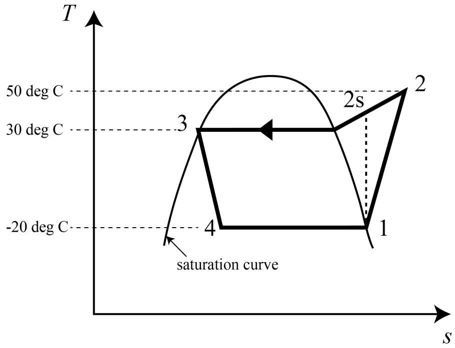
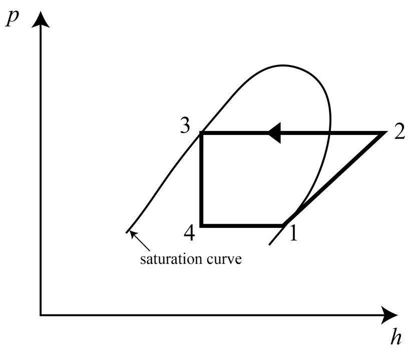
ENGINEERING TRIPOS PART IB 2014

SOLUTIONS TO PAPER 4 – THERMOFLUID MECHANICS

Question 1

(a)(i) The processes between the following states are:
1 to 2 compressor;
2 to 3 condenser (constant pressure);
3 to 4 throttle (assume isenthalpic, i.e. adiabatic with negligible change in kinetic energy);
4 to 1 evaporator (constant pressure).





(a)(ii) To find compressor specific work, w_c
From R-134a Table in Data Book:
$h_1 = 386.5$ kJ/kg
$h_2 = 435.4$ kJ/kg
$w_c = h_2 - h_1 = 48.9$ kJ/kg (work input)

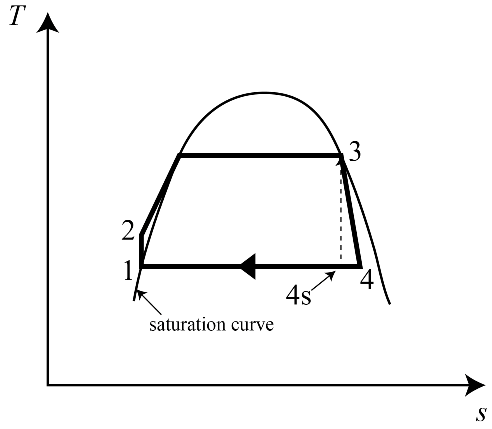
To find compressor isentropic efficiency, η
$p_2 = 7.70$ bar (table)
$s_{2s} = s_1 = 1.7410$ kJ/kg K (table)
$h_{2s} = 420$ kJ /kg (from chart in Data Book)

$$\eta = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{420 - 386.5}{435.4 - 386.5} = 0.69$$

(a)(iii) COP_R = specific heat input in evaporator / specific work input in compressor
$h_4 = h_3 = 241.7$ kJ/kg (table)

$$COP_R = \frac{h_1 - h_4}{h_2 - h_1} = \frac{386.5 - 241.7}{48.9} = 2.96$$

(b)(i) The processes between the following states are:
1 to 2 feed pump;
2 to 3 heat addition in boiler (constant pressure);
3 to 4 turbine;
4 to 1 heat rejection in condenser (constant pressure).



