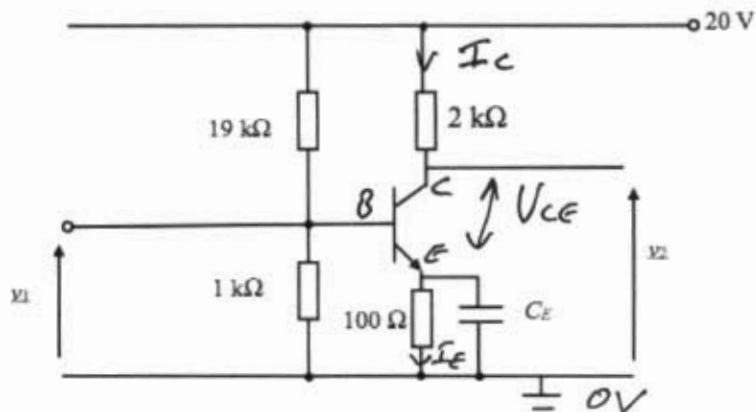


Part IB Paper 5 2014 Solutions

1.



(a) h_{FE} is infinite. As $I_C = h_{FE}I_B$ and I_C is finite, we can say that $I_B = 0$. Also, $I_E = I_C + I_B \sim I_C$. If $I_B = 0$, then the $19\text{ k}\Omega / 1\text{ k}\Omega$ base bias circuit acts as a potential divider, i.e. both resistors carry the same current.

$$\Rightarrow V_B = 20\text{V} \times \frac{1\text{k}\Omega}{1\text{k}\Omega + 19\text{k}\Omega} = 1\text{V}.$$

$$\text{As } V_{BE} = 0.7\text{V}, \Rightarrow V_E = V_B - V_{BE} = 0.3\text{V}.$$

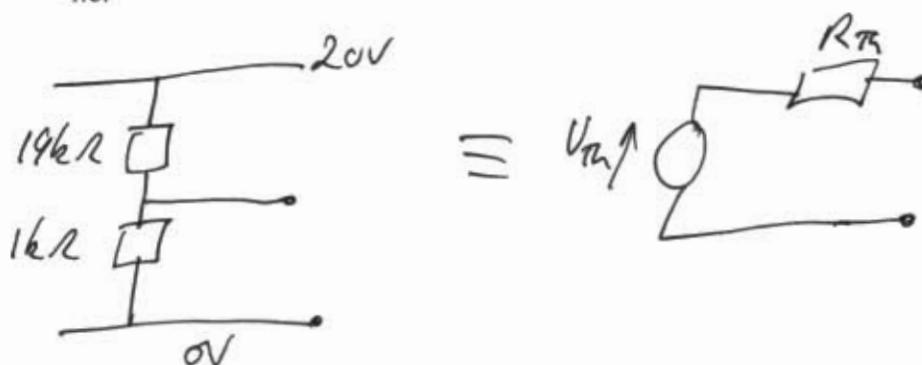
$$\Rightarrow I_E = \frac{V_E}{100\Omega} = 3\text{ mA}.$$

Therefore, the voltage drop across the $2\text{ k}\Omega$ resistor $= 2\text{ k}\Omega \times 3\text{mA} = 6\text{ V}$.

As the supply is 20 V , this means $V_C = 20\text{ V} - 6\text{ V} = 14\text{ V}$

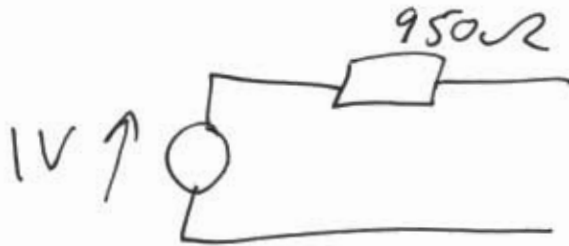
(b) In reality, $h_{FE} \neq \infty$, but is 150 . This means $I_B \neq 0$. To determine the new voltages, convert the base bias circuit to its Thevenin equivalent.

i.e.

