Engineering Tripos, Part IA, 2023

Paper 3 Electrical and Information Engineering

Solutions

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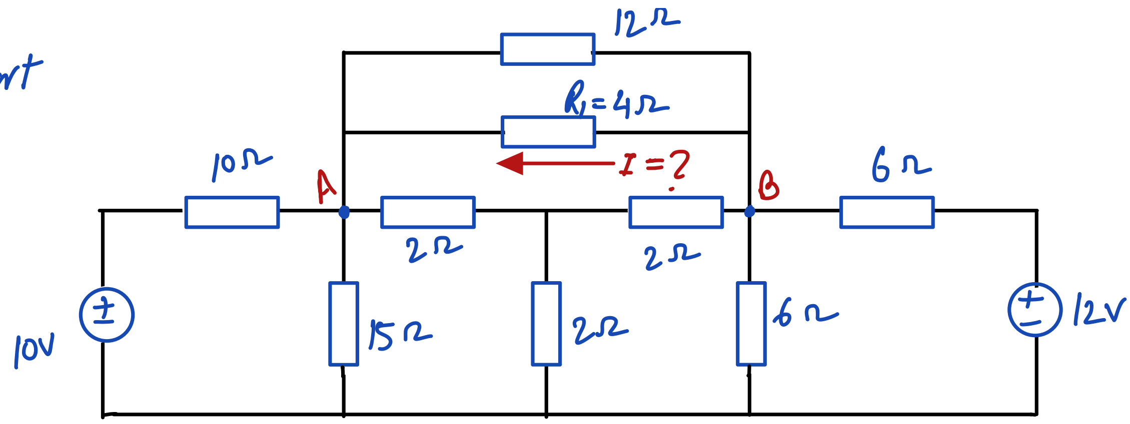
SECTION A

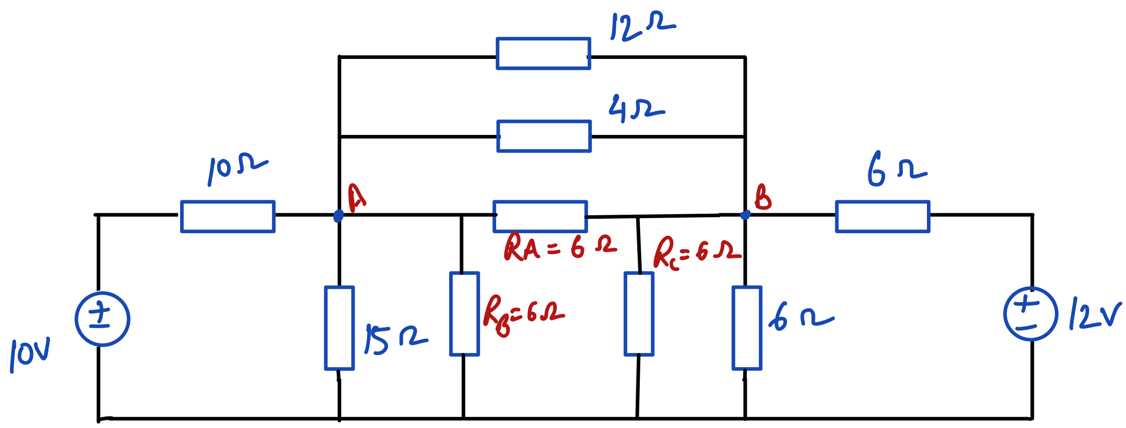
1. (short)

① Short

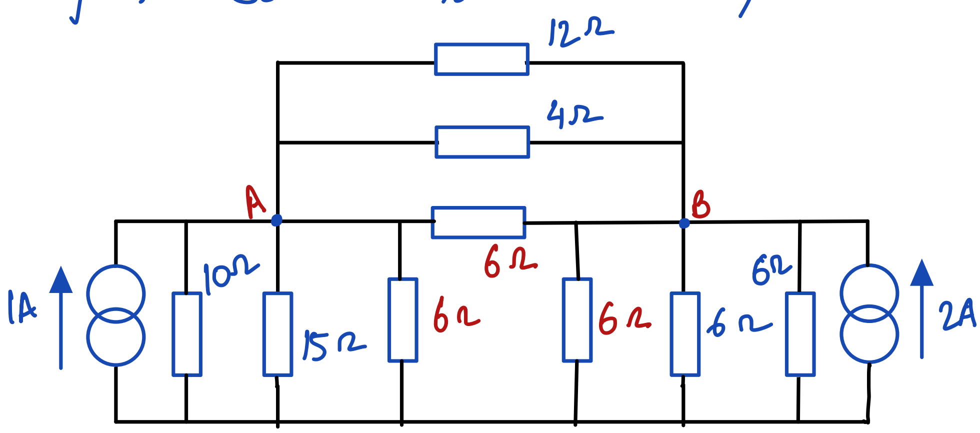


Converting the central 2Ω combination :

