Version TOL/4

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 24 April 2019 9.30 to 11.10

Module 3F1

SIGNALS & SYSTEMS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

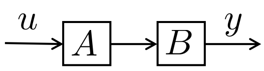
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed

10 minutes reading time is allowed for this paper at the start of the exam.

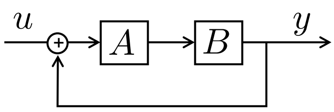
You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) State and prove the final value theorem for discrete time systems. [30%]

(b) Consider the open loop and closed loop discrete time systems shown in Figures (a) and (b):



(a) open loop system



(b) closed loop system

with transfer functions $A(z) = \frac{1-3z}{3(1-z)}$ and $B(z) = \frac{6}{(1-6z)}$.

(i) Is the open loop system BIBO stable? Justify your answer. [10%]

(ii) Compute the pulse response of the open loop system. [20%]

(iii) Verify the final value theorem for the pulse response of the open loop system. Explain why the existence of a final value for a pulse input is consistent with your answer to part (i). [15%]

(iv) Compute the transfer function and pulse response of the closed loop system. Is the closed loop system stable? [25%]

2 (a) Using the inverse Fourier transform derive the discrete-time impulse response $\{h_k\}$ of the ideal low pass filter $\mathcal{H}$ with cutoff normalized frequency at $\pi/2$. Why is the filter not realizable? [20%]

(b) Using the impulse response of $\mathcal{H}$ and the Hamming window

$$w_k = \begin{cases} 0.54 - 0.46\cos(2\pi k/M) & 0 \leq k \leq M \\ 0 & \text{otherwise,} \end{cases}$$

derive the FIR low pass filter $\mathcal{G}$ with impulse response $\{g_k\}$ for $0 \leq k \leq M = 2$. Write down the numerical values of the samples and the difference equation of $\mathcal{G}$. [15%]

(c) What are the advantages of using the Hamming window over the rectangular window? Explain why windows introduce frequency distortion. [20%]

(d) Compute the transfer function $G(z)$ of the FIR filter $\mathcal{G}$ and use it to derive explicit expressions of magnitude and phase of the filter. Sketch magnitude and phase. [25%]

(e) What is the steady state response of $\mathcal{G}$ to

(i) the sinusoidal input $2\sin\left(\frac{2\pi}{3}k\right)$ and

(ii) the unit step input?

(iii) Sketch the complete response to a unit step input (assume $u_{-2} = u_{-1} = 0$).

[20%]

3 Modern high-end subwoofers typically operate in the frequency range 20 – 200 Hz and use filtering and feedback to improve acoustic performance. The dynamics of a subwoofer are captured by the continuous transfer function $\bar{y}(s) = G_c(s)\bar{v}(s)$ from input voltage v to cone position y, given by

$$G_c(s) = \frac{10^5}{s(s^2 + 50s + 10^5)} \,.$$

(a) For the sampling period $T = 10^{-3}$, using backward difference on $G_c(s)$ derive the corresponding digital transfer function $G_d(z)$. [15%]

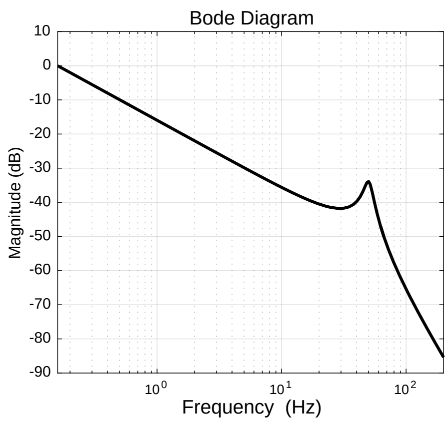
(b) Discuss the phenomenon of aliasing. Is the sampling period $T = 10^{-3}$ sufficient to avoid aliasing for real signals band-limited between 20 – 200Hz? What is the maximum admissible sampling period? [20%]

(c) The subwoofer has a resonance around 50Hz, as shown in Figure (a). Show that the resonance can be removed, improving acoustic performance, by pre-filtering every signal to the subwoofer with the filter

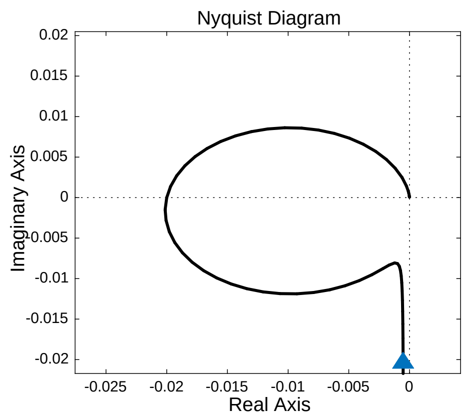
$$F(z) = 10\frac{(z - 0.975e^{j0.1\pi})(z - 0.975e^{-j0.1\pi})}{z^2} \,.$$

[30%]

(d) Servo-subwoofers use feedback to improve performance. Consider the proportional feedback controller $v = k(r - y)$ where r is the reference input and k is the proportional gain. Complete the Nyquist diagram of $G_d(z)$ in Figure (b) (sketch the complete diagram). State the Nyquist stability criterion and use it to determine the range of gains k that guarantee closed loop stability. [35%]



(a) Frequency response of $G_c(s)$



(b) Nyquist diagram of $G_d(z)$

4 (a) Let $\{X(t)\}$ be a continuous-time random process.

(i) Define the autocorrelation function of $\{X(t)\}$. [5%]

(ii) State and justify sufficient conditions on the autocorrelation function for $\{X(t)\}$ to have a well-defined power spectral density, $\mathcal{S}_X$. [10%]

(iii) Prove that if $\{X(t)\}$ is Wide-Sense Stationary then $\mathcal{S}_X$ must be real-valued. [15%]

(iv) Derive conditions on the autocorrelation function of a continuous time random process that would give rise to a purely imaginary power spectral density. [15%]

(b) A microscopic particle is confined in a microfluidic capillary and is modelled with the following equation of motion

$$m\ddot{x} + \gamma\dot{x} = W(t)$$

where m is the particle's mass, x is distance along the capillary, γ is viscous drag and $\{W(t)\}$ is a zero-mean white noise process with $\mathcal{S}_W(\omega) = W_0$ that models the random impacts of fluid molecules on the particle.

(i) With reference to the process $\{W(t)\}$, explain why the above model is necessarily an approximation of a true physical system. [5%]

(ii) Express the autocorrelation function of the particle's velocity in terms of W_0. [10%]

(iii) Suppose the particle's velocity is measured for a long time while filtering with an ideal filter, H, that has frequency response

$$H(\omega) = \begin{cases} 1 & |\omega| < \omega_0 \\ 0 & \text{otherwise.} \end{cases}$$

Derive a formula for the measured mean-square velocity of the particle. [25%]

(iv) Sketch the relationship between the measured mean-square velocity and the filter cutoff frequency, ω_0. Use this sketch to explain why the approximate nature of the model might be reasonable. [15%]

END OF PAPER

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