EGT0

ENGINEERING TRIPOS PART IA

Wednesday 4 June 2014 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (**short**)

(a) The open-ended U-tube shown in Fig. 1 is used to measure the density ρ_1 of a liquid of constant density using another liquid of known constant density ρ_2. Both ρ_1 and ρ_2 are much larger than the density of air. Stating your reasoning, find ρ_1 in terms of ρ_2, h_1 and h_2. [4]

(b) The density of a fluid (salt stratified water) is given by $\rho = 1100 + 500y$ kg m^{-3} where y is measured in metres vertically downwards from the free surface of the fluid.

(i) Find an expression for the gauge pressure in the fluid at depth y. [3]

(ii) Calculate the horizontal component of the hydrostatic force per unit width acting on one side of a rectangular gate submerged in the fluid. The top of the gate is at a depth $y = 1.0$ m and the bottom is at a depth of $y = 2.0$ m. [3]

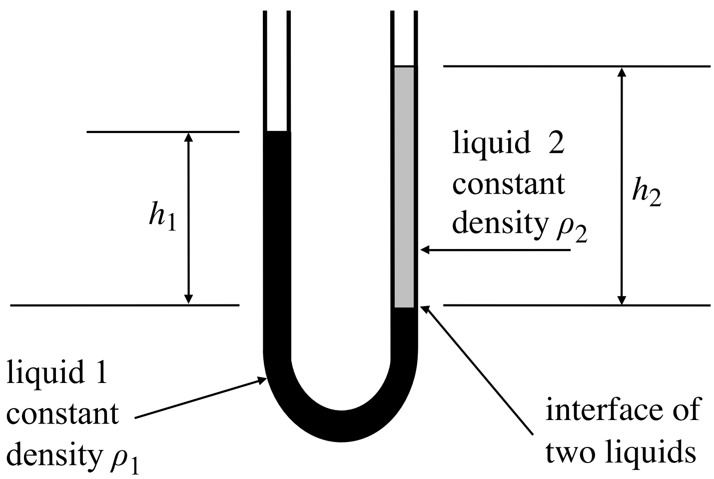

Fig. 1

2 (**short**) A two-dimensional air flow has circular stream lines, as shown in Fig. 2. The velocity distribution is given by $V(r) = c/r$, where c is a constant and r is the radial distance. At radius $r_0 = 0.2$ m the velocity is $V_0 = 20.0$ m s^{-1}. Assume air to be incompressible with density $\rho = 1.2$ kg m^{-3}.

(a) Calculate the pressure difference between the streamlines at $r = r_0$ and $r = r_1 = 0.1$ m. [6]

(b) Prove whether or not Bernoulli's equation can be applied between the streamlines at different radii. [4]

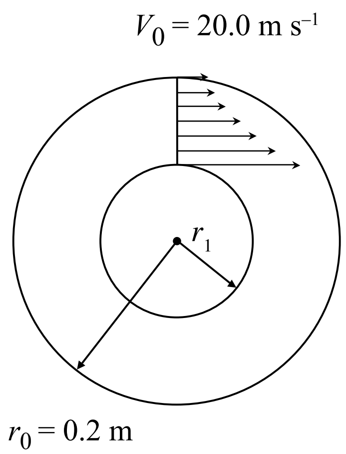

Fig. 2

3 (**long**) Figure 3 shows the entrance part of the water flow in a horizontal circular pipe of constant radius R. The pipe wall boundary layer develops immediately downstream of the entrance (plane 0), where the flow is uniform with velocity V_0. It reaches the centre of the pipe at plane 1 so that an inviscid core no longer exists downstream of this point. At plane 1, the flow has an axisymmetric parabolic velocity profile,

$$V(r) = V_c\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where V is the velocity at radius r and V_c is the velocity at the centre line of the pipe. The density of water is ρ.