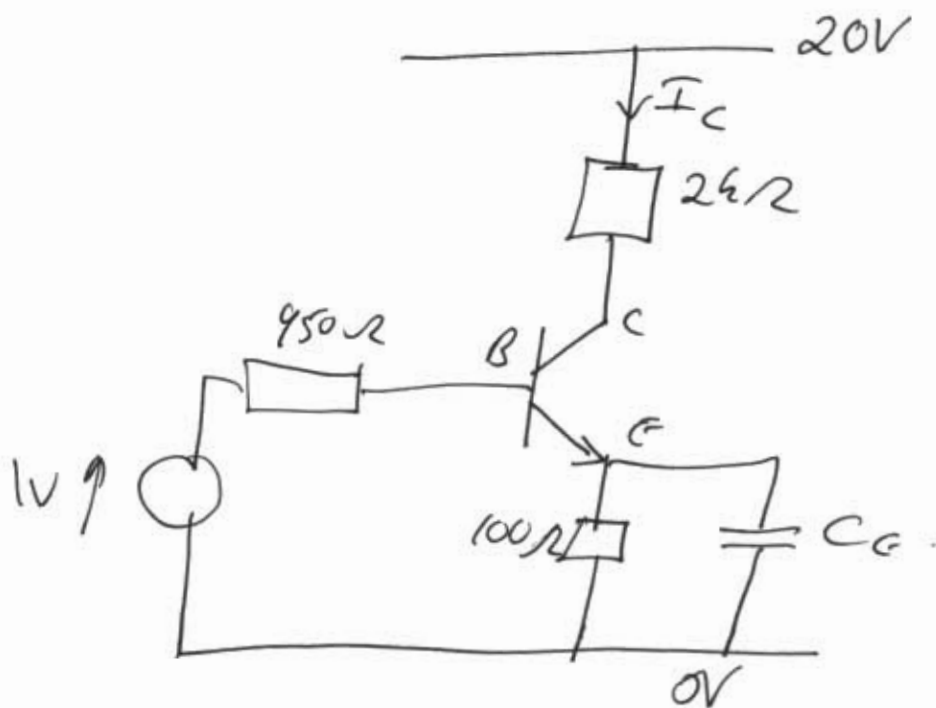


Where $V_{Th} = 1V$, $R_{Th} = 1\text{ k}\Omega \parallel 19\text{ k}\Omega = 950\ \Omega$.

⇒ The new bias circuit is



i.e. the entire circuit can be re-drawn as:



$$I_B = \frac{1V - V_B}{950\Omega}$$

$$V_B = V_E + V_{BE} = V_E + 0.7$$

$$\Rightarrow I_B = \frac{1V - (V_E + 0.7)}{950\Omega}$$

Now, in terms of I_B , $V_E = (1 + h_{FE}) \times I_B \times 100\Omega$

If we substitute this in the expression for I_B , we find

$$I_B = \frac{1 - (100I_B(1 + h_{FE}) + 0.7)}{950}$$

$$\Rightarrow 950I_B = 1 - 100I_B(1 + h_{FE}) - 0.7$$

$$\Rightarrow I_B(950 + 100(1 + h_{FE})) = 0.3$$

$$\Rightarrow I_B = \frac{0.3}{950 + 100(1 + h_{FE})} = 18.7 \mu\text{A}$$

Now we can find $V_E = (1 + h_{FE}) \times I_B \times 100\Omega$
 $= 100(1 + 150) \times 18.7 \mu\text{A} = 0.28 \text{ V}$

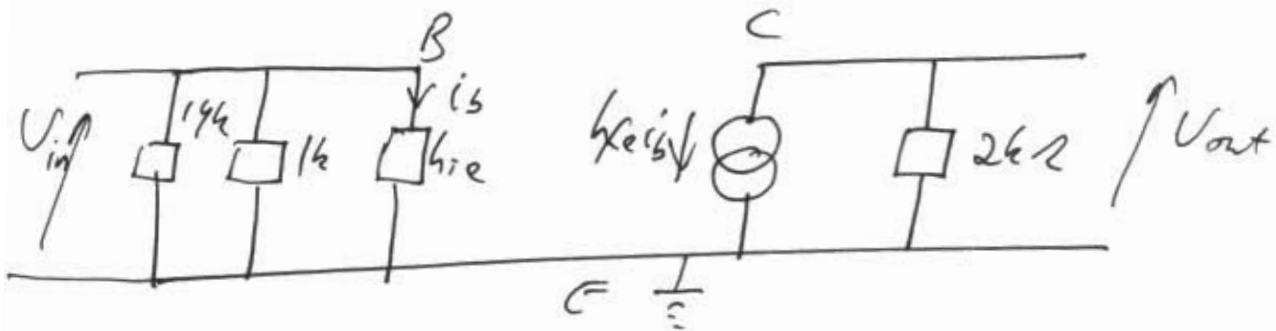
As for V_C ; $V_C = 20 - I_C \times 2\text{k}\Omega$

Now, $I_C = h_{FE}I_B = 150 \times 18.7 \mu\text{A} = 2.8 \text{ mA}$

$$\Rightarrow V_C = 20 - 2.8\text{mA} \times 2\text{k}\Omega = 14.4 \text{ V}$$

- (c) The capacitor is used to bypass the 100Ω resistor for ac signals, and is called a *bypass capacitor*. The resistor is there to allow the dc operating point to be achieved and to allow the gain to be stabilized and predictable. Putting in a capacitor enables a much higher gain to be achieved without compromising the dc stability of the circuit.
- (d) At mid-band, C_E short-circuits the 100Ω resistor, so in the small-signal equivalent circuit, the emitter is grounded.

