

Thursday 5 June 2014 9 to 12

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (**short**) An inextensible flexible cable is suspended between two rigid arms as shown in Fig. 1. Both arms can pivot freely and form angle θ to the horizontal. The cable is loaded as shown by a continuously distributed uniform load of ω per unit horizontal distance.

(a) Show that the force in each arm has magnitude $R = \omega L / \sin \theta$. [4]

(b) Derive an expression for the shape $v(x)$ of the cable in terms of x , θ and L using the co-ordinate system shown in the figure. [6]

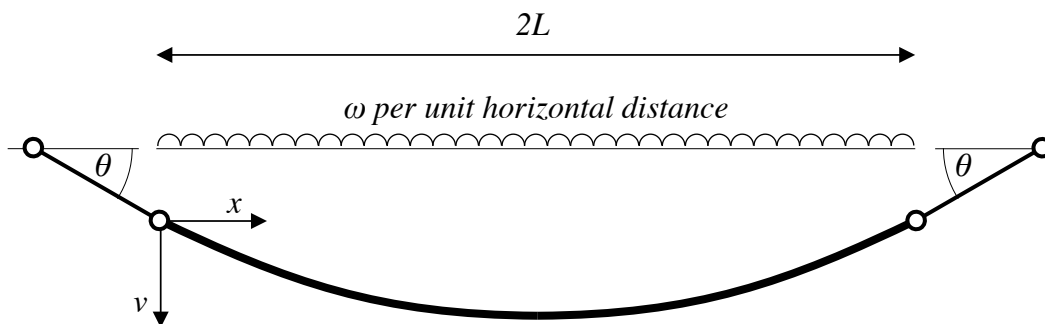


Fig. 1

2 (short) The test frame shown in Fig. 2a experiences loads H and V as shown, applied at the pin-joint P . The frame comprises two identical struts, of Young's Modulus E , cross-sectional area A , second moment of area I and length L . The struts are rigidly clamped at the ends opposite the pin joint, and are perpendicular.

(a) Figure 2b shows the horizontal strut on its own. Write down the relationships between forces T and S , applied in parallel and perpendicular directions at the tip of the strut as shown, and the tip deflections t and s in the same directions. [4]

(b) By rearranging your answers to part (a) to predict the forces required to give certain tip deflections, derive an expression for the displacements h and v arising from application of the forces H and V to the structure in Fig. 2a. [6]

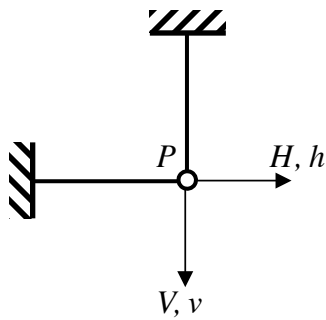


Fig. 2a

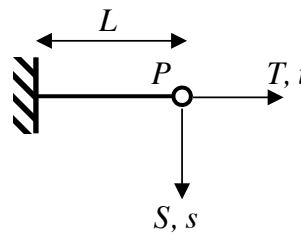


Fig. 2b

3 (short) Figure 3 shows the displacement diagram for a pin-jointed truss comprising five nodes, drawn to scale. Lines representing displacements due to bar extensions are solid, and those representing displacement due to bar rotations are dashed. Nodes A and B are restrained horizontally and vertically, and node B is horizontally a distance L to the right of A. Node C is vertically above node B. The only external force applied to the structure is at node E. The structure comprises horizontal and vertical members of length L and diagonal members of length $\sqrt{2}L$, all of which have the same Young's modulus E and cross-sectional area A .

(a) Draw the structure. [6]

(b) Find all the bar forces within the structure and the external force acting on it. [4]