(a) Find V_c in terms of V_0. [6]

(b) Explain why Bernoulli's equation can be used to determine the pressure difference $(p_0 - p_1)$ between plane 0 and plane 1. Use Bernoulli's equation to find $(p_0 - p_1)$ in terms of ρ and V_0. [8]

(c) Find expressions for the momentum fluxes at plane 0 and plane 1 in terms of R, V_0 and ρ. [10]

(d) Find an expression for the horizontal force that the flow exerts on the pipe between plane 0 and plane 1. Comment on the physical origin of this force. [6]

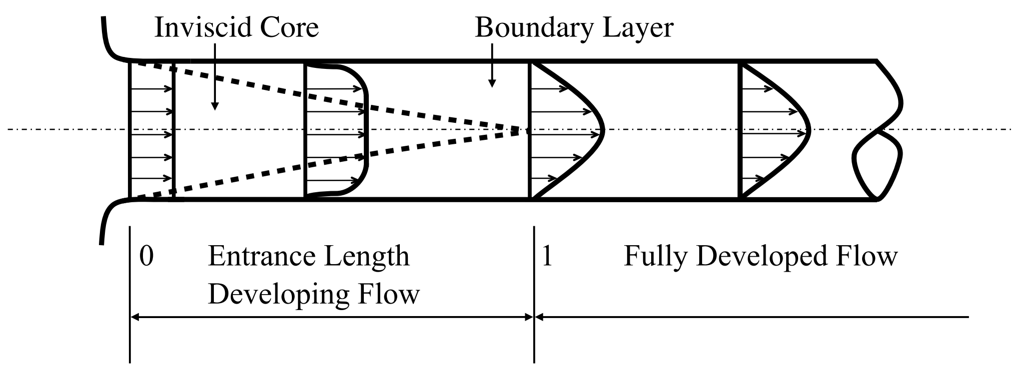

Fig. 3

4 (**short**) As shown in Fig. 4, helium gas of mass 0.075 kg is maintained at a constant pressure $p = 5$ bar by means of a weighted frictionless piston in a cylinder. It may be assumed that helium behaves as a perfect gas.

(a) In its initial state, the helium occupies a volume of 0.5 m^3. Calculate the initial temperature of the gas. [3]

(b) The helium is cooled until its temperature reaches 0 °C. Taking the helium as the thermodynamic system, calculate the work done by the system, the change in internal energy of the system, and the heat transferred to the system, during the cooling process. [7]

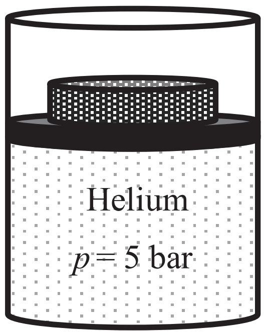

Fig. 4

5 (**short**) An air compressor operates in steady flow with air entering with pressure p_1 and temperature T_1, and leaving at pressure p_2. Within the compressor the air undergoes a reversible polytropic process, $pv^n =$ constant, where v is the specific volume and n is a constant. Assume air behaves as a perfect gas and neglect changes in kinetic and potential energy between the inlet and the outlet.

(a) Starting from the expression $dw_x = -v\,dp$ for a reversible steady flow, show that w_x (the shaft work transfer from the compressor per unit mass of air) is given by

$$w_x = -\frac{nR}{n-1}(T_2 - T_1)$$

where R is the gas constant per unit mass of air. [5]

(b) Derive an expression for the heat transfer to the compressor per unit mass of air in terms of R, T_1, T_2, n and the ratio of the specific heat capacities γ. [5]

6 (**long**) Figure 5 shows a gas turbine power plant. Air enters the compressor at State 1 ($p_1 = 1$ bar, $T_1 = 300$ K) with a mass flow rate of 10 kg s^{-1}. It is compressed to State 2 ($p_2 = 10$ bar), where the flow is divided into two streams. The main stream passes through a heater to State 3 ($p_3 = 10$ bar, $T_3 = 1400$ K) and then through a turbine to State 4 ($p_4 = 1.2$ bar). A bypass stream passes through a throttle valve to State 5 ($p_5 = 1.2$ bar). The two streams are mixed adiabatically to produce a single flow at State 6 ($p_6 = 1$ bar). The turbine and compressor are both adiabatic and reversible, and the net power output from the plant is $\dot{W}_{\mathrm{net}} = 3$ MW. Changes in kinetic and potential energy between the numbered states may be neglected and air may be treated as a perfect gas. The plant operates steadily.