Small-signal equivalent circuit:



$$\text{Input resistance} = 19\text{k}\Omega \parallel 1\text{k}\Omega \parallel h_{ie} = 228 \Omega$$

$$\text{Voltage gain} = v_2/v_1.$$

v_2 is created by the current $h_{fe}i_b$ flowing up through the $2 \text{ k}\Omega$ resistor, so

$$v_2 = -h_{fe}i_b \times 2\text{k}$$

Likewise, v_m causes a current i_b to flow through the resistor h_{ie} , so

$$v_1 = h_{ie} i_b$$

$$\Rightarrow \text{gain} = -\frac{h_{fe} i_b \times 2k}{h_{fe} i_b} = -200 \times \frac{2000}{300} = -1333$$

2.

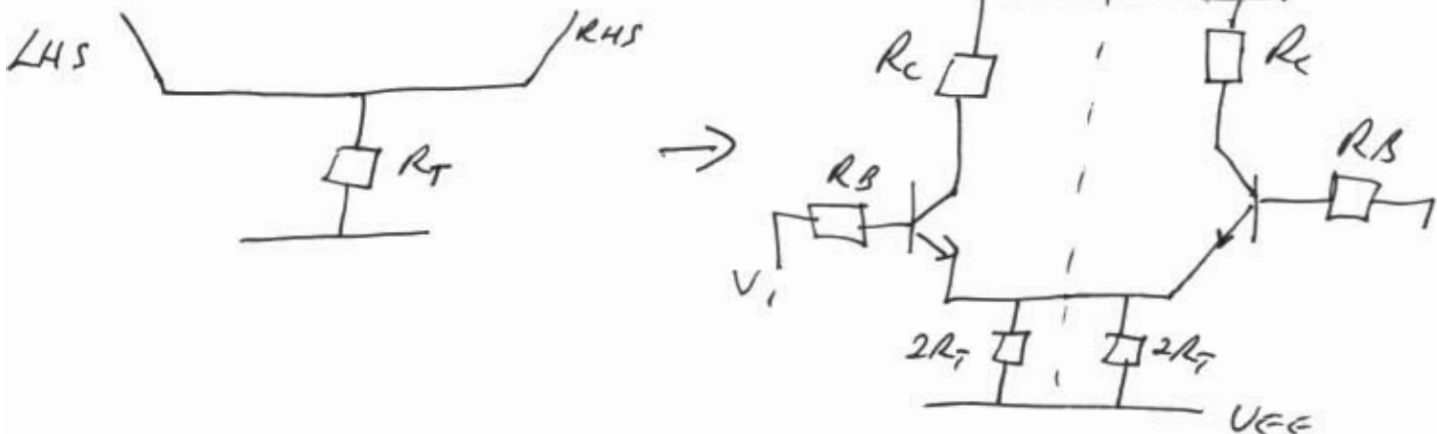
(a) The CMRR is defined as the ratio between the common-mode gain and the differential gain. We can express electrical signals on input terminals as \bar{V}_1 and \bar{V}_2 . For common-mode signals, $\bar{V}_1 = \bar{V}_2$ whereas for differential signals, $\bar{V}_1 = -\bar{V}_2$.

Common-mode signals, as they appear on both input terminals, are considered to be noise, and so we want to amplify them as little as possible while amplifying differential signals as much as possible. Thus, the larger the CMRR the better.

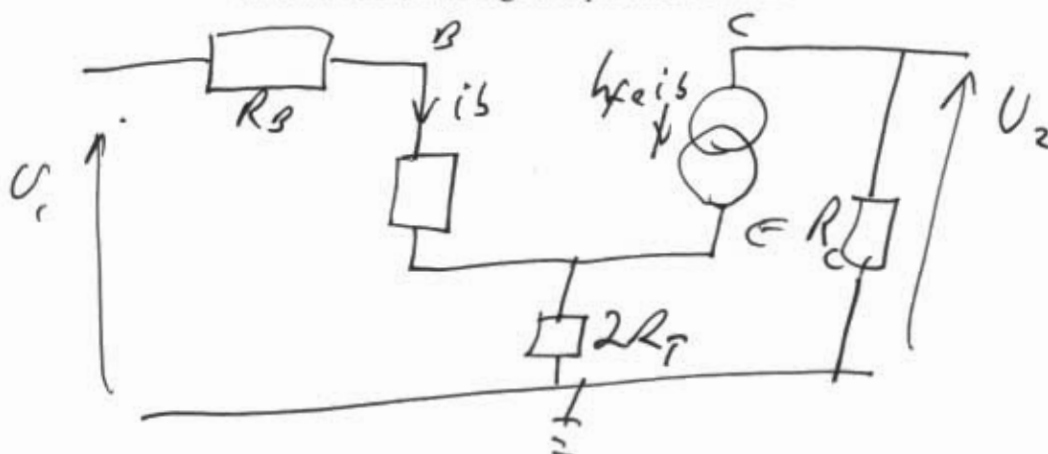
(b) The circuit is symmetric, i.e. Q1 has exactly the same characteristics as Q2, which means we can take the half-circuit approach.