(b)(ii) At turbine inlet (from table),

$h_3 = 428.8$ kJ/kg

$s_3 = 1.6848$ kJ/kg K

saturation conditions at 10°C (from table):

$s_{4f} = 1.0486$ kJ/kg K

$s_{4g} = 1.7222$ kJ/kg K

$h_{4f} = 213.6$ kJ/kg

$h_{4g} = 404.3$ kJ/kg

$s_{4s} = s_3$ (isentropic expansion)

dryness fraction at 4s is x_{4s},

$$x_{4s} = \frac{s_{4s} - s_{4f}}{s_{4g} - s_{4f}} = \frac{1.6848 - 1.0486}{1.7222 - 1.0486} = 0.94$$

enthalpy at 4s,

$$h_{4s} = h_{4f} + x_{4s}(h_{4g} - h_{4f}) = 213.6 + 0.94(404.3 - 213.6) = 393.7\text{kJ/kg}$$

isentropic efficiency of the turbine is 0.85,

$$0.85 = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

so the turbine specific work output, w_t is given by

$$w_t = h_3 - h_4 = 0.85(h_3 - h_{4s}) = 29.8\text{kJ/kg}$$

(b)(iii)

Superheating will:

- increase the turbine specific work output (increase h_3);
- increase the cycle efficiency (average temperature of heat addition has increased so Carnot efficiency (of reversible cycle) will increase);

- flow in turbine will be drier. Wetness in turbine reduces turbine isentropic efficiency and also erodes turbine blades. Hence, drier flow in turbine will raise turbine isentropic efficiency and hence raise the cycle efficiency.

Question 2

(a)(i) SFEE (steady flow energy equation): $h_2 - h_1 = -w_x + q$

SFSE (steady flow entropy equation): $s_2 - s_1 = \int_1^2 \frac{dq}{T} + \Delta s_{irrev}$

From availability definition: $b_2 - b_1 = h_2 - h_1 - T_0(s_2 - s_1)$

hence:

$$b_2 - b_1 = -w_x + q - T_0 \int_1^2 \frac{dq}{T} - T_0 \Delta s_{irrev}$$

$$b_2 - b_1 = -w_x + \int_1^2 \left(1 - \frac{T_0}{T}\right) dq - T_0 \Delta s_{irrev}$$

Note: the quantities in the above derivation are all specific (per kg) and are therefore written in lower case letters with no dots above them. Dots are reserved for quantities that are rates (per s).

(a)(ii) First term on rhs is change in power potential due to shaft work output.
Second term on rhs is change in power potential due to heat input.
Third term on rhs is change in power potential due to irreversibilities in the process (e.g. viscous dissipation or heat transfer across a finite temperature difference)

(b)(i) Control volume around whole heat exchanger and assume no heat flow in/out of control volume.
Stream 1 enters at 900K, stream 2 enters at 1800K,

$$\dot{m}_1 \Delta h_1 + \dot{m}_2 \Delta h_2 = 0$$

and $\Delta h = c_p \Delta T$ so,

$$20 \times 1.01 \times (T_m - 900) + 80 \times 1.01 \times (T_m - 1800) = 0$$

$$T_m = 1620\text{K}$$

(b)(ii) Lost power potential is due to irreversibilities (heat transfer across a finite temperature difference).

Control volume around whole heat exchanger, no heat transfer in/out of control volume so only change in entropy is due to irreversibilities:

$$\Delta S_{irrev} = \dot{m}_1 \Delta s_1 + \dot{m}_2 \Delta s_2 = 20 c_p \ln\left(\frac{1620}{900}\right) + 80 c_p \ln\left(\frac{1620}{1800}\right) = 3.36\text{kJ/K}$$

Lost power potential is given by

$$T_0 \Delta S_{irrev} = 300 \times 3.36\text{E}3 = 1.01\text{MW}$$

(b)(iii)

$$\dot{W}_x = \left(\dot{m}_1 + \dot{m}_2\right) c_p T_m \left(1 - \left(\frac{1}{20}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$\dot{W}_x = 100 \times 1.01\text{E}3 \times 1620 \left(1 - \left(\frac{1}{20}\right)^{\frac{\gamma-1}{\gamma}}\right) = 94.1\text{MW}$$