(a) Calculate the compressor exit temperature T_2 and the power input to the compressor. [5]

(b) Calculate the turbine exit temperature T_4, the mass flow rate through the turbine, and the efficiency of the power plant. [10]

(c) Calculate the mixer exit temperature T_6 and the rate of entropy generation in the mixer. [10]

(d) Sketch a T-s diagram indicating all processes undergone by the streams (*i.e.*, states 1–6). Use solid lines to denote paths through equilibrium states and dotted lines where there are no paths through equilibrium states. [5]

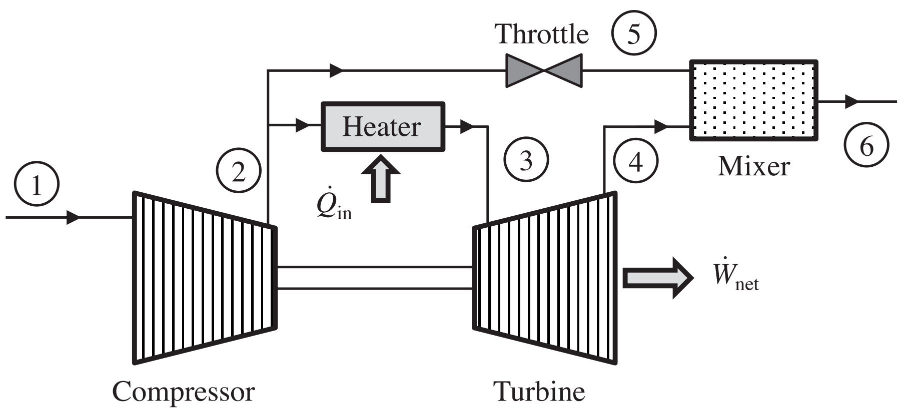

Fig. 5

SECTION B

7 (**short**) As shown in Fig. 6, a spinning ice skater is modelled using a central point mass, M (representing the body), connected by light links of length R to two point masses, each of mass m (representing the outstretched arms). The skater's initial angular velocity is Ω, and the frictional torque between the skates and the ice can be neglected.

(a) The skater pulls in her arms, after which the masses representing them are at a distance $R/2$ from the axis of spin. Find an expression for the new angular velocity of the skater. [5]

(b) If it takes time T for the skater to pull in her arms, find an expression for the average power required. [5]

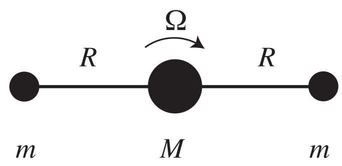

Fig. 6

8 (**short**) The mechanism shown in Fig. 7 is driven at joint A such that the member AB turns with angular velocity ω.

(a) Using the method of instantaneous centres, or otherwise, derive an expression for the velocity of the slider C. [5]

(b) If a friction force F acts at slider C and friction torques Q act at all joints A,B and C, find an expression for the torque needed to drive the mechanism. [5]

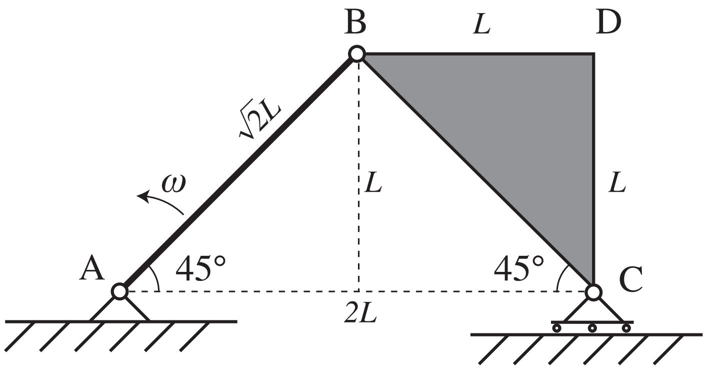

Fig. 7

9 (**long**) A heavy chain of length L and mass per unit length ρ is resting on a table, as sketched in Fig. 8.