(i) Common-mode gain.

The circuit can be split into:



This has the small-signal equivalent circuit



Here, $v_0 = -h_{fe}i_bR_c$

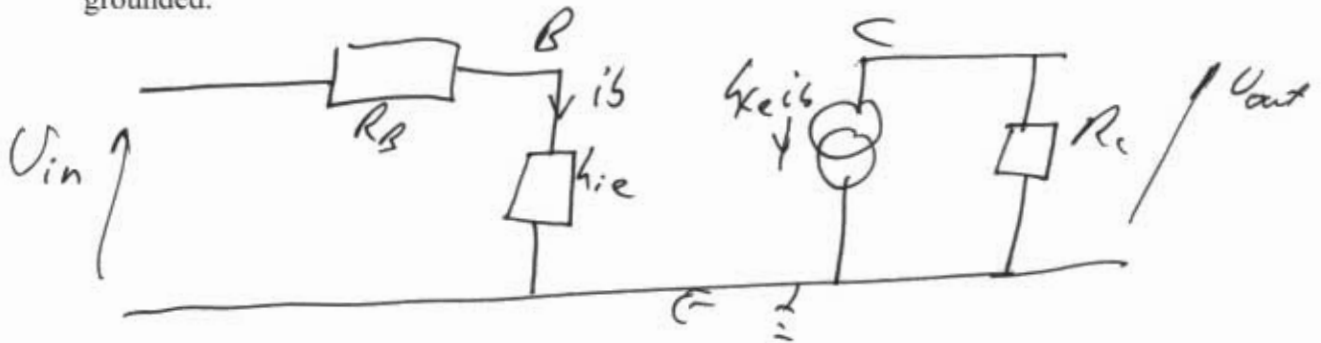
and $v_1 = i_b(R_B + h_{ie}) + 2R_T i_b(h_{fe} + 1) = i_b(R_B + h_{ie} + 2R_T(h_{fe} + 1))$

=> the common-mode gain,

$$\frac{v_0}{v_{1cm}} = -\frac{h_{fe}i_bR_c}{i_b(R_B + h_{ie} + 2R_T(h_{fe} + 1))} = -\frac{h_{fe}R_c}{R_B + h_{ie} + 2R_T(h_{fe} + 1)}$$

(ii) Differential-mode gain.

In this case, as v_1 is increasing, v_2 is decreasing, so the *net* current through R_T is constant, which means that in the small-signal equivalent circuit, the emitter is grounded.



Here, $v_0 = -h_{fe}i_bR_c$

and $v_1 = i_b(R_B + h_{ie})$

=> the differential-mode gain, $\frac{v_0}{v_{1diff}} = -\frac{h_{fe}i_bR_c}{i_b(R_B + h_{ie})} = -\frac{h_{fe}R_c}{R_B + h_{ie}}$

Therefore, the CMRR = the ratio of the differential gain to the common-mode gain:

$$CMRR = \frac{R_B + h_{ie} + 2R_T(h_{fe} + 1)}{R_B + h_{ie}}$$

(c) The differential gain has a value of $200 = \frac{h_{fe}R_c}{R_B + h_{ie}}$

From which we find that $R_B = \frac{h_{fe}R_c}{200} - h_{ie} = 100 \Omega$

To determine R_T , we know that the CMRR is 10,000, which we can use in the expression above as follows:

$$CMRR = 10,000 = \frac{R_B + h_{ie} + 2R_T(h_{fe} + 1)}{R_B + h_{ie}}$$

$$= \frac{100 + 200 + 2R_T(201)}{100 + 200} = \frac{3 + 4.02R_T}{3}$$

$$\Rightarrow 30,000 = 3 + 4.02R_T$$

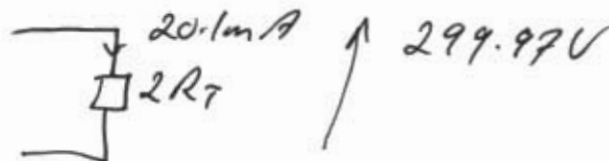
$$\Rightarrow R_T = \frac{30,000 - 3}{4.02} = 7461.9 \Omega$$

Now, the collector current, $I_C = 20 \text{ mA}$, $\Rightarrow I_E = (1 + 1/h_{fe})I_C = 20.1 \text{ mA}$

Therefore, the voltage dropped across the resistor $2R_T = V_{EE}$

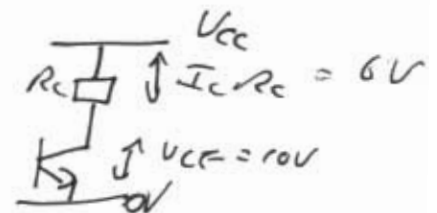
Where $V_{EE} = 2R_T I_E = 2 \times 7461.9 \times 20.1 = 299.97 \text{ V}$.

i.e.