(c)

$$\dot{W}_x = \dot{W}_{x1} + \dot{W}_{x2} = \dot{m}_1 c_p T_1 \left(1 - \left(\frac{1}{20}\right)^{\frac{\gamma-1}{\gamma}}\right) + \dot{m}_2 c_p T_2 \left(1 - \left(\frac{1}{20}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$\dot{W}_x = \left(\dot{m}_1 c_p T_1 + \dot{m}_2 c_p T_2\right) \left(1 - \left(\frac{1}{20}\right)^{\frac{\gamma-1}{\gamma}}\right) = \left(\dot{m}_1 + \dot{m}_2\right) c_p T_m \left(1 - \left(\frac{1}{20}\right)^{\frac{\gamma-1}{\gamma}}\right) = 94.1\text{MW}$$

This is the same power output as part (b)(iii). However, the power potential of the combined exhaust flows in part 2(c) will be greater than that of the exhaust of the turbine of part 2(b) due to the lost power potential in part 2(b)(ii).

(d) The two streams would need to be brought to the same temperature by running a reversible heat engine between the hot stream and cold stream. The power potential that was lost in part 2(b) would then be obtained as a power output from the heat engine.

Question 3

(a) The radial heat flux per unit area is given by

$$\dot{q}_r = -\lambda \frac{dT}{dr}$$

The total heat flux at each radius is constant at

$$\dot{Q}_r = 2\pi\lambda r \frac{dT}{dr}$$

so that,

$$T_{r=r2} - T_{r=r1} = \frac{\dot{Q}_r}{2\pi\lambda}\int_1^2 \frac{dr}{r} = \frac{\dot{Q}_r}{2\pi\lambda}\ln\left(r_2/r_1\right)$$

and,

$$R_T = \frac{\Delta T}{\dot{Q}} = \frac{\ln\left(r_2/r_1\right)}{2\pi\lambda}$$

(b)(i) One correlation is for laminar flow, one is for turbulent flow.
In laminar flow, heat is transferred by diffusion. In turbulent flow, heat is transferred by the fluid motion of the turbulent eddies. The turbulent flow has a higher radial temperature gradient at the wall and a higher surface heat transfer coefficient.

(b)(ii) From table for transport properties of air in Data Book, at 50 °C:
μ = 19.5E-6 kg/sm
Pr = 0.71
λ = 0.028 W/mK

$$\rho = \frac{p}{RT} = \frac{101325}{287 \times 323.15} = 1.09\text{kg/m}^3$$

$$V = \dot{m}/(\rho A) = 1/(1.09 \times \pi 0.1^2) = 29.1\text{m/s}$$

Reynolds number,

$$\text{Re} = \frac{\rho V d}{\mu} = \frac{1.09 \times 29.1 \times 0.2}{19.5 \times 10^{-6}} = 326000 \Rightarrow \text{ turbulent}$$

Using turbulent correlation in Data Book,

$$\text{Nu} = 0.023\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023 \times 326000^{0.8} \times 0.71^{0.4} = 517$$

$$h_{\text{int}} = \text{Nu}\frac{\lambda}{d} = 517\frac{0.028}{0.2} = 72.3\text{W/m}^2\text{K}$$

(b) (iii)

$$R_{TOT} = \frac{1}{h_{\text{int}}2\pi r_1} + \frac{\ln(r_2/r_1)}{2\pi\lambda_{pipe}} + \frac{\ln(r_3/r_2)}{2\pi\lambda_{ins}} + \frac{1}{h_{ext}2\pi r_3}$$

$$R_{TOT} = \frac{1}{2\pi}\left(\frac{1}{72.3\times 0.1} + \frac{\ln(0.11/0.1)}{0.2} + \frac{\ln(0.16/0.11)}{0.05} + \frac{1}{200\times 0.16}\right)$$

