ENGINEERING TRIPOS PART IA 2014

Paper 1 Mechanical Engineering

Solutions

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Engineering Tripos Part IA, 2014

Section A, Q1- Q3

Q1. a). $\rho_1 h_1 + \rho_{air}(h_2 - h_1) = \rho_2 h_2$

$\because \ \rho_{air} << \rho_1 \ \& \ \rho_2$

$\because \ |\rho_{air}(h_2 - h_1)| << \rho_1 h_1 \ \& \ \rho_2 h_2$

$$\rho_1 h_1 = \rho_2 h_2 \quad \Rightarrow \quad \underline{\underline{\rho_1 = \rho_2 \frac{h_2}{h_1}}}$$ [4]

b) (i). $\rho = 1100 + 500y$ kgm^{-3} ; $dp = \rho g dh$

$$p(y) = \int_0^y (1100 + 500y) g dy \ ; \ = \underline{\underline{(1100y + 250y^2)g}} \text{ pa}$$ [3]

(ii) $F = \int p \cdot dA$; $F/W = \int p \cdot dy$

$$\frac{F}{W} = \int_1^2 (1100y + 250y^2) g dy = \left[\frac{1100}{2} y^2 + \frac{250}{3} y^3\right]_1^2 \cdot g$$

$$= (550 \cdot 3 + \frac{250}{3} \cdot 7) g \cong 21909 \text{ N} \cong \underline{\underline{21.91 \text{ kN}}}$$ [3]

Q2. (a) $\frac{dp}{dr} = \rho \frac{V^2}{r} = \rho (V_0 r_0)^2 \frac{1}{r^3}$; $dp = \rho (V_0 r_0)^2 \frac{dr}{r^3}$

$$\int_{r_0}^{r_1} dp = \rho V_0^2 r_0^2 \cdot \int_{r_0}^{r_1} \frac{dr}{r^3} = \rho V_0^2 r_0^2 \cdot (-\frac{1}{2}) \left| \frac{1}{r^2} \right|_{r_0}^{r_1}$$

$$p_0 - p_1 = \frac{\rho V_0^2 r_0^2 \cdot}{2} \left[\frac{1}{r_1^2} - \frac{1}{r_0^2}\right] = \frac{\rho V_0^2}{2} \left[\left(\frac{r_0}{r_1}\right)^2 - 1\right] = \frac{1.2 \cdot 20.0^2}{2} (2^2 - 1) = \underline{\underline{720.0 \text{ pa}}}$$ [6]

(b) Bernoulli's Equation applicable if the Bernoulli constants across streamlines at different r's remain a constant, *i.e.* need to show that

$$\frac{d(p + 0.5\rho V^2)}{dr} \equiv 0 \quad \text{for all } r\text{'s.}$$

Now, $\because \frac{dp}{dr} = \rho \frac{V^2}{r}$; and $\frac{d(0.5\rho V^2)}{dr} = \rho V \frac{dV}{dr} = \rho V V_0 r_0 \frac{d(r^{-1})}{dr} = -\frac{\rho V^2}{r}$

$$\therefore \quad \frac{d(p + 0.5\rho V^2)}{dr} = \rho \frac{V^2}{r} - \rho \frac{V^2}{r} \equiv 0 \quad q.e.d.$$

Therefore the Bernoulli constants across streamlines do not change, and Bernoulli's Equation is applicable to the whole flow field across streamlines. [4]

Q3. at “1”, $V(r) = V_c\left[1-(\frac{r}{R})\right]$.

(a). By continuity, $\dot{m}_0 = \dot{m}_1$; $\dot{m}_0 = A\rho V_0 = \pi R^2 \rho V_0$

$$\dot{m}_1 = \int_0^R 2\pi r\rho V(r)dr = \int_0^R 2\pi r\rho V_c\left[1-\left(\frac{r}{R}\right)^2\right]dr = \pi\rho V_c(R^2 - \frac{R^2}{2}) = \frac{\pi R^2 \rho V_c}{2}$$

$$\because \dot{m}_0 = \dot{m}_1 \ \therefore\ \pi R^2 \rho V_0 = \frac{\pi R^2 \rho V_c}{2} \quad \Rightarrow \quad \underline{\underline{V_c = 2V_0}}$$ [6]

(b) Along the centre line of the pipe by symmetry there exist a streamline. Between “0” up to “1”, the central streamline is not affected by the viscous effect, (but will have friction loss beyond “1”). Therefore Bernoulli’s Equation is valid from “0” to “1” along the centreline, but not any other streamline between these two planes.