As for V_{CC} , it is the sum of the voltage across the transistor, V_{CE} plus the voltage dropped across R_C due to the current I_C .

$$\text{i.e. } V_{CC} = V_{CE} + I_C R_C = 10 + 20 \text{ mA} \times 300 \Omega = 16 \text{ V}$$

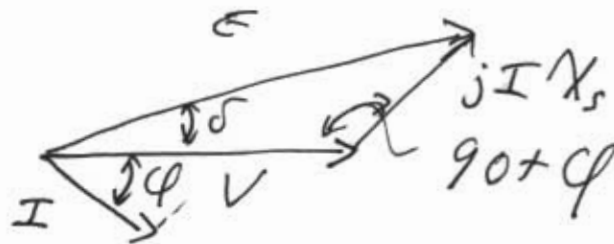


(d) V_{CC} is reasonable, but V_{EE} is very large. This is necessary given the high CMRR we wish to obtain which dictates that R_T should be as large as it is, and the high value of I_C that is needed for these transistors. The simplest modification would be to incorporate a current mirror instead of R_T , as it would have a very high impedance whilst allowing a high current to flow.

3.

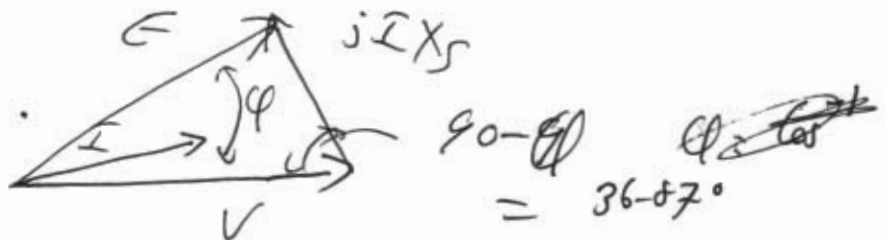
(a)

(i) Lagging



$$\phi = \cos^{-1}(0.6) = 53.13^\circ$$

(ii) Leading



In all cases, $V = V_{bus}$

E = Excitation voltage

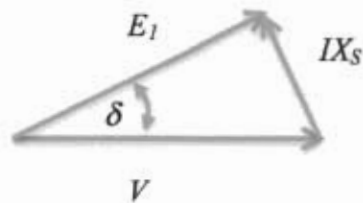
ϕ = Phase between I & V

δ = Load angle between E & V (and between the stator and rotor magnetic fields)

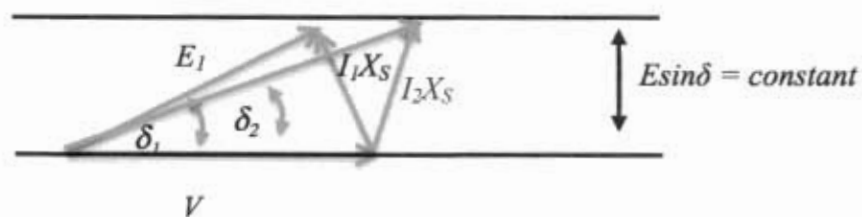
(b) The total power is $3VI\cos\phi$, which is proportional to $E\sin\delta$, which is the perpendicular height of the phasor diagram, i.e.

For constant power, $E \sin \delta$ is constant. By changing the excitation voltage, E (by altering the rotor field), the load angle will change, and the power factor can thus be varied.

i.e.



Then change E (e.g. increase the rotor field to increase E)