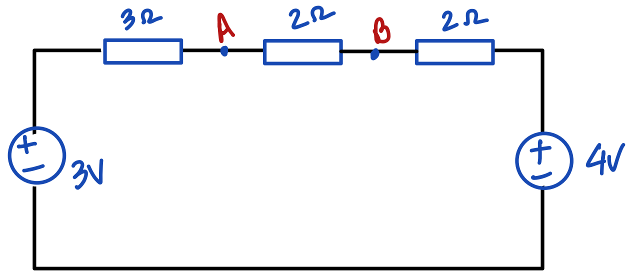
$$R_A = R_B = R_C = \frac{2\times2 + 2\times2 + 2\times2}{2} = 6\Omega$$



Converting the sources to Norton equivalent :



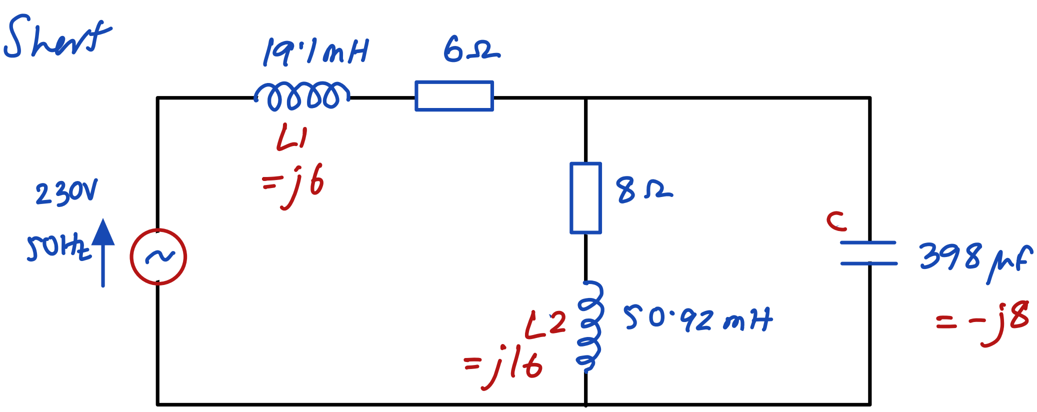
Combining the resistors in parallel & current to voltage source equivalents:



$$V_{BA} = \frac{(4-3)\times 2}{3+2+2} = \frac{2}{7}$$

Current in 4Ω is $= \frac{2}{7\times 4} = \frac{1}{14}$ Amp.

② Short



Reactances :

For $L_1 \Rightarrow j\omega L_1 = j \times 2\pi \times 50 \times 19.1 \times 10^{-3} = j6 \ \Omega$

For $L_2 \Rightarrow j\omega L_2 = j \times 2\pi \times 50 \times 50.92 \times 10^{-3} = j16 \ \Omega$

For $C \Rightarrow \dfrac{1}{j\omega C} = -j8 \ \Omega$

Total complex impedance $= 6 + j6 + \left[(8 + j16) \,//\, (-j8)\right]$

$= 6 + j6 + 4 - j12$ ← This can be done in calculator in a single step

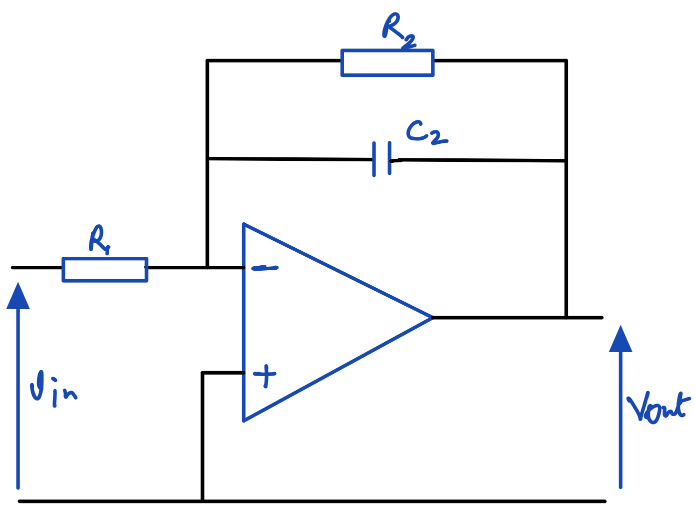
$= 10 - j6 \ \Omega$

$= 11.66 \angle -30.96^\circ$

Therefore current magnitude is $\dfrac{230}{11.66} = 19.72$ Amp

Phase $= 30.96^\circ$.

