Version JPT/5

EGT1
ENGINEERING TRIPOS PART IB

Monday 2 June 2014 2 to 4

Paper 2

STRUCTURES

*Answer not more than **four** questions, which may be taken from either section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on each cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 The support structure shown in Fig. 1 is to be designed based on a steel thin-walled circular section of 200 mm external diameter and 8 mm wall thickness. The Young’s modulus of the steel is 210 GPa, and its Poisson’s ratio is 0.3. The vertical column of the structure is embedded in the ground, which may be treated as a rigid foundation.

The structure must sustain a concentrated horizontal force F = 2 kN, which acts normal to the plane of the structure. Self-weight may be neglected.

(a) Calculate the deflection of the structure at the point of application of the force. [8]

(b) Calculate the torque, shear force and bending moment at the base of the column. Hence calculate the maximum total shear stress and the maximum bending stress at the base, and indicate on a diagram of the section the locations at which these stresses occur. [11]

(c) The limiting stress state for the design is found to comprise a total shear stress of 5 MPa, and a bending stress of 32 MPa. Use von Mises’ yield criterion, and a yield stress σ_y = 245 MPa, to determine the margin against initial yield. [6]

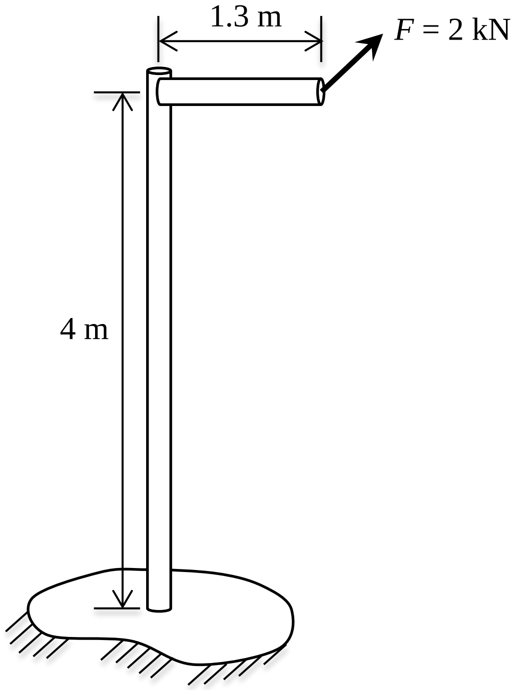


Fig. 1

2 The structure shown in Fig. 2 comprises a uniform beam AD of flexural rigidity *EI*, simply supported at points A, B, C and D. A vertical column OA is connected *rigidly* to the beam at A. The right-hand span CD carries a uniformly distributed load of total magnitude *W*, and a concentrated horizontal force *F* is applied at point O.

(a) Calculate the bending moments at B and C, and hence show that the magnitude of *F* for which the support reaction at D is zero is given by:

$$F = 19W$$ [10]

(b) Calculate the remaining support reactions for the value of *F* calculated in Part (a). Hence sketch the bending moment diagram for the beam, indicating salient values. [15]