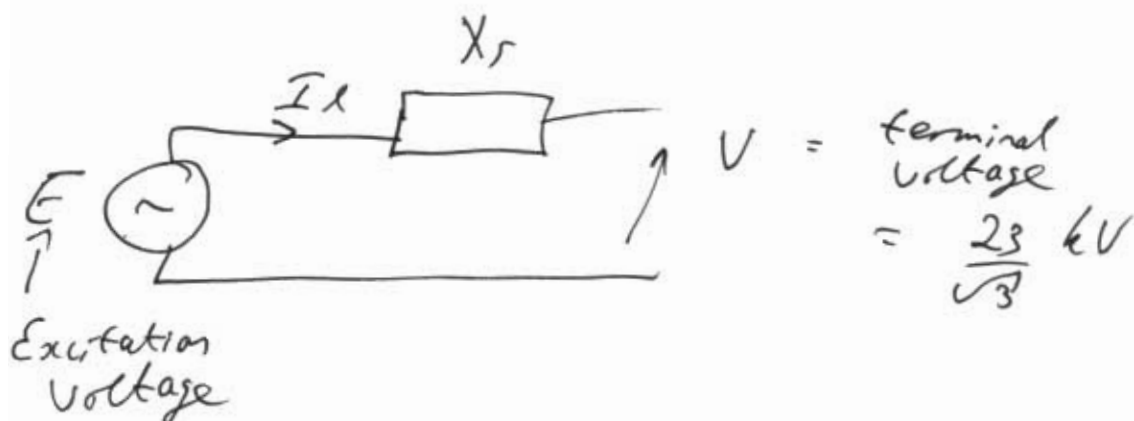


(c) Synchronous generator, operating at 600 MVA, 4-pole, line voltage of 23 kV. The generator is star-connected, so the phase voltage is $\frac{23}{\sqrt{3}}$ kV and the apparent power consumption is 200 MVA/phase.

Stator reactance = 0.2Ω / phase

Power output = $200 \text{ MW} = \frac{200}{3} \text{ MW / phase}$.

Phasor diagram:



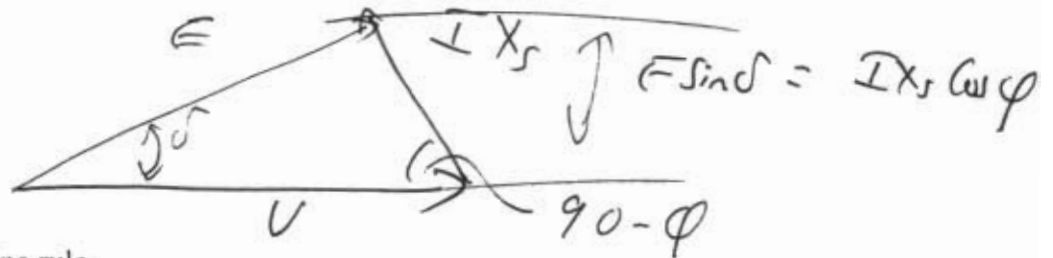
(i) $P = 3VI\cos\phi$ for the whole system

$$= \sqrt{3}V_{line}I_{line}\cos\phi$$

$$\Rightarrow 200 \text{ MW} = \sqrt{3}V_{line}I_{line}\cos\phi$$

$$\Rightarrow I_{line} = \frac{P}{\sqrt{3}V_{line}\cos\phi} = \frac{200 \times 10^6}{\sqrt{3} \times 23 \times 10^3 \times 0.6} = 8.37 \text{ kA}$$

(ii) Excitation Voltage, E .



Using the cosine rule,

$$E^2 = V^2 + (IX_s)^2 - 2VIX_s \cos(90-\phi)$$

In kV, this is

$$\left(\frac{23}{\sqrt{3}}\right)^2 + (8.37 \times 1)^2 - 2 \times \frac{23}{\sqrt{3}} \times 8.37 \times 1 \times 0.8$$

$$= 68.56$$

$$\Rightarrow E_{ph} = 8.28 \text{ kV}, E_{line} = 14.34 \text{ kV}$$

(iii) The load angle, δ

Well, from the phasor diagram, we know that $E\sin\delta = IX_s\cos\phi$

$$\Rightarrow \sin\delta = \frac{IX_s\cos\phi}{E} = \frac{8.37 \times 10^3 \times 1 \times 0.6}{8.28 \times 10^3} = 0.606$$

$$\Rightarrow \delta = 37.3^\circ$$

(iv) Synchronous speed $= f/p = 50/2 = 25$ revs/second $= 25 \times 60 = 1500$ rpm

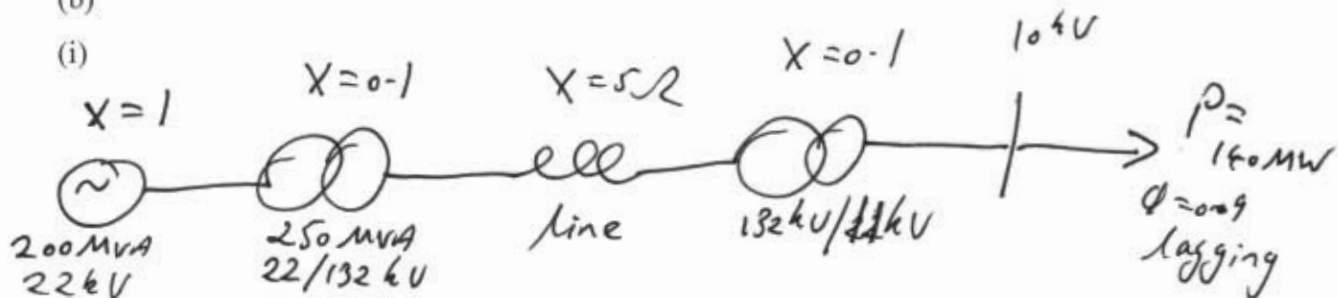
(d) From the cosine rule $IX_s^2 = V^2 + E^2 - 2VE\cos(\delta/2)$ where $\sin(\delta/2) = 250/200\sin(\delta)$
From which we get that the new line current = 17432 Amps.