③ Short



Gain expression:

$$V_{in}/R_1 = -\frac{V_{out}}{(R_2 /\!/ C_2)}$$ (equating current at the virtual ground)

$$V_{in}/R_1 = \frac{-V_{out}(1+j\omega C_2 R_2)}{R_2}$$

$$\text{Gain } V_{out}/V_{in} = \frac{-R_2}{R_1(1+j\omega C_2 R_2)}.$$

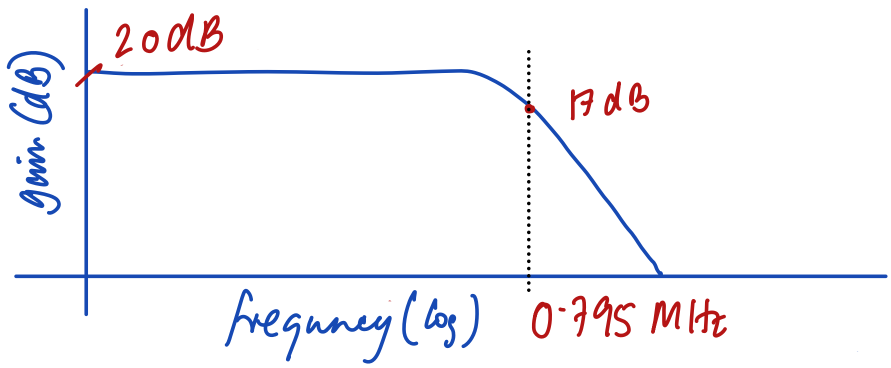
At low frequencies, the feedback is dominated by R_2.

V_{out}/V_{in} at low frequencies, $V_{out}/V_{in} = -\frac{R_2}{R_1} = -10.$

$-3dB$ point, $1 = \omega C_2 R_2 = 2\pi f C_2 R_2$

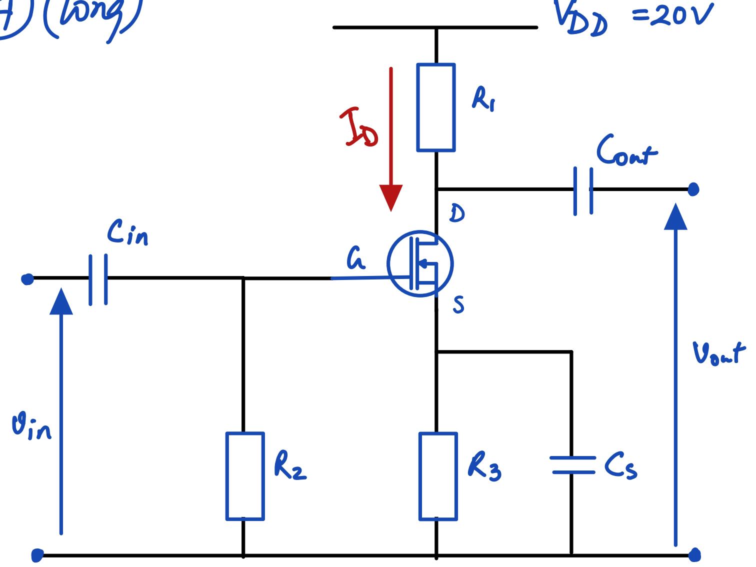
$$f = \frac{1}{2\pi C_2 R_2} = \frac{1}{2\pi \times 20\times10^{-12} \times 10\times10^3} = 0.795MHz$$

Gain at $-3dB = -\frac{10}{1.41\angle 45^\circ} = -7.09\angle -45^\circ$



20dB
17dB
gain (dB)
frequency (log)
0.795 MHz

④ (long)



ⓐ There is no gate current into the FET. C_{in} is also open to d.c. R_2 therefore puts the gate to 0V. The $I_D R_3$ drop across sets $V_{GS} = -I_D R_3$. This biases the FET. The choice of R_1, R_3, I_D dictate the operating point.

ⓑ R_1 & R_3 to achieve the operating point:

$V_{GS} = -3V \quad V_{DS} = 8V \quad I_D = 3mA$

with $g_m = 6mA/V$ & $r_d = 10k\Omega$, $V_{DD} = 20V$.