$p_0 + 0.5\rho V_0^2 = p_1 + 0.5\rho V_c^2 = p_1 + 2\rho V_0^2;\quad p_0 - p_1 = 2\rho V_0^2 - \frac{1}{2}\rho V_0^2 = \underline{\underline{\frac{3}{2}\rho V_0^2}}$ on the centre line. Also, as the streamlines inside the pipe are parallel, $\frac{\partial p}{\partial r} \equiv 0$, the static pressure is uniform on cross-sections perpendicular to the centre line, this leads to the static pressure at the centre line being constant across the whole cross-section area. [8]

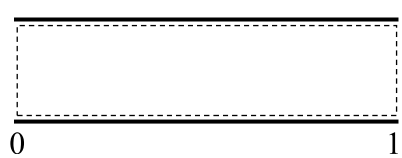
(c) At “0”, Momentum Flux $M_0 = \dot{m}V_0 = \underline{\underline{\pi R^2 \rho V_0^2}}$; as flow is uniform at “0”.

At “1”, Momentum Flux

$$M_1 = \int_0^R 2\pi r\rho V(r)^2 dr = \int_0^R 2\pi r\rho V_C^2\left[1-\left(\frac{r}{R}\right)^2\right]dr = \frac{1}{3}\pi R^2 \rho V_c^2 = \underline{\underline{\frac{4}{3}\pi R^2 \rho V_0^2}} \quad (\because V_c = 2V_0)$$

[10]

(d) Steady Flow Momentum Equation (SFME) on a C.V. between “0” and “1”: F_x is the total external force on the fluid:



$$F_x = A\cdot(p_1 - p_0) + \sum \text{Momentun Fluxes}$$

$$= \pi R^2(p_1 - p_0) + \frac{4}{3}\pi R^2 \rho V_0^2 - \pi R^2 \rho V_0^2 = -\frac{3}{2}\pi R^2 \rho V_0^2 + \frac{1}{3}\pi R^2 \rho V_0^2 = -\frac{7}{6}\pi R^2 \rho V_0^2$$

F_x is the drag force on the flow, pointing against the flow direction. The force acting on the pipe by the flow is the reaction of this force, pointing in the direction of the flow: $F_{flow} = -F_x = \underline{\underline{\frac{7}{6}\pi R^2 \rho V_0^2}}$. F_{flow} is created by the surface viscous friction drag on the pipe wall due to the movement of the flow relative to the pipe surface. [6]

Question 4

(a)

Ideal gas law as applied to a perfect gas, where R_{He} is found within the databook.
R_{He} = 2080 J/(kg K).

$$T_1 = \frac{p_1 V_1}{R_{He} m_1} = \frac{5 \cdot 10^5\,\text{Pa}\,0.5\,\text{m}^3}{2080\,\text{J/(kg K)}\,0.075\,\text{kg}} = \boxed{1602.6\ \text{K}}$$

(b)

The weighted piston dictates that $p_2 = p_1$.
Work is the $\int_1^2 PdV$ of the system, where P and m are constant, thus $p\int_1^2 dV = mR\int_1^2 dT$

$W_{12} = mR_{He}(T_2 - T_1) = 0.075\,\text{kg}\,2,080\,\text{J/(kg K)}(273.15\,\text{K} - 16026\,\text{K})$ = -207,389 J

Alternatively, the ideal gas law can be applied again to find V_2 and then used to find work.

$$V_2 = \frac{m_1 R_{He} T_2}{p_1} = \frac{0.075\,\text{kg}\,2,080\,\text{J/(kg K)}273\,\text{K}}{5 \cdot 10^5\text{Pa}} = 0.085\ \text{m}^3$$

$$W_{12} = p_1(V_2 - V_1) = 5 \cdot 10^5\,\text{Pa}\,(0.085\,\text{m}^3 - 0.5\,\text{m}^3) = -207,389\,\text{J}$$

$W_{12} = \boxed{\text{-207 kJ}}$ (work done on the system)

$$U_{12} = m_1\,c_{v,He}(T_2 - T_1) = 0.075\,\text{kg}\,3110\,\text{J/(kg K)}\,(273.15\,K - 1602.56\,K) = -310,086\,\text{J}$$