(a) The chain is being pulled up by one of its ends with a force F which varies with time t so that the chain moves with constant velocity v. Determine F, as a function of time, and sketch it on a graph. [8]

(b) The chain is lifted up again, but this time by a constant force F.

(i) Derive the differential equation obeyed by the length of chain in the air, z, and use the following trial solution to find $z(t)$:

$$z(t) = a + bt + ct^2$$

[12]

(ii) Hence, find the time it takes for the whole chain to leave the table. [5]

(iii) Determine the minimum value of F needed for the whole chain to leave the table, at least for an instant, and comment on your answer. [5]

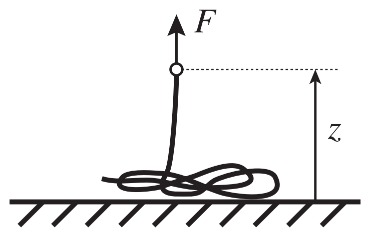

Fig. 8

10 (**short**) Figure 9 shows a smooth rod of length L rotating anti-clockwise in a horizontal plane about O at a constant angular velocity Ω. A particle P, which is free to slide along the rod, is attached to a light inextensible wire of length $L/2$ whose other end is connected to O. At the instant shown in Fig. 9, the wire snaps and the particle P slides towards the end of the rod.

(a) Show that the radial position r of P while P is still in contact with the rod, expressed as a function of time t after the wire snapped, is given by $r = A\cosh bt$ and determine A and b in terms of L and Ω. [6]

(b) Derive an expression for the time taken for P to reach the end of the rod. [4]

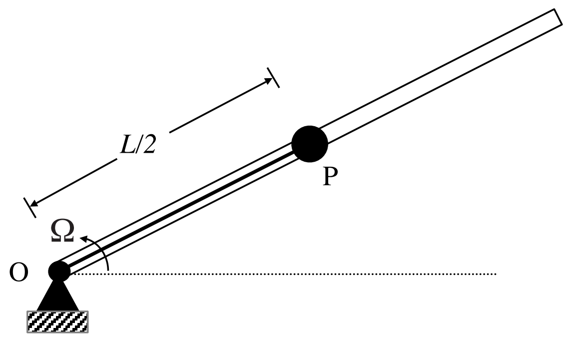

Fig. 9

11 (**short**) Figure 10 shows a rotor with moment of inertia J supported by a light elastic shaft of torsional stiffness k. The angle of rotation of the rotor from its equilibrium position is θ.

(a) Determine the natural frequency for torsional oscillations about equilibrium for $J = 1$ kg m^2 and $k = 10^4$ N m rad^{-1}. [2]

(b) If $\theta = 0.1$ rad and $\dot{\theta} = 10$ rad s^{-1} at $t = 0$, derive an expression for the amplitude of the subsequent motion in the absence of damping. [3]

(c) If the dimensionless damping factor for the system is $\zeta = 0.01$, estimate how long it would take for the amplitude to decrease to 0.1% of its initial value. [5]

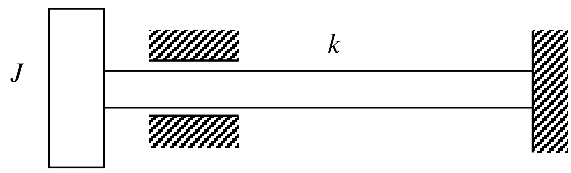

Fig. 10

12 (**long**) Figure 11 shows a schematic diagram of an instrument comprising two spring-coupled masses connected to an outer casing through additional springs. As indicated in Fig. 11, the springs connecting each mass to the outer casing are mismatched by a stiffness of value Δk from a nominally designed stiffness of value k such that $\Delta k < k$. The coupling spring between the two masses has a stiffness of value $4k$. The displacements of the two masses are x_1 and x_2, respectively as shown, and the displacement of the casing is y, all with respect to the laboratory frame of reference and the origin of each displacement is the corresponding static equilibrium position. The effects of gravity should be ignored.