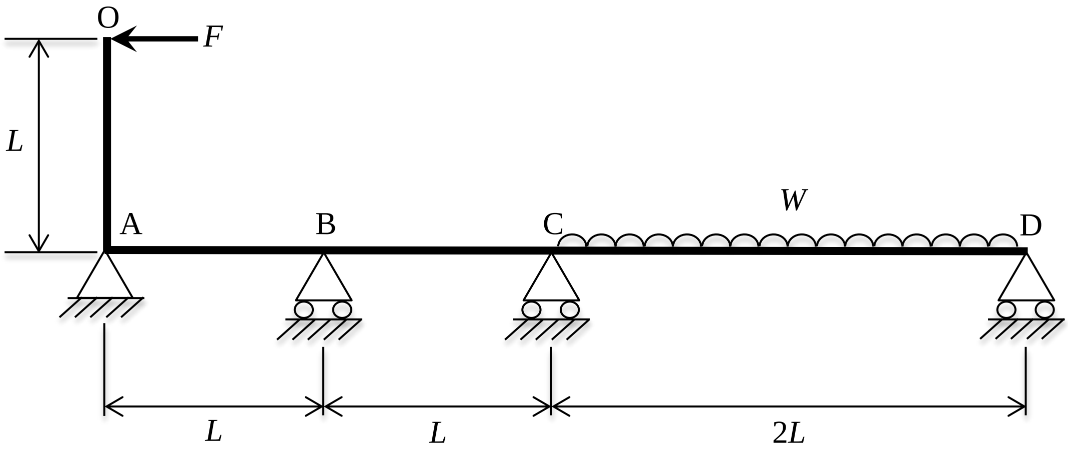


Fig. 2

3 The pin-supported portal frame shown in Fig. 3 has a flexural rigidity *EI* and coefficient of thermal expansion α. The frame is initially stress-free. A concentrated vertical load *W* is applied at the centre of the horizontal beam, as shown, and at the same time the temperature of the beam is cooled by ΔT.

Assuming that the frame remains elastic, and that axial flexibility and shear deformation may be neglected:

(a) calculate the sagging bending moment at the centre of the beam due to the combined action of the applied load and temperature drop; [10]

(b) calculate the temperature drop required to ensure that the vertical displacement at the centre of the beam remains zero, and sketch the bending moment diagram for the frame in this case. [15]

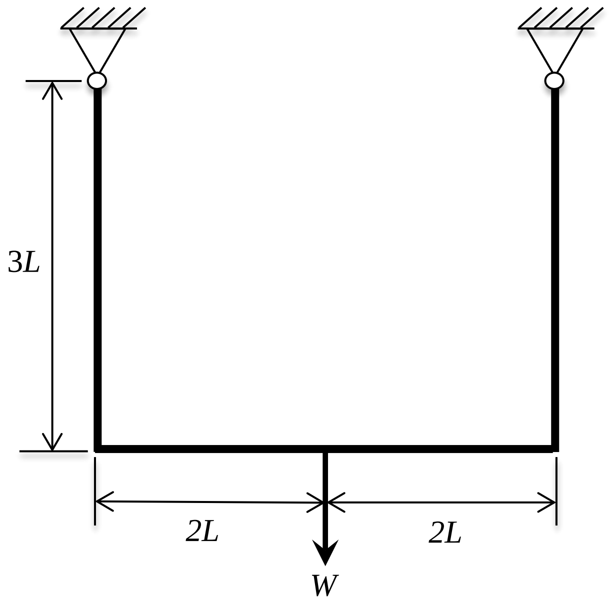


Fig. 3

SECTION B

4 The steel plate ABCD shown in Fig. 4, of length $2L$, width L and uniform thickness t, is fully clamped along the three edges AB, BC and CD, but is unsupported along the free edge AD. The plate carries a uniformly distributed load w per unit area.

(a) Assuming a yield-line pattern defined by the parameter α shown in Fig. 4, show that the plate collapses if:

$$w > \frac{12m}{L^2}\left[\frac{8\alpha+1}{\alpha(6-\alpha)}\right] \qquad [11]$$

where m is the fully plastic moment capacity per unit length of the plate.

(b) Obtain an expression for the least upper bound value for w. Hence determine the collapse load for a plate of this geometry and support condition, having dimensions L = 300 mm and t = 6 mm, and a yield stress of 250 MPa. [10]

(c) Sketch two alternative possible collapse mechanisms. [4]