$U_{12} = \boxed{\text{-310 kJ}}$ (internal energy is reduced)

$$Q_{12} = m_1\,c_{p,He}(T_2 - T_1) = 0.075\,\text{kg}\,5190\,\text{J/(kg K)}\,(273.15\,K - 1602.56\,K) = -517,474\,\text{J}$$

or $Q_{12} = U_{12} + W_{12}$ = -207 kJ - 310 kJ = - 517 kJ

$Q_{12} = \boxed{\text{-517 kJ}}$ (heat transferred out of the system)

Question 5

(a)

Given that $dw_x = -v\,dp$, the work can be found through integration $w_x = -\int_1^2 v\,dp$
Since $p\,v^n = Const$, then $v = Const^{1/n}\ p^{-1/n}$.
Substituting into the work relation

$$w_x = -\int_1^2 Const^{1/n} p^{-1/n} dp = -\Big[Const^{1/n} \frac{1}{1-1/n} p^{1-1/n}\Big]_1^2 = -\Big[\frac{n}{n-1} p^1 \underbrace{Const^{1/n} p^{-1/n}}_{=v}\Big]_1^2$$

$$w_x = -\frac{n}{n-1}\Big[pv\Big]_1^2 = -\frac{n}{n-1}(p_2 v_2 - p_1 v_1)$$

To transform the relation from pressure and volume to temperature and the ideal gas constant, the ideal gas relation is employed, $p\,v = R\,T$.

Substituting the ideal gas relation into the work equation results in
$w_x = -\frac{n}{n-1}(RT_2 - RT_1)$ or $\boxed{w_x = -\frac{nR}{n-1}(T_2 - T_1)}$

(b)

The steady flow energy equation with no kinetic or potential energy changes
$q - w_x = (h_2 - \cancel{V_2^2/2} + \cancel{gz_2}) - (h_1 - \cancel{V_1^2/2} + \cancel{gz_1}) = dh_{1-2} = c_p(T_2 - T_1)$
The question requires that c_p be transformed to an expression in terms γ and R.

Solve $R = c_p - c_v$ and $\gamma = c_p/c_v$ for c_p.

$c_v = c_p/\gamma$ therefore $R = c_p - c_p/\gamma$. Thus, $R = c_p(1 - 1/\gamma)$ or $c_p = \frac{R}{1-1/\gamma} = \frac{\gamma R}{(\gamma - 1)}$.

Plug this result into the SFEE along with the result from part a.

$$q = w_x + c_p(T_2 - T_1) = -\frac{nR}{n-1}(T_2 - T_1) + \frac{\gamma R}{\gamma - 1}(T_2 - T_1) = \Big(\frac{\gamma}{\gamma - 1} - \frac{n}{n-1}\Big)R(T_2 - T_1)$$

$$q = \Big(\frac{\gamma(n-1) - n(\gamma - 1)}{(\gamma - 1)(n-1)}\Big)R(T_2 - T_1) = \Big(\frac{\gamma n - \gamma - n\gamma + n}{(\gamma - 1)(n-1)}\Big)R(T_2 - T_1)$$

$$\boxed{q = \Big(\frac{n - \gamma}{(\gamma - 1)(n-1)}\Big)R(T_2 - T_1)}$$

Question 6

(a)

The problem states that the compressor is adiabatic and reversible, thus isentropic. From the databook $T/p^{(\gamma-1)/\gamma} = const$. Therefore,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = 300\,\mathrm{K} \left(\frac{10}{1}\right)^{(0.4)/1.4}$$

$\boxed{T_2 = 579.2 \text{ K}}$

For an adiabatic compressor there is no heat transfer and thus neglecting kinetic and potential energy changes, the change in enthalpy is directly related to the change in work, $\cancel{dQ} - d\dot{W} = dH$. Given that the gas is perfect $dH = \dot{m}_1 c_p dT$, where c_p is constant for a perfect gas. Therefore, $\dot{W}_C = -\dot{m}_1 c_p \int_3^4 dT$, or

$\dot{W}_C = -\dot{m}_1 c_p (T_3 - T_4)$ = -10 kg/s 1.005 kJ/kg/K (579.2 K - 300 K) = $\boxed{\text{-2.806 MW}}$

Therefore, the power input (negative work) into the compressor is 2.806 MW.

