Paper 4: Mathematical Methods

Answers to 2018 Tripos Paper

1. **Complex equation**

   $z = 1 + i0$ and $z = \cos(\ln(2)) - i\sin(\ln(2))$.

2. **Limit**

   $y\log(1 + \frac{x}{y}) \to x$ as $y \to \infty$ (when log to the base e).

3. **Difference equation**

   $x_n = b(2^{n+2} + (-3)^n)$.

4. **Vectors**

   (b) Any value of β. Geometric interpretation: parabolic cylinder.

   (c) (i) $\alpha + 1$, (ii) $\alpha + 1$, distances are equal.

5. **Linear algebra**

   (a) $\lambda = 2$ and $\lambda = 2 \pm \sqrt{1 + a^2}$.

   (b) normalised eigenvector $\mathbf{x} = (0, \pm 1, 0)^\top$.

   (c) $\lambda = \frac{1}{2}$ and $\lambda = (2 \pm \sqrt{1 + a^2})^{-1}$.

   (d) $8(3 - a^2)^3$.

Part IA 2018

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Section B: Numerical Answers

6. $x(t) = \dfrac{1}{5}\left[1 - e^{-t}\left(\dfrac{1}{2}\sin 2t + \cos 2t\right)\right]$

7. $y(t) = \begin{cases} 0 & t < 0 \\ te^{-t} - e^{-t} + e^{-2t} & t \geq 0 \end{cases}$; second order.

8. $P = a^3(10 - 15a + 6a^2)$
   Check: $a = 0$, $P = 0$; $a = 0.5$, $P = 0.5$; $a = 1$, $P = 1$

9. (b) $y(x) = \dfrac{2H}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right)$

10. (a) $\cos(xy + x + y + 1 - z)\begin{bmatrix} y+1 \\ x+1 \\ -1 \end{bmatrix}$ °C/m

    (b) $x + 2y - z + 1 = 0$

    (d) $3.36\,^\circ$C/s or $1.60\,^\circ$C/m

    (e) $T = 19.1\,^\circ$C