EGT0
ENGINEERING TRIPOS PART IA

Monday 9 June 20149 to 12

## Paper 3

## ELECTRICAL AND INFORMATION ENGINEERING

Answer all questions.

The approximate number of marks allocated to each part of a question is indicated in the right margin.

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper
SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

## 1 (long)

(a) State Thevenin's theorem. Explain how loop current analysis can be used to determine currents and voltages in an electrical circuit.
(b) Figure 1(a) shows the circuit for an a.c. bridge measurement system. The voltage source is an a.c. source of frequency $\omega$. When the bridge is balanced, the current through the meter M is zero. Find the conditions for balance in terms of the resistances $\mathrm{R}_{1}, \mathrm{R}_{2}, \mathrm{R}_{3}, \mathrm{R}_{4}$ and the ratio L/C.
(c) Figure 1(b) shows a second bridge circuit, a Wien bridge. Find the frequency at which the bridge balances in terms of $\mathrm{R}_{3}, \mathrm{R}_{4}$ and C .


Fig. 1(a)


Fig. 1(b)

## 2 (long)

(a) Describe the advantages, for a circuit designer, of negative feedback on an amplifier.
(b) Draw a labelled equivalent circuit of an Op Amp with an input resistance of $\mathrm{R}_{\mathrm{in}}$, an output resistance of $\mathrm{R}_{\text {out }}$ and a gain of A .
(c) Figure 2 shows an Op Amp circuit driving a cable of resistance $R_{3}$, and a load of resistance $\mathrm{R}_{4}$ in parallel with capacitance C. Taking $\mathrm{A}=10^{4}, \mathrm{R}_{1}=4.5 \mathrm{k} \Omega, \mathrm{R}_{2}=500 \Omega$, $\mathrm{R}_{3}=75 \Omega, \mathrm{R}_{4}=150 \mathrm{k} \Omega$ and $\mathrm{C}=100 \mathrm{pF}$, calculate the approximate mid-band gain and 3 dB frequency of the circuit.


Fig. 2

3 (short) Figure 3 shows a MOSFET which is to be biased to an operating point of $\mathrm{V}_{\mathrm{GS}}=4 \mathrm{~V}, \mathrm{~V}_{\mathrm{DS}}=10 \mathrm{~V}$ and $\mathrm{I}_{\mathrm{DS}}=4 \mathrm{~mA}$.
(a) Calculate the values of $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ to achieve this.
(b) If $\mathrm{r}_{\mathrm{d}}=30 \mathrm{k} \Omega$ and $\mathrm{g}_{\mathrm{m}}=5 \mathrm{mS}$ for the MOSFET, calculate the mid-band small signal gain of the circuit.


Fig. 3

4 (short) Using Thevenin's theorem, or otherwise, calculate the current through the 6 V battery in Fig. 4.


Fig. 4

5 (short) A small factory consumes 24 kW of power with a lagging power factor of 0.7 at the factory. When it is connected to a 50 Hz supply, the rms voltage across the factory terminals is 240 V . The supply line has an impedance $Z_{s}=(0.02+\mathrm{j} 0.04) \Omega$.
(a) Draw a circuit diagram and calculate the real power lost in the line, and the voltage supplied at the input.
(b) Assuming that the rms voltage across the factory terminals continues to be 240 V , calculate the value of the capacitor connected across the factory terminals that is needed to correct its power factor to unity.

## SECTION B

6 (short)
(a) Prove that for any integer $n>0$ the Boolean expression below is true.

$$
A_{1}+A_{2}+\ldots+A_{n}=\overline{\overline{A_{1}} \cdot \overline{A_{2}} \ldots \overline{A_{n}}}
$$

(b) Draw a circuit diagram that implements the Boolean expression

$$
A . B+C
$$

by means of NOR gates only, using the least number of gates possible.

## 7 (short)

(a) Explain what is meant by static and dynamic hazards.
(b) Use a Karnaugh map to simplify the Boolean expression below.

$$
A \cdot \bar{B} \cdot C \cdot D \cdot \bar{E}+A \cdot \bar{B} \cdot \bar{C} \cdot D \cdot \bar{E}+A \cdot \bar{B} \cdot \bar{C} \cdot D \cdot E+A \cdot B \cdot \bar{C} \cdot D \cdot E
$$

Draw a circuit that implements this by means of NAND gates and inverters only, using the least number of these components possible.
(c) Explain how you would modify your simplified expression in (b) such that static 1-hazards are also avoided.

## 8 (short)

(a) A memory chip has 30 address wires and 8 data wires. Calculate its capacity in bits. State also the capacity in megabytes.
(b) (i) Consider the following commands in PIC assembly language :

```
    movlw 0x02
        movwf 0x20
lb decfsz 0x20
        goto l.b
```

Describe how the contents of the memory location $0 \times 20$ change as the commands above are executed.
(ii) Describe how the contents of the memory location $0 \times 20$ change as the commands below are executed instead :

```
    movlw 0x02
```

lb movwf 0x20
decfsz 0x20
goto lb

9 (long) Binary data are received sequentially from an input $I$ and the output $Z$ becomes 1 if the sequence 010 is detected. The output $Z$ is otherwise 0 . The detector also responds to overlapping sequences, i.e., the sequence 01010 should give $Z=1$ after bit 3 is received and also after bit 5 is received.
(a) By considering the following four states

|  | Description |
| :---: | :---: |
| state 1 | start |
| state 2 | got 0 |
| state 3 | got 01 |
| state 4 | got 010 |

draw the state diagram for the system.
(b) Two JK bistables, with outputs labelled as $Q_{A}$ and $Q_{B}$ respectively, are used to construct the detector. By using the bistable allocation below draw the state transition table.