(b)

T_4 can be found by the isentropic relation,

$$T_4 = T_3 \left(\frac{p_4}{p_3}\right)^{(\gamma-1)/\gamma} = 1400\,\mathrm{K} \left(\frac{1.2}{10}\right)^{(0.4)/1.4} \rightarrow \boxed{T_4 = 763.9 \text{ K}}$$

To calculate the mass flow rate we must incorporate the given $\dot{W}_{net}$.
$\dot{W}_T = \dot{W}_{net} - \dot{W}_C$ = 3 MW + 2.81 MW = 5.81 MW

$$\dot{W}_T = \dot{m}_{2-4} c_p (T_3 - T_4) \rightarrow \dot{m}_{2-4} = \frac{\dot{W}_T}{c_p (T_3 - T_4)} = \frac{5{,}810 \text{ kW}}{1.005 \text{ kJ/kg/K (1400 K - 763.9 K)}} \rightarrow \boxed{\dot{m}_{2-4} = 9.082 \text{ kg/s}}$$

To calculate the efficiency of the system

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{W}_{net}}{\dot{m}_{2-4} c_p (T_3 - T_2)} = \frac{3{,}000 \text{ kW}}{9.082 \text{ kg/s } 1.005 \text{ kJ/kg/K (1400 K- 579.2 K)}} \rightarrow \boxed{\eta = 0.404 \text{ or } 40.4\%}$$

(c)

Conservation of energy with no heat transfer or external work

$$\cancel{\dot{Q}} - \cancel{\dot{W}_x} = -\dot{m}_5 h_5 - \dot{m}_4 h_4 + \dot{m}_6 h_6$$

Using conservation of mass, $\dot{m}_6 = \dot{m}_5 + \dot{m}_4$ and perfect gas $h = c_p T$.
Therefore, $0 = -(\dot{m}_6 - \dot{m}_4) C_p T_5 - \dot{m}_4 C_p T_4 + \dot{m}_6 C_p T_6$, or

$$T_6 = (1 - \dot{m}_4/\dot{m}_6) T_5 + \dot{m}_4/\dot{m}_6 T_4$$

From ② → ⑤ throttling valve where $dh_{2-5} = 0 \therefore T_5 = T_2 = 579.2$K
T_6 =(1-9.082/10) 579.2 K + (9.082/10) 763.9 K = $\boxed{T_6 = 746.95 \text{ K}}$

From the 2nd Law of Thermodynamics applied to a control volume (in databook)

$$\cancel{\frac{dS_{CV}}{dt}} + \sum \dot{m}_{out} s_{out} - \sum \dot{m}_{in} s_{in} = \int \cancel{\frac{d\dot{Q}}{T}} + \dot{S}_{irrev}$$

$$\dot{S}_{irrev} = \dot{m}_6 s_6 - \dot{m}_5 s_5 - \dot{m}_4 s_4 = \dot{m}_5(s_6 - s_5) + \dot{m}_4(s_6 - s_4)$$

For a perfect gas the rate of entropy generation is given by the expression $\dot{s}_{1-2} = (c_p\, ln(T_2/T_1) - R\, ln(p_2/p_1))$ and therefore,

$$\dot{S}_{irrev} = (\dot{m}_6 - \dot{m}_4)(c_p\, ln(T_6/T_5) - R\, ln(p_6/p_5)) + \dot{m}_4(c_p\, ln(T_6/T_4) - R\, ln(p_6/p_4))$$