$$R_{TOT} = 0.022 + 0.076 + 1.193 + 0.005 = 1.30\text{K/Wm}$$

(b)(iv)
For a small element of the pipe (a slice of width δx),

$$-\dot{m}c_p\frac{dT}{dx}\delta x = \frac{(T - T_\infty)}{R_{TOT}}\delta x$$

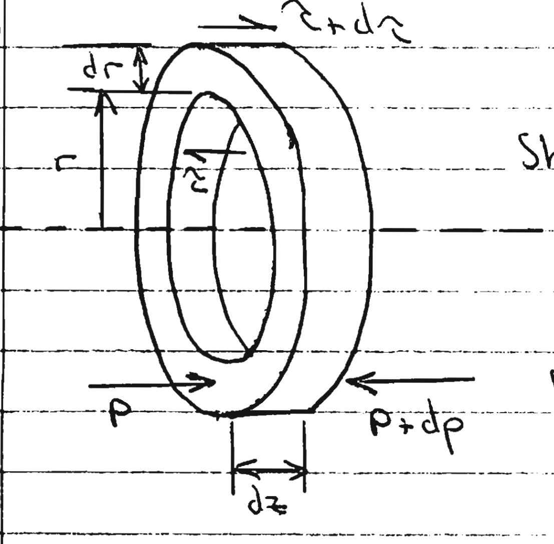
Integrating gives

$$\frac{T - T_\infty}{T_1 - T_\infty} = \exp\left(\frac{-x}{R_{TOT}\dot{m}c_p}\right)$$

When $(T - T_\infty)$ is half of the inlet value,

$$x = -\ln(0.5)\dot{m}c_p R_{TOT} = 0.693\times 1\times 1.01\times 10^3\times 1.30 = 910\text{m}$$

4 (a)



Shear force on inner surface

$$F_z = \tau_{rz} 2\pi r \, dz$$

Net shear force on element

$$dF_z = \frac{\partial F_z}{\partial r} dr = \frac{\partial}{\partial r}(\tau_{rz} 2\pi r)\, dr\, dz$$

Pressure force on front $= p \, 2\pi r \, dr$

Net pressure force on element $= -\frac{\partial}{\partial z}(p 2\pi r dr)\, dz = -\frac{dp}{dz} 2\pi r dr dz$

Fluid is not accelerating, so forces ballance:

$$-\frac{dp}{dz} 2\pi r \, dr \, dz + \frac{\partial}{\partial r}(\tau_{rz} 2\pi r)\, dr\, dz = 0$$

$$\Rightarrow \quad -\frac{dp}{dz} r + \frac{\partial}{\partial r}(r \tau_{rz}) = 0$$

$$\Rightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}(r \tau_{rz}) = \frac{dp}{dz}$$

$$\Rightarrow \quad \frac{1}{r}\frac{\partial}{\partial r}(r \tau_{rz}) = -\left|\frac{dp}{dz}\right| \quad (dp/dz < 0)$$

(b) Integrate:

$$r \tau_{rz} = +\left|\frac{dp}{dz}\right| \left\{ B - \frac{r^2}{2} \right\}$$

Newtons law of viscosity: $\tau_{rz} = \mu \frac{du_z}{dr}$

$$\Rightarrow \quad \mu \frac{du_z}{dr} = \left|\frac{dp}{dz}\right| \left\{ \frac{B}{r} - \frac{r}{2} \right\}$$

$$\Rightarrow \quad u_z = \frac{1}{\mu}\left|\frac{dp}{dz}\right| \left\{ A - \frac{r^2}{4} + B \ln r \right\}$$