$$R_3 \times I_D = -V_{GS} = 3.$$

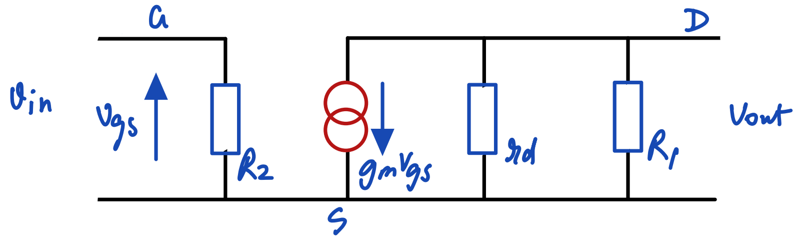
$$R_3 = \frac{3}{3mA} = 1K$$

Also, $V_{DD} = I_D R_1 + V_{DS} + V_S$

$$20 = 3 \times 10^{-3} \times R_1 + 8 + 3$$

$$R_1 = \frac{9}{3 \times 10^{-3}} = 3\ k\Omega.$$

Ⓒ Small signal model



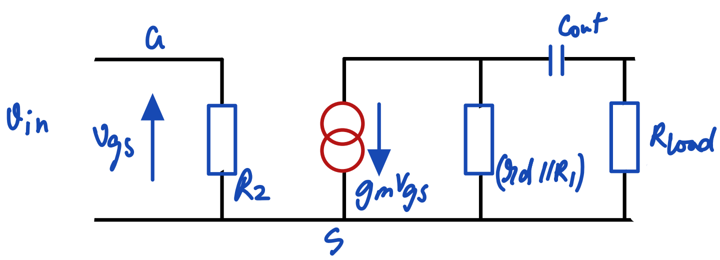
$$V_{gs} = v_{in} \qquad V_{out} = -g_m V_{gs} (R_1 \parallel r_d)$$

$$\text{gain} = \frac{V_{out}}{V_{in}} = -\frac{g_m V_{gs} (R_1 \parallel r_d)}{V_{gs}} = -g_m (R_1 \parallel r_d)$$

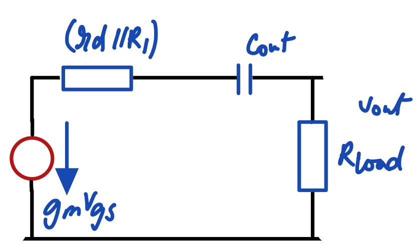
o/p impedance $= (R_1 \parallel r_d)$

i/p impedance $= R_2$

ⓓ The low -3dB cut off is 20Hz for a 5kΩ resistor.



The o/p section can be redrawn as:



$$V_{out} = \frac{-g_m V_{gs} R_{load}}{R_{load} + (R_1 \parallel r_d) + \frac{1}{j\omega C_{out}}}$$

The real & imaginary part in the denominator are same.

Therefore, $$\omega C_{out} = \frac{1}{R_{load} + (R_1 \| r_d)}$$

$$C_{out} = \frac{1}{2\pi \times 20 \times [5k + (3k \| 10k)]}$$

$$C_{out} = 1.089 \mu F$$

(e) To maximize the signal power, R_{load} = Output impedance.

C_{out} remains unchanged.

$$1.089 \times 10^{-6} = \frac{1}{2\pi \times f_{new} \times 2 \times (3k \| 10k)}$$

$$f_{new} = 31.67 \text{ Hz}.$$

(5) (long)

We know $P = VI\cos\phi$. Given, power of machine $P_m = 200$ kW

$$200 \times 10^3 = 1 \times 10^3 \times I \times 0.85$$

$$I = \frac{200}{0.85} = 235.29 \text{ Amp.}$$

Feeder power loss, $P_f = I^2 R = (239.29)^2 \times 0.8$

$= 44.290$ kW

$$Q_f = I^2 X_f = (239.29)^2 \times 1.6$$

$= 88.581$ kVAR

Q of machine $Q_M = P_M \tan\phi$

$= 200 \tan\left[\cos^{-1}(0.85)\right]$ kVAR

$= 123.94$ kVAR.

For the source, $P_S = 200 + 44.29 = 244.29$ kW.

$Q_S = 88.58 + 123.94 = 212.52$ kVAR

Apparent power $S_S = \sqrt{P_S^2 + Q_S^2}$

$= 323.8$ kVA

Source voltage $V_S = \dfrac{323.8}{235.29} = 1.376$ kV.

(b) Machine terminal voltage $= 1$ kV.

For power factor correction, capacitor must generate the same reactive power consumed by the machine, Q_m.

This is 123·94 kVAR.

Power $\frac{V^2}{X_L} = 123{\cdot}94 \times 10^3 = (1000)^2 \times 2\pi \times 50 \times C$

$$C = \frac{123{\cdot}94}{2\pi \times 50 \times 1000} = 394{\cdot}51\ \mu F$$

The new feeder current $VI = 200 \times 10^3$

$$I = \frac{200 \times 10^3}{1000}$$

$$= 200A.$$

New feeder power loss $= (200)^2 \times 0{\cdot}8$

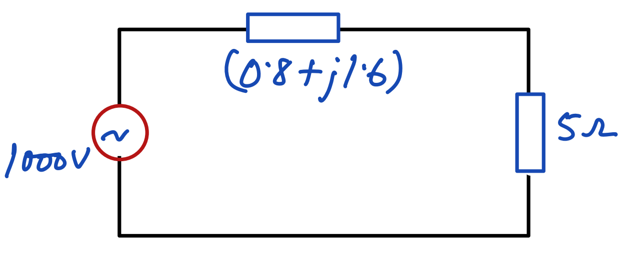
$= 32$ kW.