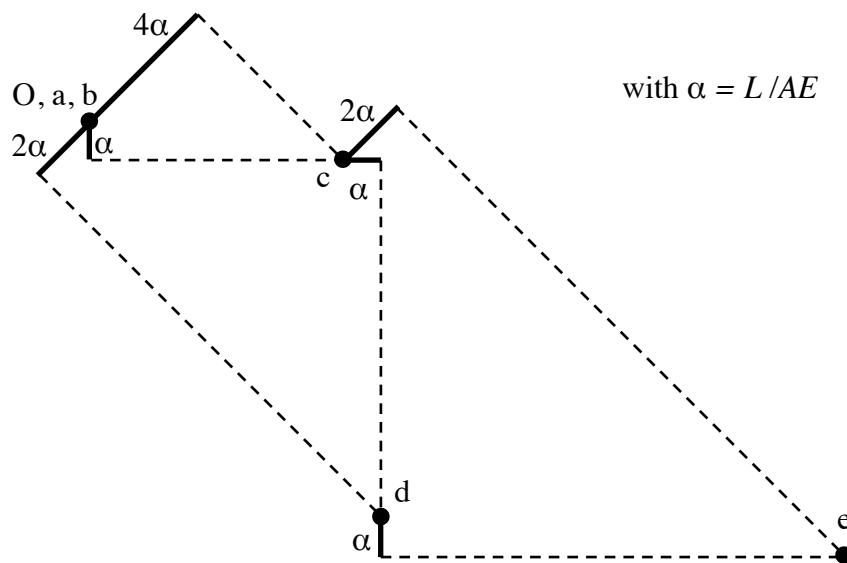


Fig. 3

4 (short) Two columns, each of length 2 m, are compressed in a testing machine as shown in Fig. 4. The ends of the columns are firmly embedded in stiff grips, and the descent of the upper grip is controlled so that the two columns always have the same length. The cross-sections of the two columns are shown in the figure. Both are made from steel with Young's Modulus of 210 GPa.

- (a) Calculate the second moments of area for the two columns, using the minor axis where there is a choice. [4]
- (b) How far will the upper grip have descended before one of the columns buckles? [6]

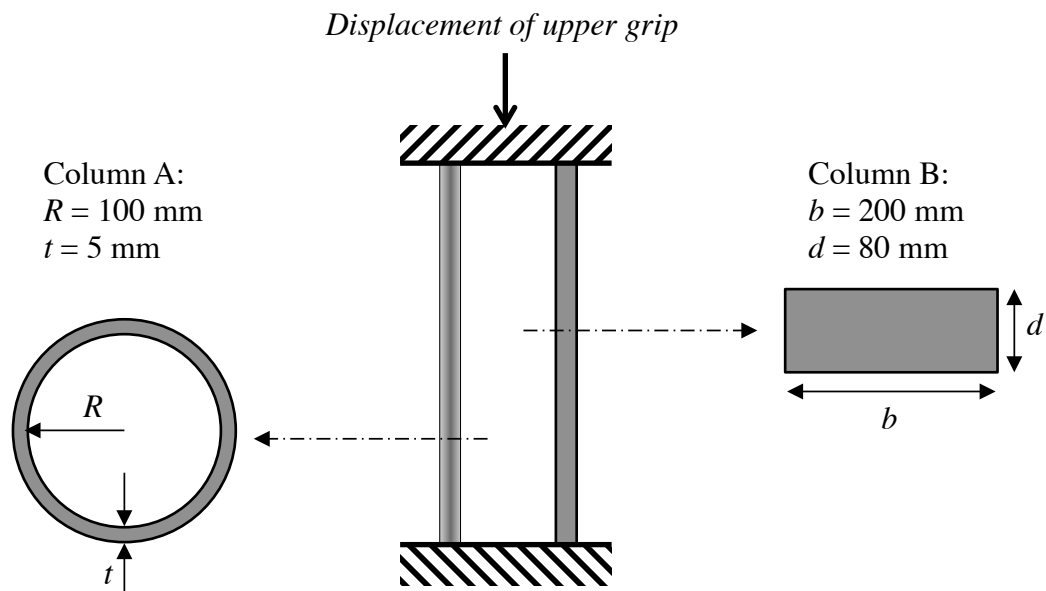


Fig. 4

NOT TO SCALE

5 (long) A crane is assembled from pin-jointed steel struts as shown in Fig. 5. The vertical struts in the tower and all horizontal struts are 5 m long, and the Young's modulus of the steel is 210 GPa. The crane is designed for a maximum load of 100 kN.