(c) No slip condition: $u_z = 0$ on $r = R_i$, $r = R_o$

$$\begin{cases} 0 = \frac{1}{\mu}\left|\frac{dp}{dz}\right|\left\{A - \frac{R_i^2}{4} + B\ln R_i\right\} \\ 0 = \frac{1}{\mu}\left|\frac{dp}{dz}\right|\left\{A - \frac{R_o^2}{4} + B\ln R_o\right\} \end{cases}$$

$$\Rightarrow \quad \begin{cases} A + B\ln R_i = R_i^2/4 \\ A + B\ln R_o = R_o^2/4 \end{cases}$$

$$\Rightarrow \quad B\ln(R_o/R_i) = \frac{1}{4}(R_o^2 - R_i^2)$$

$$\Rightarrow \quad B = \frac{R_o^2 - R_i^2}{4\ln(R_o/R_i)}$$

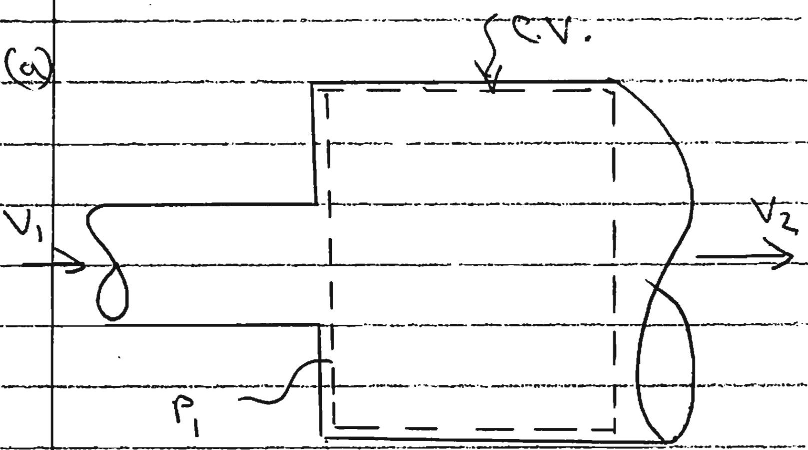
(d) Shear stress on inner surface is

$$\tau_{rz} = \frac{1}{R_i}\left|\frac{dp}{dz}\right|\left\{B - R_i^2/2\right\}$$

Net shear force $F_z = 2\pi R_i \tau_{rz} = 2\pi\left|\frac{dp}{dz}\right|\left\{B - R_i^2/2\right\}$

$$\Rightarrow \quad F_z = 2\pi\left|\frac{dp}{dz}\right|\left\{\frac{R_o^2 - R_i^2}{4\ln(R_o/R_i)} - \frac{R_i^2}{2}\right\}$$

5 (a)



Net force on C.V. $= P_1 A_2 - P_2 A_2$ (in axial direction)

Net flux of momentum out of C.V. $= \dot{m}(V_2 - V_1) = V_2 A_2 \rho (V_2 - V_1)$ (in axial direction)

Force-momentum equation:

$$(P_1 - P_2) A_2 = \rho A_2 V_2 (V_2 - V_1)$$

$$\Rightarrow \quad \underline{\underline{P_2 - P_1 = \rho V_2 (V_1 - V_2)}}$$

(b) Bernoulli: energy upstream $= P_1 + \frac{1}{2}\rho V_1^2$

energy down stream $= P_2 + \frac{1}{2}\rho V_2^2$

energy loss $= P_1 + \frac{1}{2}\rho V_1^2 - (P_2 + \frac{1}{2}\rho V_2^2)$

$$= P_1 - P_2 + \frac{1}{2}\rho (V_1^2 - V_2^2)$$

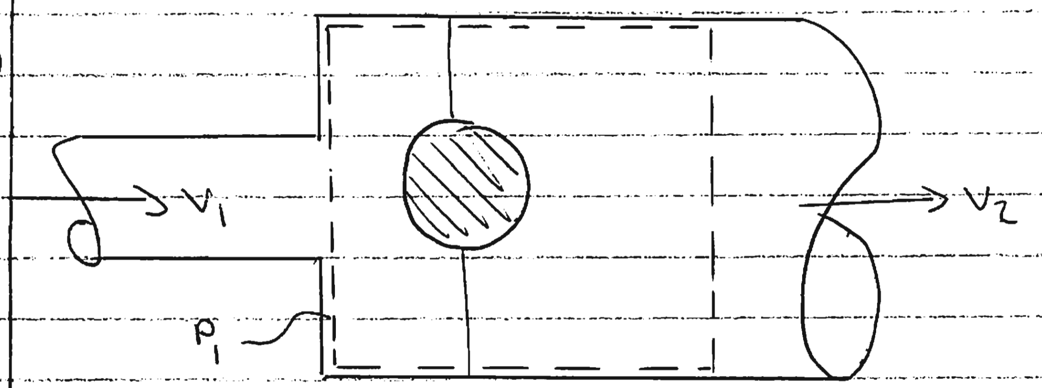
$$= -\rho V_2 (V_1 - V_2) + \frac{1}{2}\rho (V_1^2 - V_2^2)$$

$$= \frac{1}{2}\rho (V_1^2 - V_2^2 - 2V_1V_2 + 2V_2^2)$$

$$= \frac{1}{2}\rho (V_1^2 - 2V_1V_2 + V_2^2)$$

$$= \frac{1}{2}\rho (V_1 - V_2)^2$$

(c)



Net Force on CV is $= P_1 A_2 - P_2 A_2 -$ Tension in wires

$$= P_1 A_2 - P_2 A_2 - F_d$$

Net flux of momentum out of CV $= \rho A_2 V_2 (V_2 - V_1)$

(as before)

Force momentum equation:

$$(P_1 - P_2) A_2 - F_d = \rho A_2 V_2 (V_2 - V_1)$$

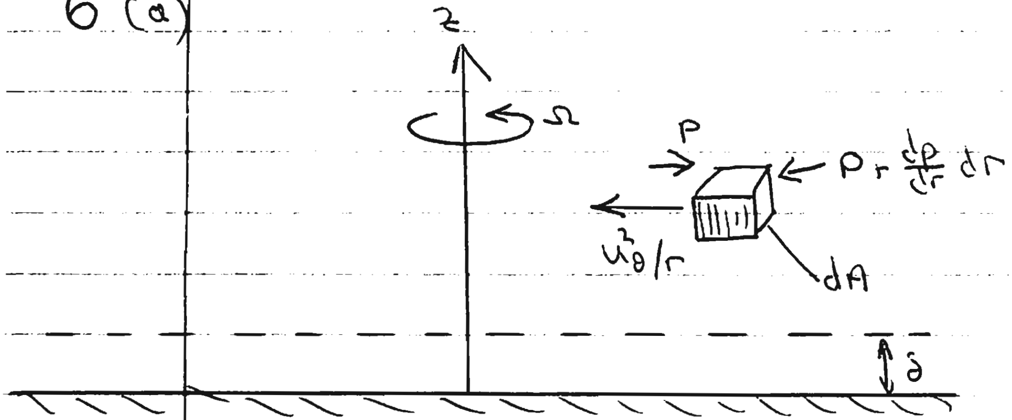
$$\Rightarrow \quad P_1 - P_2 = \frac{F_d}{\pi R_2^2} + \rho V_2 (V_2 - V_1)$$

energy loss $= P_1 + \frac{1}{2}\rho V_1^2 - (P_2 + \frac{1}{2}\rho V_2^2)$

$$= \frac{F_d}{\pi R_2^2} + \underbrace{\rho V_2 (V_2 - V_1) + \frac{1}{2}\rho (V_1^2 - V_2^2)}_{\text{same as in (b)}}$$

$$= \frac{F_d}{\pi R_2^2} + \frac{1}{2}\rho (V_1 - V_2)^2$$

6 (a)



Fluid element has radially inward acceleration of u_θ^2/r. There must be a radial force on element to maintain this acceleration.