ⓒ The source voltage is now set at 1kV.

With the capacitor still connected, the machine will now be a purely resistive load. We know that the packaging machine consumes 200kW with 1kV across it.

Hence, $200 \times 10^3 = \frac{(1000)^2}{R} \Rightarrow R = 5\,\Omega$

We can therefore draw the following equivalent circuit :



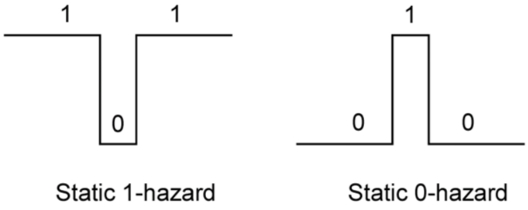
The voltage across the machine terminal is :

$$= \frac{1000 \times 5}{|5{\cdot}8 + j1{\cdot}6|} = \frac{5000}{6{\cdot}01} = 831{\cdot}94 \text{ Volts.}$$

SECTION B

6 (**short**)

(a) The signal exhibits a momentary state change when it is expected to stay intact primarily due to propagation delays in the circuit. Static 0 and 1 hazard occur when the signal is expected to stay at 0 and 1, respectively.



(b)

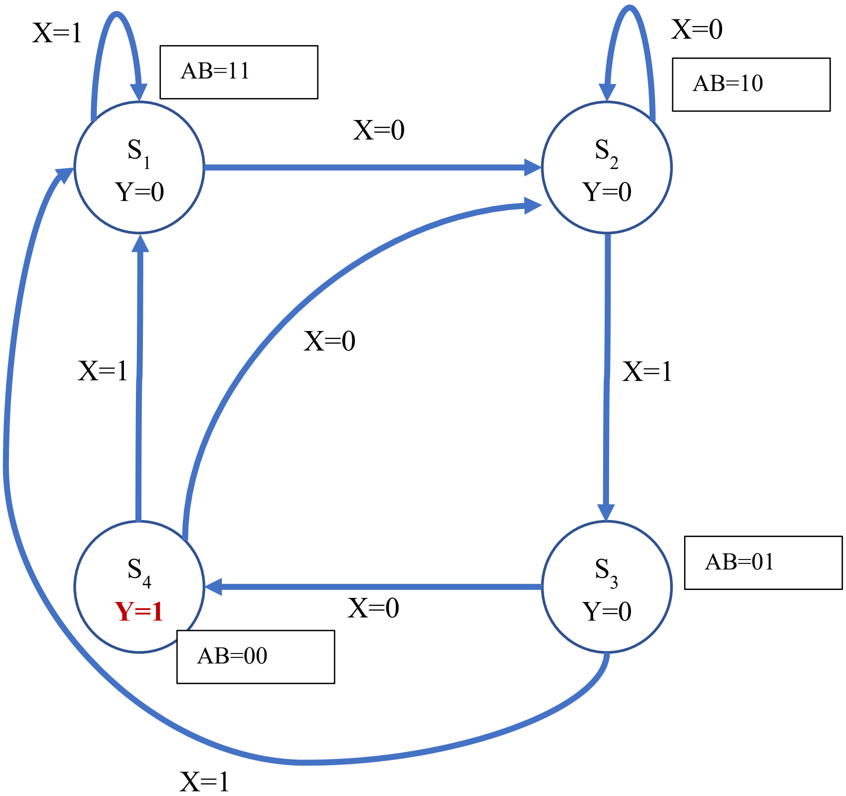
CD \ AB	00	01	11	10
00	1	1	0	0
01	1	1	1	1
11	0	0	1	1
10	0	0	0	0

This is the Karnaugh Map for function F. Clearly, for D=1 and C=0, $F = \bar{A} + A$ which corresponds to the static 1 hazard situation. To address this hazard, the terms corresponding to the red rectangle in the K-map above, i.e., $\bar{C}D$, could be added to the function F, i.e., $F = \bar{A}\bar{C} + AD + \bar{C}D$

Similarly, we can obtain the complement of the function to see if there is a static 0 hazard, i.e., $\bar{F} = \bar{A}C + A\bar{D}$ → $F = \overline{\bar{A}C + A\bar{D}} = \overline{\bar{A}C}.\overline{A\bar{D}} = A.\bar{A}$ when C=1, D=0. Hence, there is a static 0 hazard. To mitigate it, the terms corresponding to the blue rectangle in the K-map, i.e., $C\bar{D}$ could be added to the complement of the function F, i.e., $\bar{F} = \bar{A}\bar{C} + AD + C\bar{D}$.