(a) At maximum load, calculate the bar forces in the following struts:

(i) AB, AD, AC, BD. [6]

(ii) EF, EG, FG, FH. [6]

(iii) IJ, JK, IK. [6]

(b) The crane has been designed so that every loaded strut extends or contracts by exactly 10 mm at maximum load. What is the vertical deflection of the tip of the crane (node X) at maximum load, when using this design? [12]

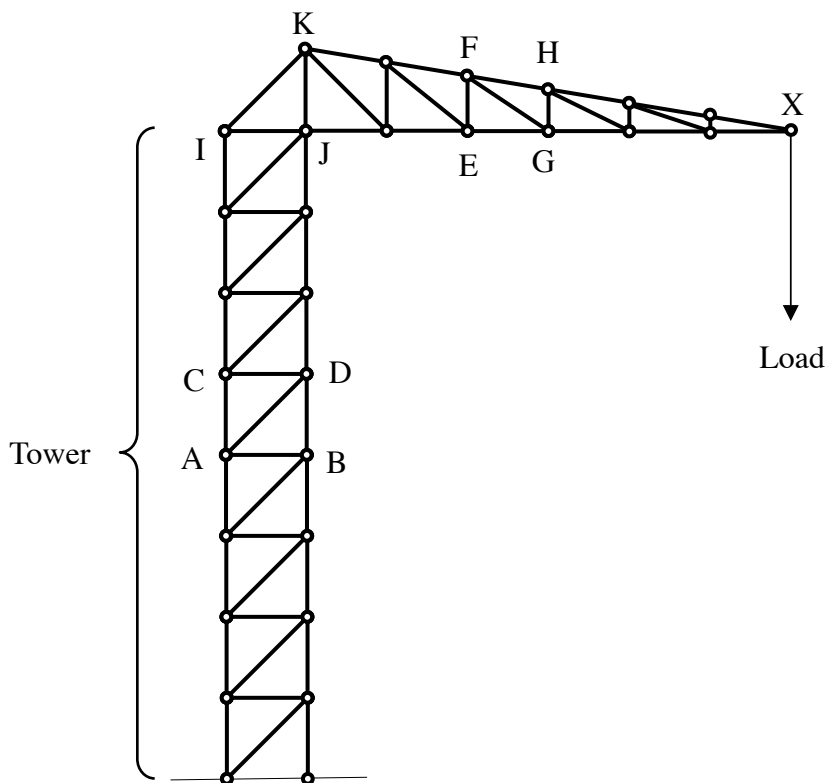


Fig. 5

6 (long) Fig. 6a shows a simplified lengthwise view of an oil tanker with two identical storage tanks, A and B. The empty ship weighs 200,000 tonnes and each tank has capacity for 160,000 tonnes of oil. The tanker is made of steel plates of a single thickness, with Young's Modulus 210 GPa and yield stress 350 MPa.

When tank A is full and tank B is empty, and the ship is floating in still water, the distributed reaction force applied by the sea to the ship has the form shown in Fig. 6b.

(a) Find the values of R_L and R_R defined in Fig. 6b, assuming the weight of the ship is uniformly distributed along its length, and using a value for g of 10 m s^{-2} . [12]

(b) For this loading, derive formulae for the shear force and bending moment distributions along the length of the ship **within tank A only**. Sketch these distributions, marking salient points. [10]

(c) If the ship has a uniform cross-section as shown in Fig. 6c, estimate the minimum thickness of steel plate required so that the steel remains elastic under this loading. Briefly state and discuss any assumptions you make. [8]

