Version DDS/6

EGT1
ENGINEERING TRIPOS PART IB

Monday 2 June 2014   9 to 11

Paper 1

MECHANICS

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

Write your candidate number ***not*** *your name on each cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section.*

1 (a) A uniform hexagonal prism rolls along a flat horizontal surface. Just before one of its faces makes contact with the surface, the angular velocity of the prism is ω_1; when the face just leaves the surface, the angular velocity is ω_2. Assuming that no slipping or bouncing occurs, find the ratio between ω_1 and ω_2 and show that it is approximately 0.647 . [10]

(b) A hexagonal pencil with sides of length a is held on a planar surface inclined at 7° with the horizontal, as shown in Fig. 1. The pencil's uppermost edge is moved very slightly to the right and released, so that it starts to roll down the surface. Using the assumptions in Part (a) above, find an expression for the angular velocity of the pencil just before the first impact occurs, in terms of a and the acceleration due to gravity g. Hence determine whether or not the pencil will continue to roll down the surface, following the impact. [15]

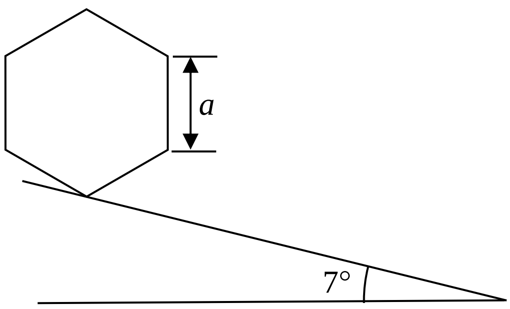

Fig. 1

2 A short uniform wooden plank of length $4a$, width $2a$ and depth a is balanced on a cylindrical surface of radius b, as shown in elevation in Fig. 2.

(a) If the plank is rolled, without slipping, away from the position shown through an angle θ, find an expression for the change in its potential energy. Hence find the smallest value of b for which the arrangement shown in Fig. 2 is stable. [12]

(b) If $b = 2a$, find the frequency of the small oscillations which would result if the plank is rolled slightly away from its equilibrium position and released. [13]

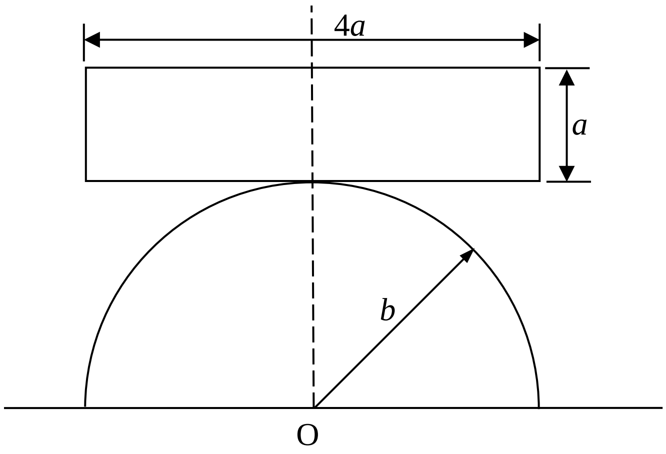

Fig. 2

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3 The blades of the wind turbine shown in Fig. 3 have length l and are rotating with a constant angular velocity Ω relative to the turbine housing. The housing itself is also rotating with a constant angular velocity ω about a vertical axis, to face the blades into the wind.

(a) Using the fixed right-handed reference frame shown in the figure, find the velocity of the point P (at the tip of one of the blades) as a function of the angle θ, when the turbine is facing in the $-\mathbf{j}$ direction, as shown. [8]

(b) Find the acceleration of the point P as a function of the angle θ, at this instant. [8]

(c) The blades each have mass m. Assuming that they are uniform along their length, find the bending moment at the root of a blade when it is pointing vertically downwards (i.e. $\theta = 180°$). [9]