7 **(short)**

(a) 4 states required as in the state diagram below and therefore 2 bistables will be sufficient to implement the circuit.



(b)

	Current State		Next State		Bistable Inputs			
X	*A*	*B*	*A*	*B*	J_A	K_A	J_B	K_B
0	1	1	1	0	X	0	X	1
1	1	1	1	1	X	0	X	0
0	1	0	1	0	X	0	0	X
1	1	0	0	1	X	1	1	X
0	0	1	0	0	0	X	X	1
1	0	1	1	1	1	X	X	0
0	0	0	1	0	1	X	0	X
1	0	0	1	1	1	X	1	X

8 (**short**)

(a) 2^{11}x8 bits = 2048 bits =2KBytes is the capacity of the memory chip.

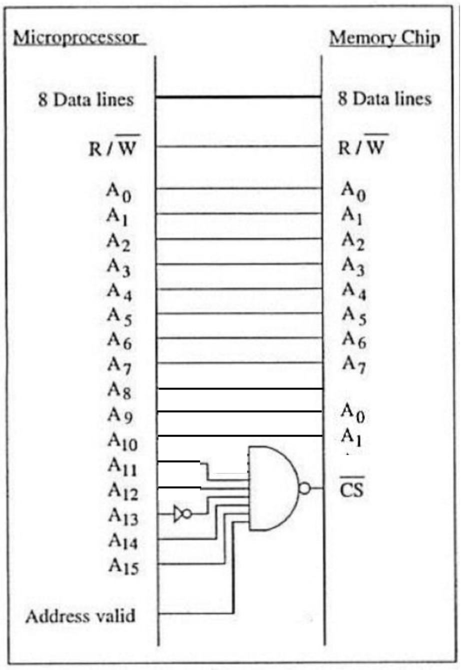
(b)

$D800_H$ → 1101 | 1000 | 0000 | 0000

$DFFF_H$ → 1101 | 1111 | 1111 | 1111

5-bits CS   11-bits the address space

→ $\overline{CS} = \overline{A_{15}A_{14}\overline{A_{13}}A_{12}A_{11}}$



9 **(long)**

(a)

Current State				Next State				J_3	K_3	J_2	K_2	J_1	K_1	J_0	K_0
Q_3 Q_2 Q_1 Q_0				Q^+_3 Q^+_2 Q^+_1 Q^+_0											
1	1	1	1	1	1	1	0	X	0	X	0	X	0	X	1
1	1	1	0	1	1	0	1	X	0	X	0	X	1	1	X
1	1	0	1	1	1	0	0	X	0	X	0	0	X	X	1
1	1	0	0	1	0	1	1	X	0	X	1	1	X	1	X
1	0	1	1	1	0	1	0	X	0	0	X	X	0	X	1
1	0	1	0	1	0	0	1	X	0	0	X	X	1	1	X
1	0	0	1	1	0	0	0	X	0	0	X	0	X	X	1
1	0	0	0	0	1	1	1	X	1	1	X	1	X	1	X
0	1	1	1	0	1	1	0	0	X	X	0	X	0	X	1
0	1	1	0	0	1	0	1	0	X	X	0	X	1	1	X
0	1	0	1	0	1	0	0	0	X	X	0	0	X	X	1
0	1	0	0	0	0	1	1	0	X	X	1	1	X	1	X
0	0	1	1	0	0	1	0	0	X	0	X	X	0	X	1
0	0	1	0	0	0	0	1	0	X	0	X	X	1	1	X
0	0	0	1	0	0	0	0	0	X	0	X	0	X	X	1
0	0	0	0	1	1	1	1	1	X	1	X	1	X	1	X

(b)

J_0

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	1	X	X	1
01	1	X	X	1
11	1	X	X	1
10	1	X	X	1

K_0

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	1	1	X
01	X	X	1	X
11	X	1	1	X
10	X	1	1	X

$J_0 = 1$

$K_0 = 1$

J1

Q3Q2 \ Q1Q0	00	01	11	10
00	1	0	X	X
01	1	0	X	X
11	1	0	X	X
10	1	0	X	X

K1

Q3Q2 \ Q1Q0	00	01	11	10
00	X	X	0	1
01	X	X	0	1
11	X	X	0	1
10	X	X	0	1

$J_1 = \overline{Q_0}$

$K_1 = \overline{Q_0}$

J2

Q3Q2 \ Q1Q0	00	01	11	10
00	1	0	0	0
01	X	X	X	X
11	X	X	X	X
10	1	0	0	0

$J_2 = \overline{Q_1}.\overline{Q_0}$

K2

Q3Q2 \ Q1Q0	00	01	11	10
00	X	X	X	X
01	1	0	0	0
11	1	0	0	0
10	X	X	X	X

$K_2 = \overline{Q_1}.\overline{Q_0}$

J3

Q3Q2 \ Q1Q0	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	X	X	X	X
10	X	X	X	X

