organisation to carry out a test market. The market research organisation can produce a favourable (f) or unfavourable (u) report. These are not infallible, the firm assesses $p(f|H)$ as 0.9 but $p(u|M)$ as 0.7. The organisation charges £5K for its survey. If the firm employs the market research organisation, solve for the firm's optimal decision. Should the firm work with the market research organisation? [25%]

(b) Briefly explain what Markowitz portfolio analysis is and how it works. [20%]

(c) Briefly explain what a Markov chain is. How can we characterise a Markov chain? [15%]

(d) Consider the following transition matrix (numbers in %):

From/To	A	B	C	D	E
A	20	80	0	0	0
B	0	90	10	0	0
C	30	0	20	10	40
D	0	0	10	90	0
E	0	25	0	0	75

Draw the transition network and determine the classes. Does the Markov chain have absorbing classes? [15%]

3 (a) One of your employees has been working on a model to schedule the production of an important part in a least-cost manner. Unfortunately, you spilled coffee on the sheet of paper which contained the solution to the dynamic programming problem (see the table below). You decide to try and reconstruct the solution yourself.

Period t

	1	2	3	4
s_t	200	200	200	200
d_t		29	30	
$Y_t(1)$		258	378	
$Y_t(2)$				
$Y_t(3)$				
$Y_t(4)$				
z_t				
δ				

$h_t =$ for all t

To assist yourself, you jot down the following notes:

- Let n^* equal the number of months worth of demand that production in the first month will cover. For example, if $n^* = 3$, then production in month 1 will meet demand for months 1, 2, and 3.
- Let s_t represent the setup cost.
- Let d_t represent the demand for month t.
- Define z_t as the cost of an optimal schedule for the first t periods.
- Define $Y_t(m)$ as the cost of meeting demand for periods 1 through t by producing in period m $(m \leq t)$ for periods m through t and following an optimal production schedule for the first $(m-1)$ periods.
- Let $\delta = (\delta_1, \delta_2, \delta_3, \delta_4)$, where $\delta_t = 0$ if no production in that period, and 1 otherwise.
- No shortages are allowed and there is no production lead time. The holding cost is incurred at the end of each period and is constant over all periods.

 (cont.

(i) Using the notation on the previous page and defining $C_t(m)$ as the cost of meeting demand for period m through t by producing in period m ($m \leq t$), write closed-form expressions for $Y_t(m)$ and z_t. [10%]

(ii) What is the value of h_t? [10%]

(iii) Using only the information readable on the solution table and data for only the first three months, determine n^* for the first three months. [15%]

(iv) You are concerned about how demand in month 4, d_4, might impact your answer to part (iii). Either find ranges of values for d_4 for which the answer to part (iii) would change or else show that the answer to part (iii) would not change whatever the value of d_4. [15%]

(v) Are there values of d_4 such that n^* would remain fixed regardless of the values of d_5? If so, what are they? If not, why not? (Please note that n^* here is not necessarily equal to the values in parts (iii) and (iv).) [15%]

(b) A manufacturer of small electric motors uses an automatic milling machine to produce slots in motor shafts. A batch of shafts is run and then checked. All shafts that do not meet required dimensional tolerances are discarded. At the beginning of each new batch, the milling machine needs to be adjusted, because the machine's cutter head wears slightly during batch production.

The manufacturer would like to predict the number of defective shafts per batch as a function of the size of the batch. To this end, the engineering department has gathered data on 30 batches; the summary statistics of the data is provided below:

	Batch size	**Number of defective shafts**
Minimum	100	5
Average	252.5	44.8
Maximum	400	112

The engineering department estimated a linear regression model to predict the number of defective shafts per batch as a function of the batch size. The output is reproduced on the next page.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.98
R Square	0.95
Adjusted R Square	0.95
Standard Error	7.56
Observations	30

ANOVA

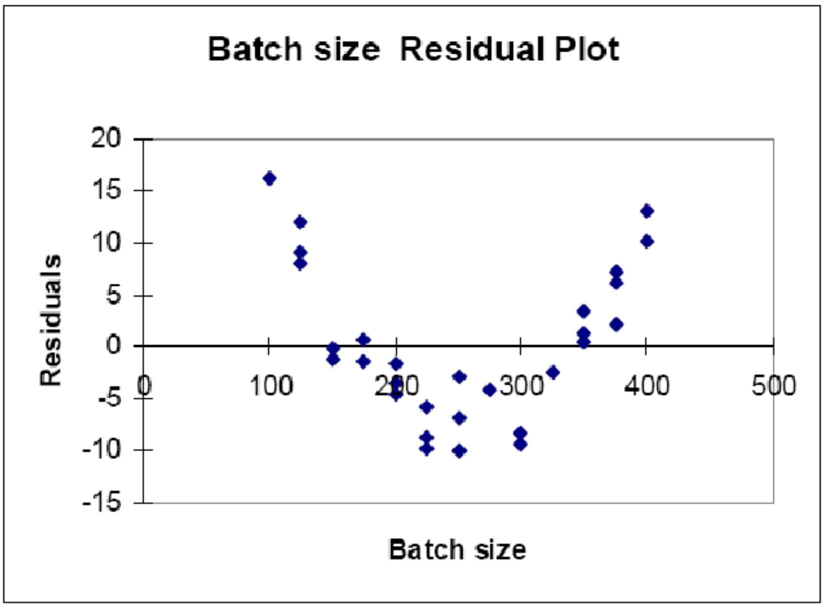
	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	32744.46	32744.46	572.9	0.00
Residual	28	1600.34	57.16		
Total	29	34344.80			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-47.9	4.11	-11.65	0.00	-56.32	-39.48
Batch size	0.37	0.02	23.94	0.00	0.34	0.40

(i) What is the regression equation produced by the linear regression model? [5%]

(ii) According to this model, what is the predicted number of defective shafts per batch if the size of the batch is 50? What can you say about the validity of this prediction? [10%]

(iii) Look at the residual plot that is produced with the regression output. What can you say about the model? Do you recommend using this model? [20%]



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