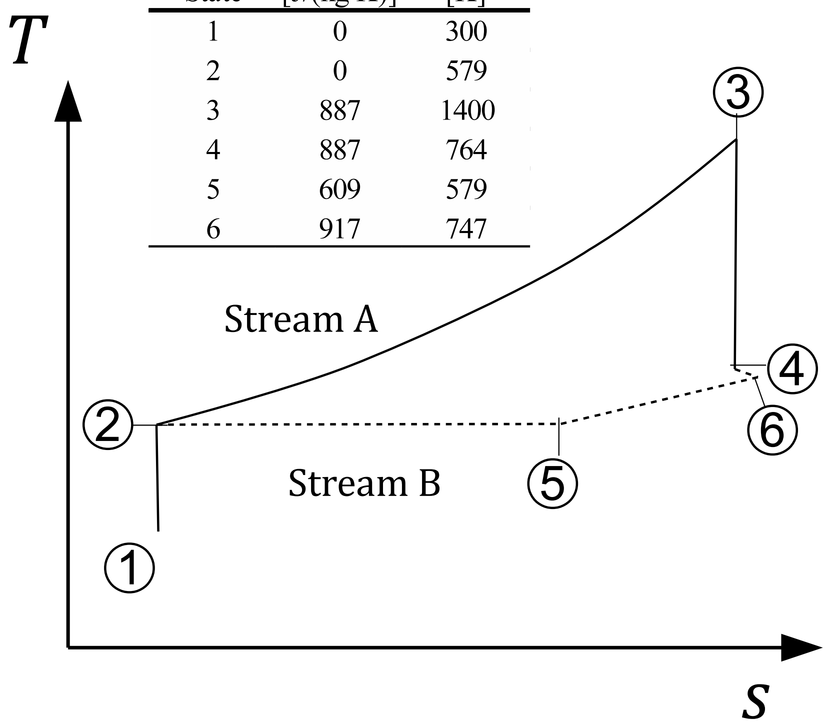
$\dot{S}_{irrev}$ = (10 - 9.082) kg/s (1005 J/(kg K) ln(747/579)- 287 J/(kg K) ln(1/1.2))
+ 9.082 kg/s (1005 J/(kg K) ln(747/764)- 287 J/(kg K) ln(1/1.2))

$\boxed{\dot{S}_{irrev} = 553.04 \text{ J/(K s)}}$

(d)

Table not needed

State	s [J/(kg K)]	T [K]
1	0	300
2	0	579
3	887	1400
4	887	764
5	609	579
6	917	747



Temperature versus entropy diagram of cycle.

2014 IA Mechanical Engineering Section B solutions

Gábor Csányi & Ashwin Seshia

Q7 Ice skater

a) Angular momentum is conserved during the process.

$$H_{\text{initial}} = 2m(\Omega R)R$$
$$H_{\text{final}} = 2m(\omega \frac{R}{2})\frac{R}{2}$$

Therefore,

$$\boxed{\omega = 4\Omega}$$

b) One way to work out the average power exerted, is to consider the change in energy. Rotational kinetic energy is not part of the IA syllabus, but since we have only two point masses, we can use the kinetic energy formula for linear motion. The speed of the outer masses is $v_1 = R\Omega$ before and $v_2 = \omega R/2$ after the pulling in of the arms. Therefore the change in kinetic energy is

$$\begin{aligned}\Delta E &= \frac{1}{2}2mv_2^2 - \frac{1}{2}2mv_2^2 \\ &= 2\frac{1}{2}m\left(\frac{4\Omega R}{2}\right)^2 - 2\frac{1}{2}m(R\Omega)^2 \\ &= 3m\Omega^2 R^2\end{aligned}$$

So the average power exerted is

$$\boxed{P = \frac{3m\Omega^2 R^2}{T}}$$

Another possible solution is to integrate the force that is needed to pull in the arms. When the length of the arms is r, the force exerted is

$$F(r) = 2m\omega(r)^2 r$$

where, generalising the answer to part a), the angular velocity as a function of r is

$$\omega(r) = \Omega\frac{R^2}{r^2}$$

Therefore

$$\begin{aligned}P &= \frac{1}{T}\int_R^{R/2} F(r)\,dr \\ &= \frac{2m\Omega^2 R^4}{T}\int_R^{R/2}\frac{dr}{r^3} = \frac{2m\Omega^2 R^4}{T}\left[-\frac{1}{2}\frac{1}{r^2}\right]_R^{R/2} \\ &= \frac{m\Omega^2 R^4}{T}\left(\frac{4}{R^2} - \frac{1}{R^2}\right) = \frac{3m\Omega^2 R^2}{T}\end{aligned}$$

Putting realistic numbers in, e.g. $T = 1$ s, $\Omega = 12$ rad s^{-1}, $m = 4$ kg, $R = 0.5$ m, we get 432 W of power!