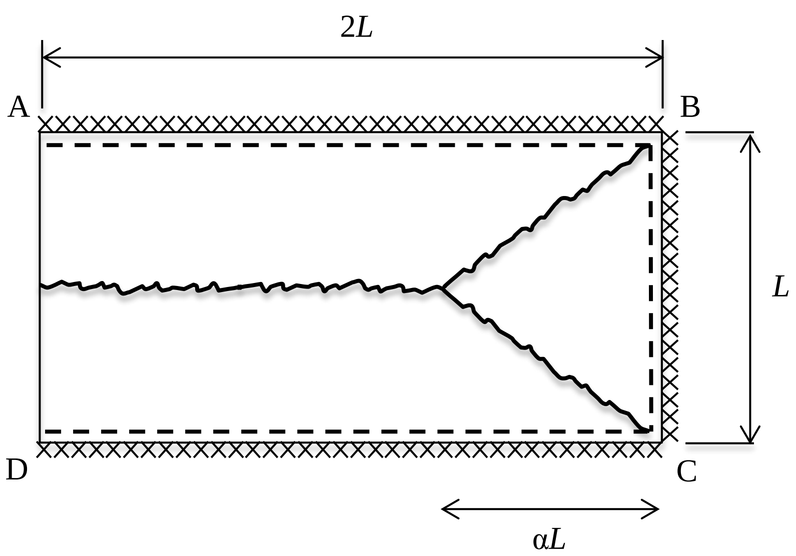


Fig. 4

 (TURN OVER

5 Figure 5(a) shows a long strip foundation that is partially embedded in a long embankment of soil with its free surface inclined at 45° to the horizontal. The foundation is assumed to behave as a rigid block, and the soil as a rigid-plastic continuum of uniform isotropic material with a shear yield stress k. A uniform line-load F per unit length is applied vertically to the centreline of the footing.

(a) The plane sliding block mechanism shown in Fig. 5(a) is proposed as a possible collapse mechanism. Assuming that the block moves parallel to AC, and that no work is done when the soil separates from the foundation along the line AE:

(i) obtain an upper bound estimate for the limiting value of F in terms of the shear yield stress k, the half-width of the footing b and the block height h; [7]

(ii) determine the optimum upper bound estimate for this mechanism by considering how F varies with h. [4]

(b) Figure 5(b) shows an alternative mechanism, based on rotation about point I, which is assumed to lie on the line AB. Calculate a second upper bound estimate for the limiting value of F, and comment on the result. [14]

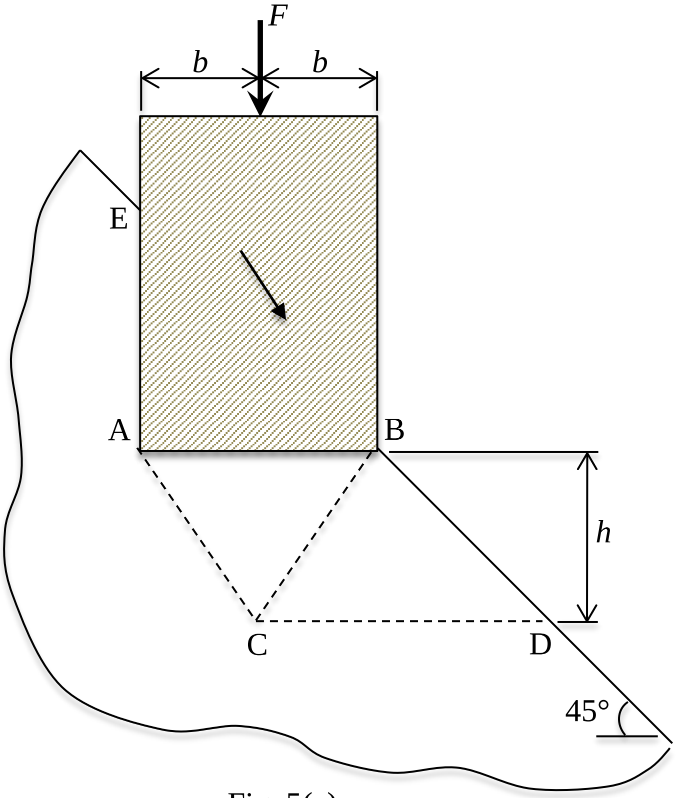


Fig. 5(a)

