Version GV/3

EGT2
ENGINEERING TRIPOS PART IIA

Monday 6 May 2019 2 to 3.40

Module 3F2

SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) What does it mean for a system to be *controllable*? [10%]

(b) Describe two different ways in which the controllability of a system can be tested, being careful to define the matrix appropriate to each test. What are the benefits of each test? [20%]

(c) A model of the longitudinal dynamics of an aircraft is given by $\dot{\underline{x}} = A\underline{x} + B\underline{u}$ where the elements of the state vector $\underline{x}$ are forward velocity, vertical velocity, pitch rate and pitch angle respectively. The first input is aileron angle and the second is thrust. The state-space matrices are given by

$$A = \begin{bmatrix} 0 & 1.0 & 0 & -3.0 \\ 0 & -3.0 & 0.5 & 0 \\ 1.0 & -10.0 & -4.0 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$

Find a state feedback gain matrix to place the closed-loop poles at $s = -3.5 \pm j$, $-0.05 \pm 0.05j$, using the thrust input alone. [40%]

(d) Can you infer from your working in part (c) that the system is controllable from the thrust input alone *without* carrying out the tests in part (b)? [10%]

(e) What would be the limitations of controlling the aircraft in this way, and how might the design be improved? [20%]

2 Consider the system with transfer function

$$G(s) = \frac{s^2 - 12s + 48}{s^2 + 12s + 48}$$

which is to be controlled in a negative feedback configuration by a controller $K(s)$.

(a) (i) Show that the root locus diagram for this feedback system, for $K(s) = k$, consists of arcs of a circle centred at the origin and consequently [20%]

(ii) sketch the root locus diagram for this system. [15%]

(iii) Describe the behaviour of the feedback system as k is increased from zero to a large positive value. [15%]

(b) Now consider the controller $K(s) = \frac{k}{s}$.

(i) Sketch the root-locus diagram of the new feedback system. [20%]

(ii) For what range of k are all the poles of the closed-loop system real valued? [20%]

(c) For this system, which controller is preferable, and why? Are your conclusions surprising? [10%]

[You may use the fact that the roots of polynomial $z^4 - 24 * z^3 - 48 * z^2 + 1152 * z + 2304$ are at $z \approx 23.8,\ 8.1,\ -5.9,\ -2.0$]

3 (a) Discuss the role of linearised models in control system design. When is the use of such models justified? [15%]

(b) Consider the system

$$\ddot{x} + \dot{x} - (1 - x^2)x = u$$

(i) Find all equilibria of this system when $u = 0$. [15%]

(ii) Linearise the system about each equilibrium. [20%]

(iii) Sketch the state trajectories of this system, being careful to indicate any asymptotes. [20%]

(iv) Calculate the u that shifts one of the equilibria to $x = 1.5$, what happens to the other equilibria? [10%]

(v) What state will the system be left in if u starts at 0, slowly increases to the value calculated in (iv) and then slowly decreases again to 0? [10%]

(c) Is the use of a model linearised around the origin justified in terms of understanding the overall behaviour of the system described in (b)? [10%]

4 (a) Explain, with the aid of block diagrams, how the combination of a state observer and estimated state feedback can be used as the basis of control system design, when it is required that the output of the system follows a given reference signal. [30%]

(b) A system is given as

$$\dot{x} = 2x + u, \; y = x + w$$

where w represents measurement noise. The input $u = -k\hat{x} + r$, where r is a reference signal and $\hat{x}$ is the estimated state given by an observer with observer gain h.

(i) Find the closed-loop transfer functions from $\bar{r}(s)$ and $\bar{w}(s)$ to $\bar{y}(s)$ in terms of k and h. [30%]

(ii) Find the *open-loop* transfer function of the controller, from $\bar{y}(s)$ to $\bar{u}(s)$. [20%]

(iii) Discuss the role of k and h with reference to your answers to (i) and (ii). [20%]

END OF PAPER