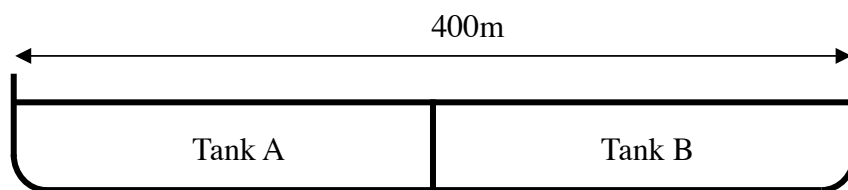


Fig. 6a

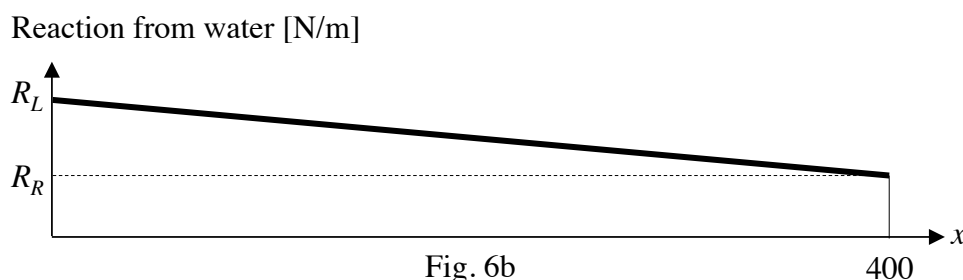


Fig. 6b

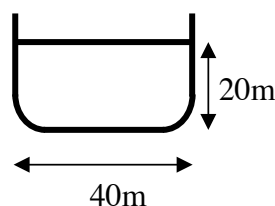


Fig. 6c

NOT TO SCALE

SECTION B

7 (short)

(a) An isotropic, linear elastic material with Young's modulus E and Poisson's ratio ν is subjected to tensile stresses σ_1 , σ_2 and σ_3 along its principal axes. Show that the resulting strains ε_1 , ε_2 and ε_3 are given by

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E}(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) \\ \varepsilon_2 &= \frac{1}{E}(-\nu\sigma_1 + \sigma_2 - \nu\sigma_3) \\ \varepsilon_3 &= \frac{1}{E}(-\nu\sigma_1 - \nu\sigma_2 + \sigma_3)\end{aligned}\tag{4}$$

(b) A cube of the material in part (a) is compressed along the 1 direction but is constrained in both transverse directions. Show that the "effective modulus" (σ_1/ε_1) relating the stress σ_1 and strain ε_1 along the 1 direction is given by

$$\frac{\sigma_1}{\varepsilon_1} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\tag{6}$$

8 (short) The data below describe the oxidation of a steel at 1220 °C in an atmosphere containing oxygen.

Time, t (s)	500	1000	2000	5000	10000
Mass gain per unit area, Δm (g m ⁻²)	4.5	6.0	8.5	13.5	19.5

(a) Demonstrate that, at 1220 °C, the kinetics of oxidation are approximately parabolic such that $(\Delta m)^2 = kt$ and that the parabolic rate constant k is about 0.038 g²m⁻⁴s⁻¹. [4]

(b) Assuming the oxide has the composition Fe₃O₄, could a sheet of this steel of thickness 1 mm, totally immersed in this environment, be used for 300 days without being oxidised throughout its entire thickness?

Relative atomic weight of oxygen = 16.

Relative atomic weight of iron = 56.

Density of the steel = 7800 kg m⁻³. [6]

9 (short)

(a) Consider the halite structure AB shown in Fig. 7a. Find the ratio of the diameters of A and B ions such that the A and B ions are touching on the edges of the unit cell and the A ions are touching on the face diagonal.

[4]

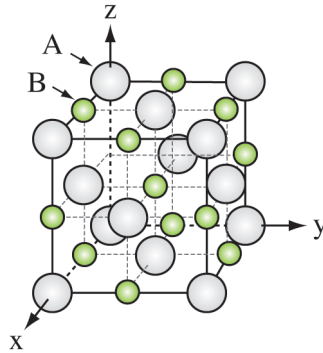


Fig. 7a

(b) Figure 7b shows the unit cell of diamond cubic (DC) silicon carbide. The lattice constant of silicon carbide is $a = 0.436 \text{ nm}$. The atomic mass of silicon is $28.09 \text{ kg kmol}^{-1}$ and that of carbon is $12.01 \text{ kg kmol}^{-1}$. Calculate the theoretical density of the silicon carbide.

[6]

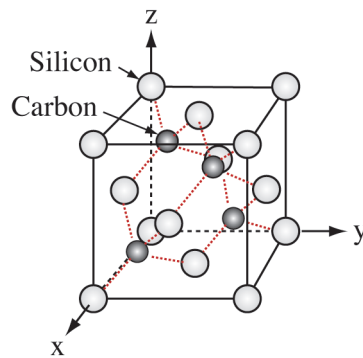


Fig. 7b

10 (**short**) Figure 8 shows scanning electron micrographs of fracture surfaces obtained from metallic samples tested in tension.