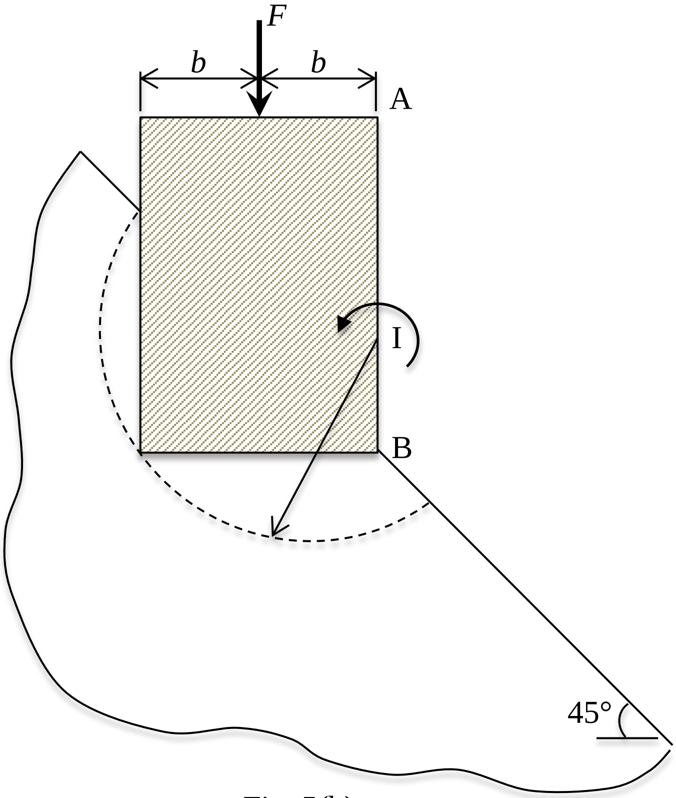


Fig. 5(b)

6 Figure 6(a) shows a uniform beam of length L and fully plastic moment capacity M_p. It is fully clamped at both ends and carries a concentrated load W at a fixed distance αL from its left-hand end.

(a) Use the *lower bound* method to calculate the collapse load of the beam in terms of L, α and M_p. [8]

(b) Use the *upper bound* method to calculate the collapse load of the beam in terms of L, α and M_p. Comment on the result in comparison with your lower bound estimate. [6]

(c) The steel beam section shown in Fig. 6(b) is to be used as a floor beam, as indicated in Fig. 6(a), with a total span of L = 6 m. Given the steel yield stress σ_y = 245 MPa, calculate the maximum safe working load of the beam for a load factor of 2 against collapse. [7]

(d) An initial lack of fit at one of the supports meant that the beam had to be forced into place during installation, generating a peak bending stress of 10% of the value at yield. Explain how the maximum safe working load should be modified and why. [4]

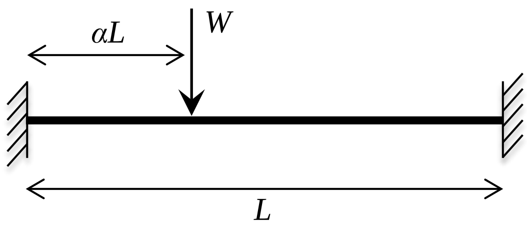
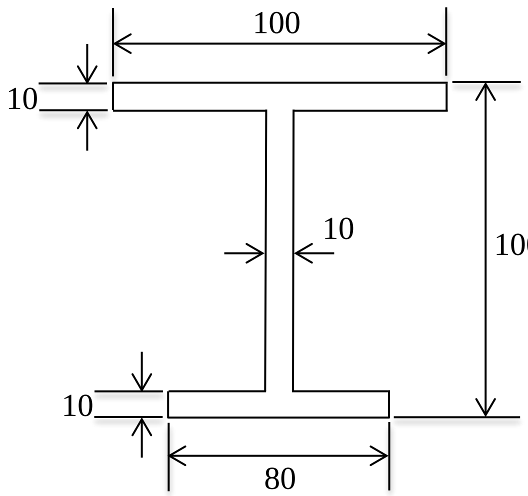


Fig. 6(a)



(All dimensions in mm)

Fig. 6(b)

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