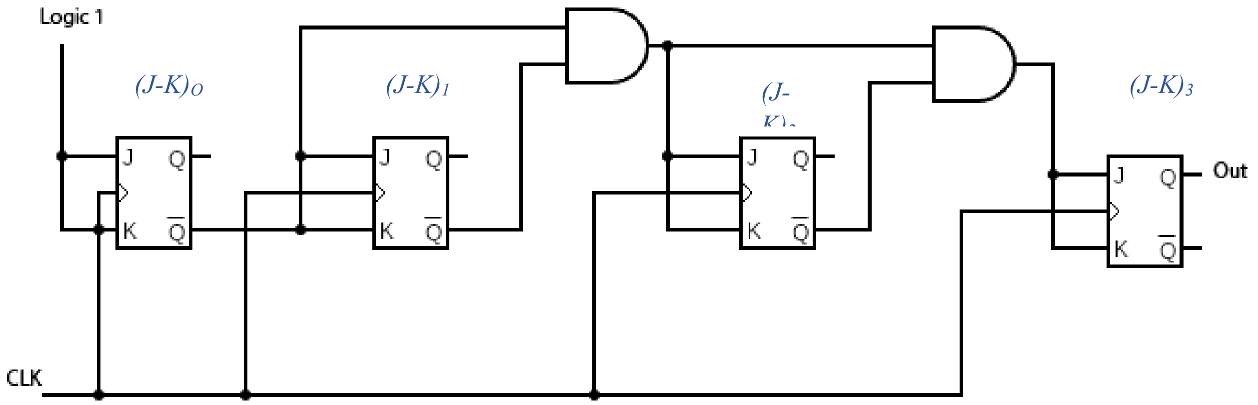
K3

Q3Q2 \ Q1Q0	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	0	0	0	0
10	1	0	0	0

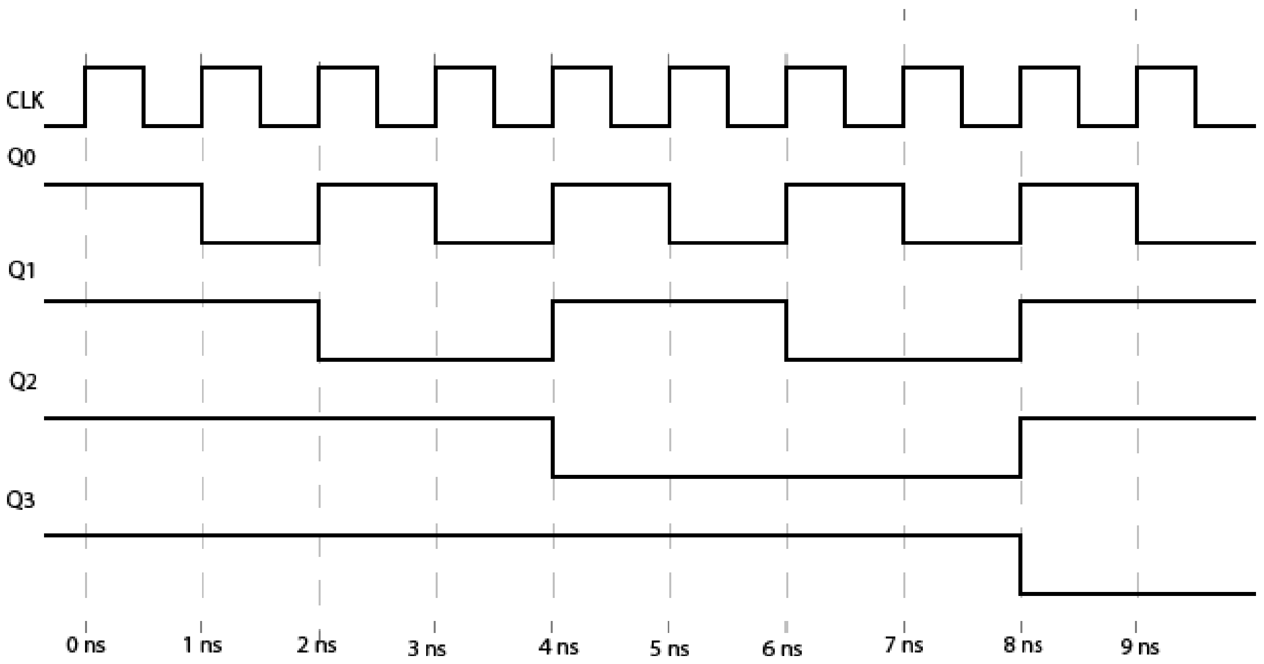
$J_3 = \overline{Q_2}.\overline{Q_1}.\overline{Q_0}$