Newton II : $$\frac{u_\theta^2}{r}(\rho dV) = \left(p + \frac{dp}{dr}dr\right)dA - p\,dA$$

$$\Rightarrow \quad \frac{u_\theta^2}{r}(\rho\, dr\, dA) = \frac{dp}{dr}dr\, dA$$

$$\Rightarrow \quad \frac{dp}{dr} = \rho\frac{u_\theta^2}{r}$$

$$u_\theta = \Omega r \Rightarrow \frac{dp}{dr} = \rho\Omega^2 r \Rightarrow p - p_0 = \tfrac{1}{2}\rho\Omega^2 r^2$$

(p_0: centreline pressure.)

(b) $\delta = f(\Omega, \nu)$ (No other variables at our disposal)

δ: m, Ω: s^{-1}, ν: m^2/s

Π Theorem: number of variables = 3

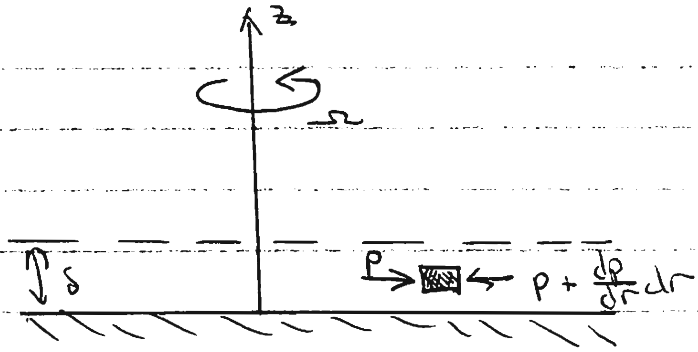
number of dimensions = 2

$\Rightarrow$ number of dimensionless groups = 1

By inspection, $\dfrac{\delta^2\Omega}{\nu}$ is dimensionless.

Since there are no other dimensionless groups we have $\delta^2\Omega/\nu$ = constant, or $\delta \sim \sqrt{\nu/\Omega}$.

(c)



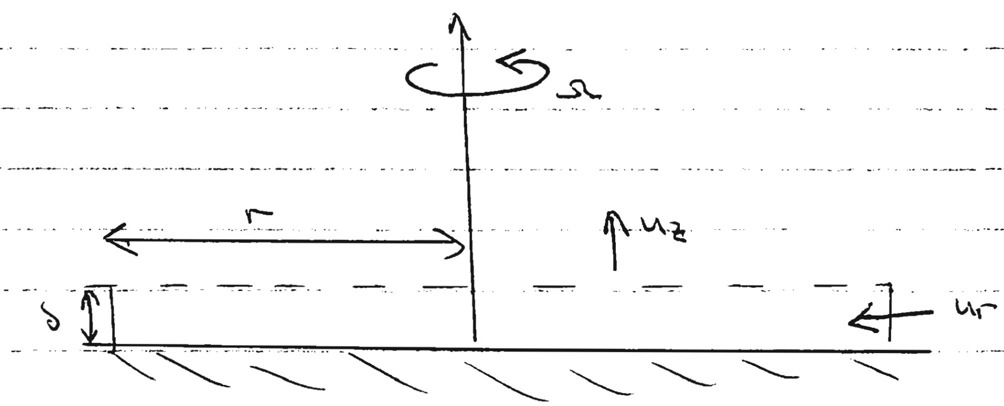
Consider a fluid element in boundary layer.

There is a radial pressure ~~force of~~ gradient of

$$\frac{dp}{dr} = \rho \Omega^2 r .$$

The centripetal acceleration within the boundary layer is lower because u_θ is smaller. Thus the radial pressure force is greater than u_θ^2/r, and this excess force drives a radial inflow.

(d)



Conservation of mass

$$2\pi r \delta u_r \sim \pi r^2 u_z$$

$$\Rightarrow \quad \delta u_r \sim r u_z$$

But $u_r \sim u_\theta \sim \Omega r \Rightarrow r u_z \sim \delta \Omega r$

$$\Rightarrow \quad \underline{\underline{u_z \sim \delta \Omega}}$$