Q8 Mechanism

a) The instantaneous centre of motion, I, is directly above C at a distance of $2L$, on the extension of the line AB. The velocity of B is along BC, so due to symmetry, the angular velocity around the instantaneous center is also ω. Therefore the velocity of C is

$$\boxed{v_c = 2\omega L}$$

to the left.

b) Suppose a torque T acts at A, so the power supplied is $T\omega$. This has to counteract all frictional torques and frictional forces. The joint B is turning with angular velocity 2ω (the rate of change of the angle ABC), so the power balance is

$$T\omega = Q(\omega + 2\omega + \omega) + F2\omega L$$

so

$$\boxed{T = 4Q + 2FL}$$

Q9 chain

Let us define the linear density of the chain as $\rho = M/L$. Denoting the height of the chain above the table by z, the mass of the chain in the air is ρz.

a) The chain is being pulled at a constant speed v, so we have

$$z = vt$$

The gravitational pull on the chain is $\rho z g = \rho v g t$, so the total force is $F - \rho v g t$ that has to equal the rate of change of momentum,

$$F - \rho v g t = \frac{d}{dt}(mv)$$

where $m = \rho z = \rho v t$ is the mass in the air. While the chain is being lifted, the mass in the air is changing,

$$\frac{d}{dt}(\rho v t v) = \rho v^2$$

After the chain is fully lifted, its momentum does not chain any more, so the force required is just that to balance the weight. Therefore

$$\boxed{F = \begin{cases} \rho g v t + \rho v^2 & 0 < \ t < \ L/v \\ Mg \quad (= \rho L g) & L/v < \ t \end{cases}}$$

b) For the case of constant pulling force F, again the total force must equal the rate of change of momentum. Momentum is still mass $\times$ velocity, but the velocity is not a constant any more, so $mv = \rho z \times \dot{z}$, and therefore

$$\begin{aligned} F - \rho z g &= \frac{d}{dt}(\rho z \dot{z}) \\ F &= \rho g z + \ \rho z \ddot{z} + \ \rho \dot{z}^2 \end{aligned}$$

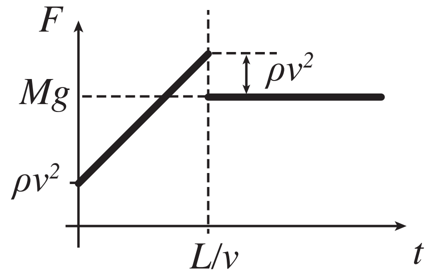


Figure 1: Force on the chain in part a), the case of constant velocity.

The proposed solution is the quadratic polynomial $z = a + bt + ct^2$. Using the initial condition $z = 0$ at $t = 0$ yields $a = 0$. Substituting solution into the differential equation, we have

$$(a + \ bt + \ ct^2)2c + (b + 2ct)^2 + g(a + bt + ct^2) = F/\rho.$$

We equate coefficients of powers of t,

$$\begin{aligned} 2ac + b^2 + ga &= F/\rho \\ 2bc + \ 4bc + \ gb &= 0 \\ 2c^2 + 4c^2 + gc &= 0 \end{aligned}$$

Notice that the second and third equations are redundant. Together with the first equation, they give b and c,

$$\begin{aligned} c &= -g/6 \\ b &= \sqrt{F/\rho} \end{aligned}$$

So the complete solution is

$$\boxed{z = \sqrt{F/\rho}\, t - gt^2/6}$$

c) The chain leaves the table when $z = L$, so the corresponding time t satisfies

$$-\frac{g}{6}t^2 + \sqrt{F/\rho}\, t - L = 0$$

Using the the quadratic formula gives

$$\boxed{t = \frac{\sqrt{F/\rho} - \sqrt{F/\rho - 2Lg/3}}{g/3}}$$

d) A real solution exists when $\boxed{F > \ 2L\rho g/3}$. This is less than the force required to balance the weight of the chain, $L\rho g$. Using this minimum force, the chain would momentarily leave the table and then fall back down again.

10 (a) Radial acceleration = 0 after wire snaps

$$\therefore \ddot{r} - r\Omega^2 = 0$$

solving above for $r(0) = L/2$, $\dot{r}(0) = 0$ we get

$$r(t) = A \cosh bt$$

where $A = \frac{L}{2}$

$b = \Omega$.