(a) For each of the micrographs, briefly describe the main features observed. Comment on the relative magnitude of the energy expended during fracture in each case. Sketch the *nominal stress – nominal strain* curves for each case. Account for the main difference between the shapes of the two curves. [5]

(b) If the micrographs were obtained from specimens of the same steel, suggest a possible explanation for the observed differences. Explain your reasoning. Using sketches to illustrate your answer, explain what would be the macroscopic shape of the samples after failure in each case. [5]

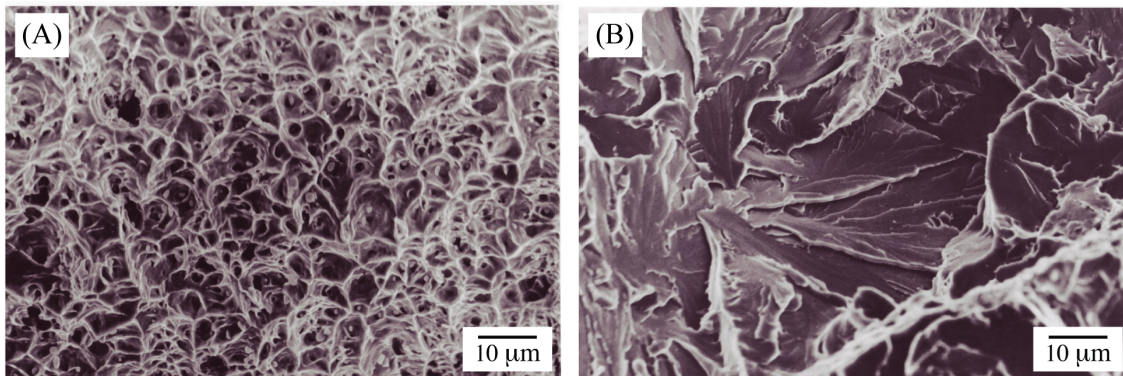


Fig. 8

11 (long)

(a) Figure 9 shows *nominal stress – nominal strain* curves I, II, III, IV obtained from tensile tests for four alloys A, B, C and D described in Table 1. Identify which curve corresponds to which alloy, explaining your reasoning and discussing the reasons for the differences in the curves. [8]

(b) A ductile wire with a cross-section initially uniform is loaded in tension. At the maximum load, necking is observed to start. A criterion for the onset of necking is

$$\frac{d\sigma_t}{d\varepsilon_t} = \sigma_t$$

where σ_t is the true stress and ε_t is the true strain. The relationship between true stress σ_t and true strain ε_t for an annealed alloy wire undergoing a tensile test approximates to

$$\sigma_t = 250 \varepsilon_t^{1/2} \text{ MPa}$$

Determine:

- (i) the true strain in the wire when the tensile load reaches its maximum value; [4]
- (ii) the tensile strength of the alloy. [6]

(c) Alloy wire samples, with tensile strength σ_{ts} , are subjected to an alternating stress about a zero mean stress. Under these test conditions a stress range of $0.50\sigma_{ts}$ gives a fatigue life of 10^6 cycles. Repeating the test with a stress range of $0.65\sigma_{ts}$ reduces the fatigue life to 10^4 cycles. In a second series of tests, a stress range of $0.50\sigma_{ts}$ is applied at a mean stress of $\sigma_m = 0.10\sigma_{ts}$ for 4×10^4 cycles. The stress range is then increased to $0.55\sigma_{ts}$ and testing is continued until failure occurs. Estimate the total fatigue life of the samples in the second series of tests. [12]

(cont.