|  | $Q_{A}$ | $Q_{B}$ |
| :--- | :---: | :---: |
| state 1 | 0 | 0 |
| state 2 | 0 | 1 |
| state 3 | 1 | 0 |
| state 4 | 1 | 1 |

(c) Use Karnaugh maps to deduce simplified expressions for the connections between the bistables. Draw also the corresponding circuit diagram of the detector.
(d) State how many bistables would be needed if the detector is now designed to detect,
(i) the sequence 01 ,
(ii) the sequence 101 .

## SECTION C

10 (short)
(a) Describe and justify the method of images as applied to point charges near infinite metal sheets.
(b) Two semi-infinite metal sheets meet at an angle of $60^{\circ}$ as shown in Fig. 5. Describe the arrangement of image charges that is needed to determine the electric field around a point charge that is placed at point P on the plane that bisects the $60^{\circ}$ wedge.
(c) If the point charge in Fig. 5 is free to move, which way would it move initially?


Fig. 5

## 11 (short)

(a) Starting from the Biot-Savart law derive an expression for the magnetic flux density $B$ on the axis of a circular coil of radius $a$, at a distance $z$ from the plane of the coil, when the coil carries a current $I$. Sketch the result as a function of $z$ for both positive and negative values of $z$.
(b) Two such coils, carrying current $I$ in the same direction, are placed a distance $d$ apart on the same axis, as shown in Fig. 6. Sketch $B(z)$ as a function of $z$, along the axis of the coil.
(c) When $d=a$, the magnetic flux density on the axis mid-way between the two coils has the property that $\mathrm{d}^{2} B / \mathrm{d} z^{2}=0$. What does this mean for the field around the midpoint? Comment on the usefulness of this result for experimental work.


Fig. 6

## 12 (long)

(a) In Fig. 7, an aluminium bar of cross-sectional area $1 \mathrm{~cm}^{2}$ is suspended between the pole pieces of a magnet that produces a uniform magnetic flux density $B=0.5 \mathrm{~T}$. In order to insert the bar into the gap between the pole pieces a force $F$ is required as shown. It may be assumed that the energy stored in a magnetic field per unit volume is given by $B^{2} / 2 \mu$, where, for aluminium, $\mu=1.000023 \mu_{0}$. By equating the work done by the force $F$ to the change in stored magnetic energy, calculate the value of $F$.
(b) A plane circular search coil is used to measure the magnetic flux density $B$. The coil consists of one turn, of area $1 \mathrm{~cm}^{2}$, and the total resistance of the coil is $R=1 \mathrm{k} \Omega$. Calculate the magnetic flux threading the coil, if it is placed between the pole pieces with the plane of the coil perpendicular to the magnetic field. Also, calculate how much charge flows through the coil as it is removed from the field to a place where the magnetic flux density is zero. How could you measure the charge?
(c) An alternative method of measuring $B$ would be to have the search coil rotating about an axis through one diameter. Show that the flux cutting the coil is sinusoidal with time, and calculate the emf if the coil rotates at 3600 revolutions per minute when it is placed between the pole pieces.
(d) In practice, which method is more useful for validating the design of a magnet, and why?


Fig. 7

## END OF PAPER

## Answers

1. (b) $\mathrm{R}_{1} \times \mathrm{R}_{4}=\frac{\mathrm{L}}{\mathrm{C}} \quad$ (c) $\omega=\frac{1}{\mathrm{C} \sqrt{\mathrm{R}_{3} \times \mathrm{R}_{4}}}$
2. (c) Gain $\cong 10, \quad f \cong \frac{1}{2 \pi \mathrm{CR}_{3}} \cong 21 \mathrm{MHz}$
3. 

(a) $40 \mathrm{k} \Omega, 2.5 \mathrm{k} \Omega$
(b) -11.5
4. 0.525 A
5.
(a) $400 \mathrm{~W}, 246 \mathrm{~V}$
(b) 1.35 mF
6. (b) $Y=\overline{\overline{A+C}+\overline{B+C}}$
7. (b) $Y=A \cdot D \cdot \overline{C \cdot E} \cdot \overline{B \cdot \bar{E}}$
8. (a) $2^{33}$ bits $=2^{30} / 10^{6}$ Mbytes
(b) (i) Decremented until zero (ii) Infinite loop
9.
(c) $J_{A}=I \cdot Q_{B}, K_{A}=\bar{I} \cdot Q_{B}+I \cdot \overline{Q_{B}}, \quad J_{B}=\bar{I}, \quad K_{B}=I$
(d) (i) 2 bistables
(ii) 2 bistables
10. (c) Vertically downwards
11. (a) $B=\frac{\mu_{0} I a^{2}}{2\left(z^{2}+a^{2}\right)^{3 / 2}}$
12.
(a) $2.28 \times 10^{-4} \mathrm{~N}$
(b) $5 \times 10^{-5} \mathrm{~Wb}, \quad 0.05 \mu \mathrm{C}$
(c) 0.019 V (amplitude)