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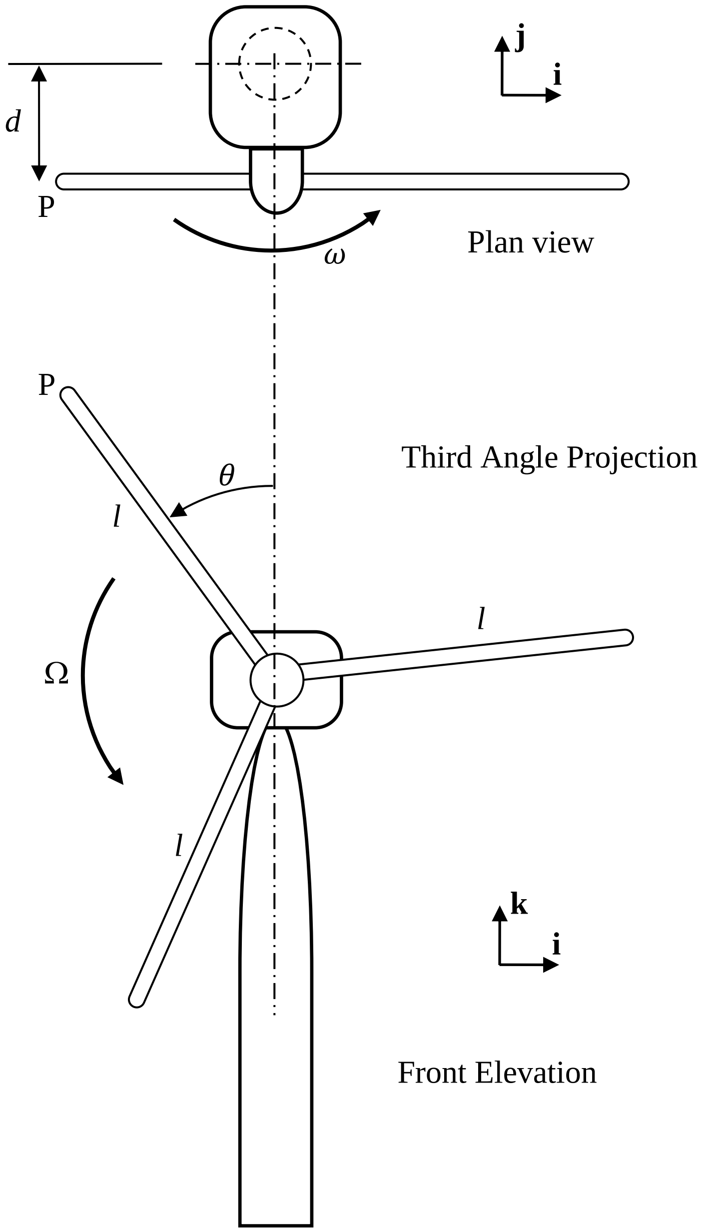

Fig. 3

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SECTION B

*Answer not more than **two** questions from this section.*

4 Figure 4 shows a side elevation of the mechanism of an "up-and-over" garage door. The 2 m tall door is supported by fixed horizontal rails at point A, and by two spring loaded arms BD (one on each side of the door), hinged to the door at B and to the frame at C. The door can be considered as a uniform lamina of mass 50 kg, the other moving parts have negligible mass. Ignore friction.

Consider the instant when BCD is horizontal (as shown in Fig. 4) and point A is moving to the left at a constant speed of 3 m s^{-1}. At this instant the combined force from both springs DE is 600 N.

(a) Find the velocity of the centre of mass of the door and the rate of extension of the springs DE. [8]

(b) (i) Find the acceleration of the centre of mass. [8]

(ii) Hence calculate the vertical force F required at B at this instant. [9]

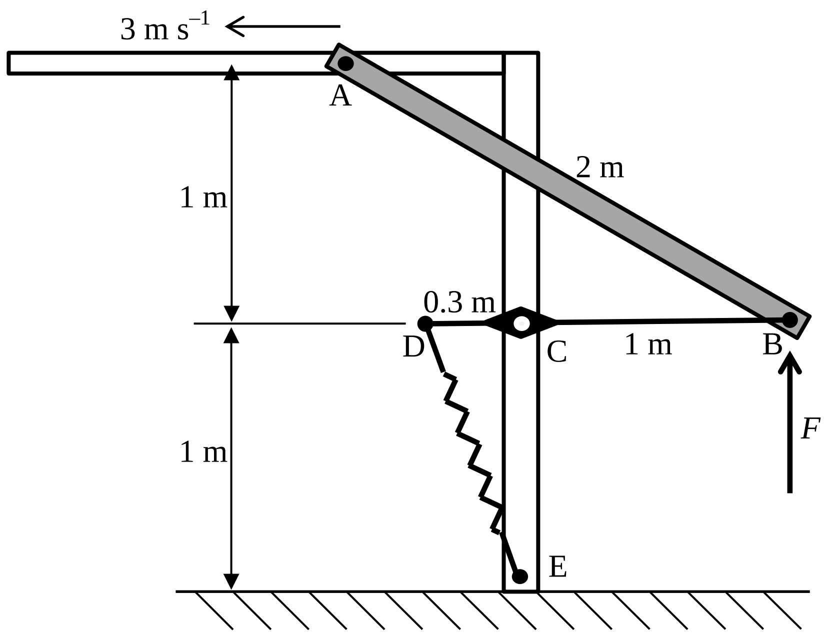

Fig. 4

5 Figure 5 is a schematic of a centrifuge that rotates with angular velocity Ω. Solid material is first fed into the centrifuge at a radius r_o where it is accelerated up to the speed $r_o \Omega$ by friction with the rough inner wall. The material is then pushed by a separate mechanism F "up" the smooth inner surface of a conical section to exit the centrifuge at a radius r_i. The centrifuge spins sufficiently fast that you may neglect the acceleration due to gravity in all your calculations.