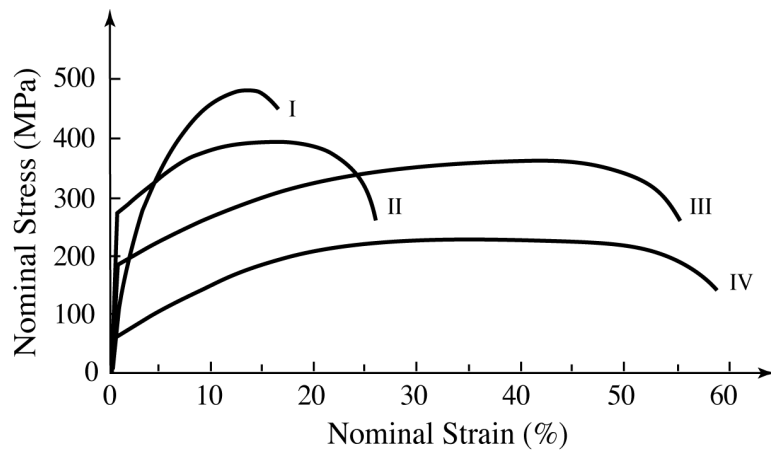


Fig. 9

A	Brass, annealed
B	Brass, drawn
C	Heat-treatable Al alloy
D	Copper, annealed

Table 1

12 (long)

(a) Discuss the *leak-before-break* criterion in connection with pressure vessel design. [6]

(b) A steel thin-walled spherical pressure vessel is to be used in a chemical processing plant. The vessel has a wall thickness of 10 mm and a diameter 2 m and is required to operate safely at a pressure of 3 MPa. The steel was specified with a fracture toughness K_{IC} of $85 \text{ MPa}\sqrt{\text{m}}$.

Non-destructive testing revealed that in the finished vessel there are small cracks in the inner surface with their planes normal to the wall surface. The stress intensity factor K for such cracks is given by

$$K = 1.13\sigma\sqrt{\pi a}$$

where σ is the tensile stress in the wall and a is the crack depth.

(i) Show that a leaking vessel satisfies the leak-before-break criterion. [6]

(ii) Under fatigue loading by a stress intensity range ΔK , the cracks grow at a rate

$$\frac{da}{dN} = A\Delta K^4$$

where $A = 5 \times 10^{-16} (\text{m/cycle})(\text{MPa}\sqrt{\text{m}})^{-4}$ and N is the number of fatigue cycles. Estimate how many cycles it will take for leakage to occur if the steel contains a crack of depth 2 mm. [8]

(c) Consider a thin-walled spherical pressure vessel. For a fixed vessel radius, derive a *material merit index* in terms of the fracture toughness K_{IC} and the yield stress of the wall material σ_y , which should be maximised in order to operate at a maximum pressure consistent with the leak-before-break criterion. It is required that plastic deformation should not occur. Use the data in the table below to select the best material according to the merit index you have derived.

Material	σ_y (MPa)	K_{IC} ($\text{MPa}\sqrt{\text{m}}$)
Medium carbon steel	603	75
Titanium alloy	748	65
Aluminium alloy	265	30
Magnesium alloy	235	20

[10]

END OF PAPER

Answers

1. (b) $v(x) = x \tan \theta \left(1 - \frac{x}{2L}\right)$
2. (a) $t = \frac{LT}{EA}$, $s = \frac{5L^3}{3EI}$
(b) $h = \frac{HL^3}{E(AL^2 + 3I)}$, $v = \frac{VL^3}{E(AL^2 + 3I)}$
3. (b) External force at E is 1 to the right, horizontally.
4. (a) $I_{tube} = 16.9 \times 10^{-6} \text{ m}^4$, $I_{bar} = 8.53 \times 10^{-6} \text{ m}^4$
(b) 10.5 mm
5. (a) (i) 0, 0, 600, -700 kN
(ii) 0, -600, 0, 608 kN
(iii) -700, -700, 849 kN
(b) 1.98 m
6. (a) $R_R = 3 \times 10^6 \text{ Nm}^{-1}$, $R_L = 15 \times 10^6 \text{ Nm}^{-1}$
(b) $S = (2x - 0.015x^2) \text{ MN}$, $M = (x^2 - 0.005x^3) \text{ MNm}$
(c) $t > 21.2 \text{ mm}$
8. (b) Thickness of metal lost on either side 0.33 mm (< 1 mm). Material can be used.
9. (a) $D_B/D_A = 0.414$
(b) Theoretical density = 3.21 Mg m^{-3}
11. (b) (i) $\epsilon_t = 0.5$ (ii) $\sigma_n \cong 107 \text{ MPa}$
(c) $N_f \cong 6.25 \times 10^4 \text{ cycles}$
12. (b) (ii) $N_f \cong 9.81 \times 10^7 \text{ cycles}$
(c) K_{IC}^2/σ_y , Best material: medium carbon steel