4.

(a) The per-unit system sales all generators to the same voltage and simplifies power network analysis.

(b)

(i)



Choose base MVA = 250 MVA

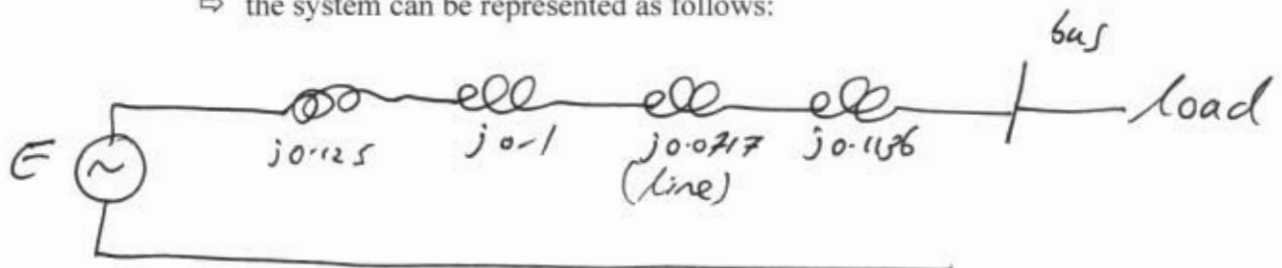
⇒ Generator reactance becomes $0.1 \times 250 / 200 = 0.125$

⇒ The second transformer reactance becomes $0.1 \times 250 / 220 = 0.1136$

For the line, $Z_b = \frac{V_b^2}{VA_b} = \frac{(132 \times 10^3)^2}{250 \times 10^6} = 69.7 \Omega$

Therefore, the per-unit line impedance is $Z_{line}/Z_b = 5/69.7 = 0.0717$ p.u.

⇒ the system can be represented as follows:



$$P_{load} = \sqrt{3} I_{load} V_1 \cos \phi$$

$$\Rightarrow I_{load} = \frac{P}{\sqrt{3} V_1 \cos \phi} = \frac{140 \times 10^6}{\sqrt{3} \times 10 \times 10^3 \times 0.9} = 8.98 \text{ kA}$$

$$\text{However, } I_b^{load} = \frac{MVA_b}{\sqrt{3} V_b} = \frac{250 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 13.12 \text{ kA}$$

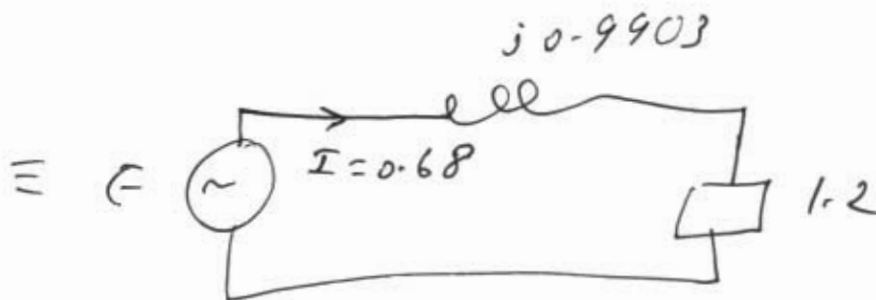
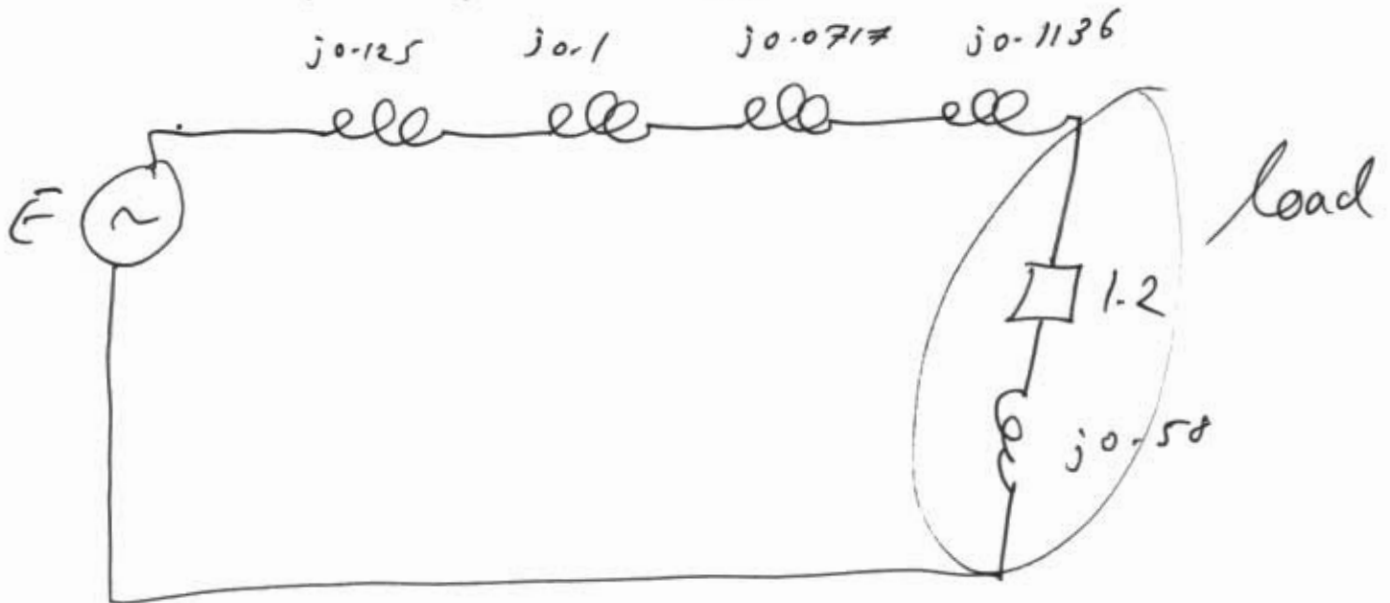
$$\Rightarrow I^{load} = 8.98 / 13.12 = 0.68 \text{ p.u.}$$