(b) when $r(t) = L$, $t = t_f$ say

$$L = \frac{L}{2} \cosh \Omega t_f$$

$$t_f = \frac{1}{\Omega} \cosh^{-1}(2).$$

11 (a)

$$\omega_n = \sqrt{\frac{K}{J}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{10^4}{1}} = \frac{100}{2\pi} = 15.9\,Hz$$

(b)

$$\theta = A\cos\omega_n t + B\sin\omega_n t$$

@ $t = 0$, $\theta = 0.1$ rad, $\dot{\theta} = 10$ rad s^{-1}

$\Rightarrow$ $A = 0.1$, $B = 0.1$

$$\therefore \theta = 0.1(\cos 100t + \sin 100t).$$

(c)

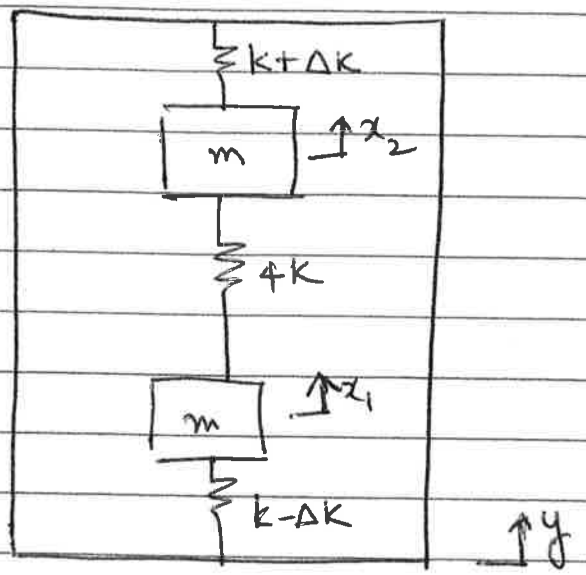
$$N\delta = \ln\frac{\theta_1}{\theta_2}$$

$$2\pi\zeta N = \ln\frac{1}{0.001}$$

$N \simeq 110$ cycles

$$\tau \simeq \frac{110}{15.9} \simeq 6.9\,s.$$

Q12



(a)

$$m\ddot{x}_1 = -(k-\Delta k)(x_1-y) - 4k(x_1-x_2)$$
$$m\ddot{x}_2 = -(k+\Delta k)(x_2-y) - 4k(x_2-x_1)$$

$$-\Delta k\, x_1 + m\ddot{x}_1 + 5k\, x_1 - 4k x_2 = (k-\Delta k)y$$
$$+\Delta k\, x_2 + m\ddot{x}_2 + 5k\, x_2 - 4k\, x_1 = (k+\Delta k)y$$

or $$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} 5k-\Delta k & -4k \\ -4k & 5k+\Delta k \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} (k-\Delta k)y \\ (k+\Delta k)y \end{bmatrix}$$

(b) $$((5k-\Delta k) - m\omega^2)(5k+\Delta k - m\omega^2) - 16k^2 = 0$$

$$(5k - m\omega^2 - \Delta k)(5k - m\omega^2 + \Delta k) = 16k^2$$
$$(5k - m\omega^2)^2 = 16k^2 + (\Delta k)^2$$

$$5k - m\omega^2 = \pm\sqrt{16k^2 + \Delta k^2}$$
$$\omega^2 = \frac{5k \pm \sqrt{16k^2 + \Delta k^2}}{m}$$

Plugging in $x_1 = X_1 \cos\omega t$, $x_2 = X_2 \cos\omega t$, $y = Y\cos\omega t$ we g[et]

(c).

$$[(5k - \Delta k) - m\omega^2] X_1 - 4k X_2 = (k - \Delta k) Y \quad -(1)$$

$$[(5k + \Delta k) - m\omega^2] X_2 - 4k X_1 = (k + \Delta k) Y \quad -(2)$$

(1) + (2)

$$(k - m\omega^2)(X_1 + X_2) - \Delta k (X_1 - X_2) = 2kY \quad -(3)$$

(1) − (2)

$$(9k - m\omega^2)(X_1 - X_2) - \Delta k (X_1 + X_2) = -2\Delta k Y \quad -(4)$$

Substituting (3) in (4)

$$(9k - m\omega^2)(X_1 - X_2) - \Delta k \left[\frac{2kY + \Delta k (X_1 - X_2)}{k - m\omega^2}\right] = -2\Delta k Y$$

$$(9k - m\omega^2)(k - m\omega^2)(X_1 - X_2) - (\Delta k)^2 (X_1 - X_2) = 2k\Delta k Y + (k - m\omega^2)(-2\Delta k Y)$$

$$(X_1 - X_2) = \frac{2\Delta k Y m\omega^2}{[(9k - m\omega^2)(k - m\omega^2) - \Delta k^2]}.$$

$\Delta k = 0 \Rightarrow$ system is symmetric & $X_1 - X_2 = 0$
There is no differential motion of the two masses

(d)