(a) Derive the equation of motion for this system. [4]

(b) If the casing is held fixed at $y = 0$, find expressions for the resonant frequencies of the system. [8]

(c) The external casing is driven such that it vibrates harmonically with frequency ω so that its displacement is given by $y(t) = Y\cos\omega t$. The resulting displacement of the two masses can be expressed as $x_1(t) = X_1\cos\omega t$ and $x_2(t) = X_2\cos\omega t$. Derive an expression for $(X_1 - X_2)$ as a function of ω. Briefly discuss the physical interpretation of this result when $\Delta k = 0$. [10]

(d) Sketch the variation of the displacement ratio $(X_1 - X_2)/Y$ with ω. [8]

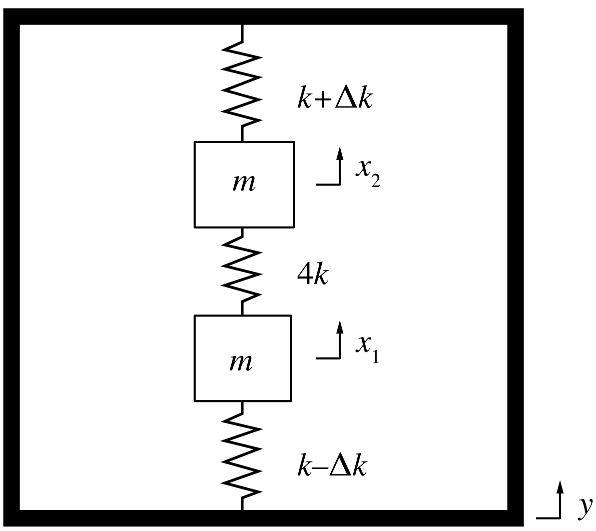

Fig. 11

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Answers

1. (b) (i) $(1100y + 250y^2)g$ (ii) 21.9 kN

2. (a) 720 pa

3. (a) $2V_0$ (b) $\frac{3}{2}\rho V_0^2$ (c) $\frac{4}{3}\pi R^2 \rho V_0^2$ (d) $\frac{7}{6}\pi R^2 \rho V_0^2$

4. (a) 1603 K (b) −207 kJ , −310 kJ , −517 kJ

5. (b) $\frac{(n-\gamma)}{(\gamma-1)(n-1)} R(T_2 - T_1)$

6. (a) 579 K , 2.81 MW (b) 764 K , 9.08 kg s^{-1}, 40.4%
   (c) 747 K , 553 $\mathrm{J\,K^{-1}s^{-1}}$

7. (a) 4Ω (b) $\frac{3m\Omega^2 R^2}{T}$

8. (a) $2\omega L$ (b) $4Q + 2FL$

9. (a) $F = \rho gvt + \rho v^2$ for $0 < t \le L/v$ and $F = \rho Lg$ for $L/v < t$

   (b) $z = t\sqrt{\frac{F}{\rho}} - \frac{t^2 g}{6}$

   (c) $t = \frac{3}{g}\left(\sqrt{\frac{F}{\rho}} - \sqrt{\frac{F}{\rho} - \frac{2Lg}{3}}\right)$

   (d) $F > \frac{2L\rho g}{3}$

10. (a) $A = \frac{L}{2}$, $b = \Omega$ (b) $\frac{1}{\Omega}\cosh^{-1}(2)$

11. (a) $f = \frac{100}{2\pi}$ (b) $0.1\sqrt{2}$ (c) 6.9 s

12. (b) $\omega^2 = \frac{5k \pm \sqrt{16k^2 + (\Delta k)^2}}{m}$

    (c) $X_1 - X_2 = \frac{2Y\,\Delta k\, m\omega^2}{[(9k - m\omega^2)(k - m\omega^2) - (\Delta k)^2]}$