(a) Determine the work done by the centrifuge to accelerate a point mass m to the speed $r_o \Omega$. The mass is initially stationary. The cylindrical section of the centrifuge has a rough inner surface with a coefficient of friction μ. [7]

(b) (i) Find the work done by F to "lift" the mass m from the radius r_o to the smaller radius r_i. Note that although the inner surface of the conical part of the centrifuge is smooth, it has longitudinal internal ribs that prevent tangential slip. [8]

(ii) By reference to the tangential force on the mass, or otherwise, calculate any further work done by the drum during this lifting process. [6]

(c) Hence determine the total work done in the machine to process a mass m. [4]

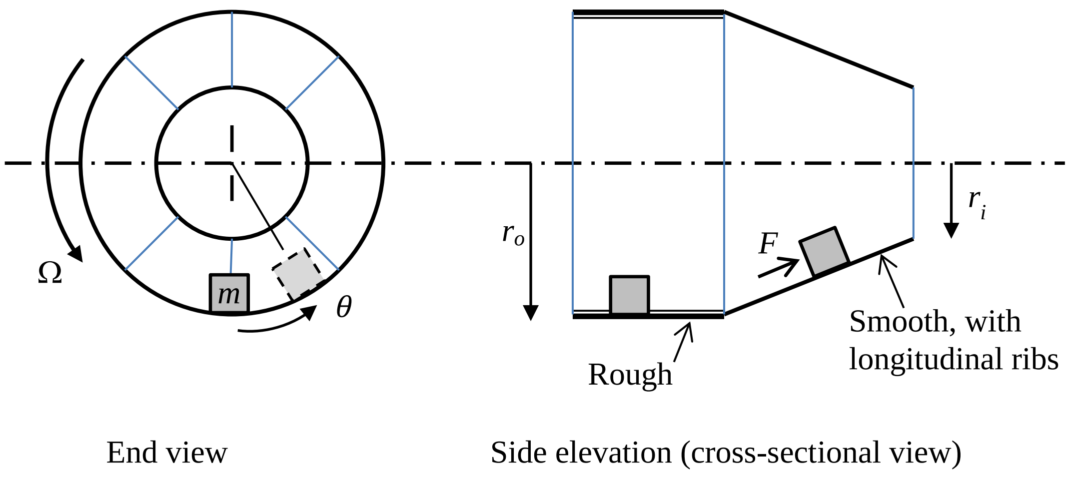

Fig. 5

6 The three main moving parts (two compressors, A & B, and one turbine, C) of an aircraft engine may be represented by three solid discs, each of radius 0.3 m, thickness 0.1 m and mass 100 kg. The three discs are mounted on a single light shaft of length 1.5 m supported by two bearings 1 m apart (see Fig. 6). The shaft is orientated longitudinally in the aircraft. Discs A and C are mounted at each end of the shaft while disc B is mounted 0.5 m behind A. The shaft spins at 10,000 rpm.

(a) Find the magnitude and direction of the forces on the bearings when:

(i) the aircraft rolls about its longitudinal axis at 5 rpm; [6]

(ii) the aircraft yaws about its vertical axis at 2 rpm. [6]

(b) The end discs, A and C, are out of balance to the extent of 0.015 kg m and 0.025 kg m respectively. Disc B is out of balance by 0.02 kg m. The discs are fixed to the shaft so as to provide static balance.

(i) Find the angular relationships between the out-of-balance masses of the three discs. [6]

(ii) Find the dynamic loads on the bearings when the aircraft is flying straight ahead. [7]

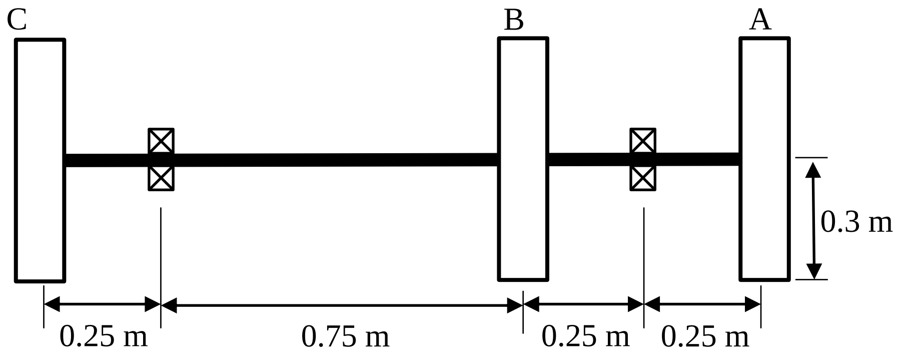

Fig. 6

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