$K_3 = \overline{Q_2}.\overline{Q_1}.\overline{Q_0}$

(c)

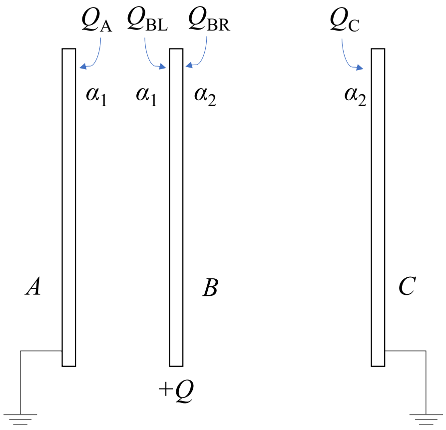

Logic 1
(J-K)O
(J-K)1
(J-K)3
J
Q
K
Q
Out
CLK

(d)


CLK
Q0
Q1
Q2
Q3
0 ns
1 ns
2 ns
3 ns
4 ns
5 ns
6 ns
7 ns
8 ns
9 ns

SECTION C

10 **(short)**



$Q_{BL} + Q_{BR} = Q$

$Q_A = - Q_{BL}$; $Q_C = - Q_{BR}$

(a) $D_A * A = Q_A = \alpha_1 * A$

$\varepsilon_r * E_{AB} = \alpha_1$; $\varepsilon_r * E_{BC} = \alpha_2$

$V_{AB} = E_{AB} * d_1$; $V_{BC} = E_{BC} * d_2$;

$E_{AB} * d_1 = E_{BC} * d_2$;

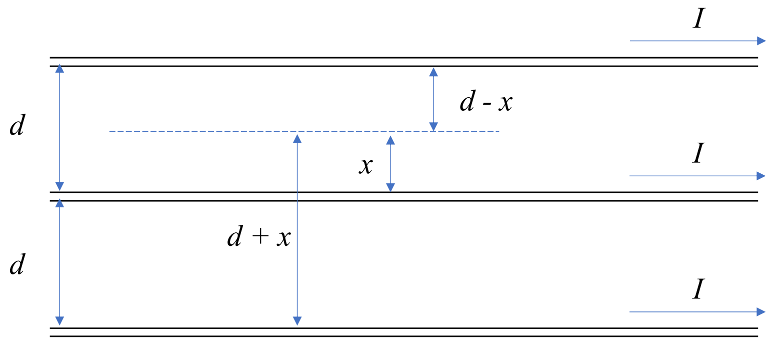
$$\frac{\alpha_1}{\alpha_2} = \frac{d_2}{d_1} = 2$$

$Q_A = Q_{BL} = -2.0 \times 10^{-7}$ C

$Q_C = Q_{BR} = -1.0 \times 10^{-7}$ C

(b) $V_{AB} = V_{BC} = E_{AB} * d_1 = (\alpha_1 * d_1)/ \varepsilon_r = 2.3 \times 10^3$ V

11 **(short)**



The location that B=0 must be within the same plane with the three wires. Assure the location that B=0 is x away from the central wire, and so the total B can be given by:

$B_{total} = B_1 + B_2 + B_3$

$$B_{total} = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d+x)} - \frac{\mu_0 I}{2\pi(d-x)}$$

Let $B_{total} = 0$, $x = \pm\frac{1}{\sqrt{3}}d$ = ±0.5773 d;

$x = 0$ would also be a solution.

12 **(long)**

(a) Electric filed:

At R=1.0 cm, inside the conducting sphere, so E=0;
At R=3.0 cm,

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} = 3 \times 10^5 V/m$$

At R=6.0 cm, outside the outer surface that got connected to earth,
So E=0.

(b) Capacitor of the concentric spheres:

$$Q = CV;$$

$$C = \frac{Q}{V}$$

$$V = \int \frac{Q}{4\pi\varepsilon_0 R^2} dr = \frac{Q}{4\pi\varepsilon_0}(\frac{1}{R_1} - \frac{1}{R_2})$$

$$C = \frac{4\pi\varepsilon_0}{\frac{1}{R_1} - \frac{1}{R_2}} = 4.49 \times 10^{-12} F$$

(c) Total electrostatic energy:

$$W = \frac{1}{2}CV^2 = \frac{1}{2}QV = 1.01 \times 10^{-4}\ \mathrm{J}$$