$$V^{load} = 10 \text{ kV} / 11 \text{ kV} = 0.909 \text{ p.u.}$$

$$\Rightarrow |Z^{load}| = \frac{V^{load}}{I^{load}} = \frac{0.909}{0.68} = 1.337 \text{ p.u.}$$

$$\begin{aligned} \text{Now, } Z^{load} &= |Z^{load}| \cos\phi + j |Z^{load}| \sin\phi \\ &= 1.2 + j0.58 \text{ p.u.} \end{aligned}$$

Therefore, the entire system reduces down to :



$$\Rightarrow E = IZ = 0.68 \times \sqrt{(0.9903)^2 + 1.2^2} = 1.058 \text{ p.u.}$$

As the base voltage at the generator end is 23 kV, this means the generator excitation,

$$E = 23 \text{ kV} \times 1.058 = 23.28 \text{ kV/line.}$$

(ii)

The generator line current is 0.68 p.u.

$$I_b^{generator} = \frac{MVA_b}{\sqrt{3}V_b} = \frac{250 \times 10^6}{\sqrt{3} \times 22 \times 10^3} = 4159 \text{ A}$$

$$\Rightarrow \text{the generator line current} = 4159 \times 0.68 = 2.83 \text{ kA}$$

(c) For a short-circuit at the bus end, the impedance before it is

$$j(0.125 + 0.1 + 0.0717 + 0.1136)$$

$$= j0.41$$

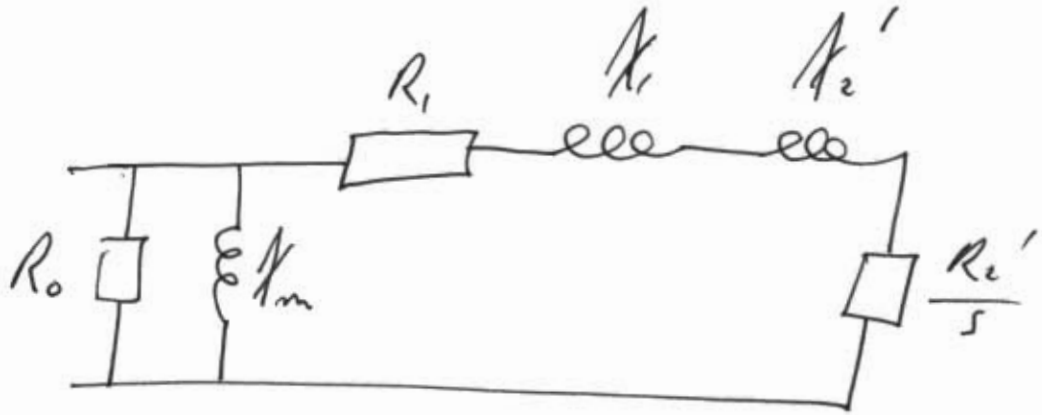
$$\Rightarrow MVA_{SC} = 1/0.41$$

$$= 2.44 \text{ p.u.}$$

$$= 2.44 \times 250 \text{ MVA} = 610 \text{ MVA}$$

$$\text{The short-circuit current, } I_{SC} = \frac{MVA_{SC}}{\sqrt{3}V_b} = \frac{610 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 32 \times 10^3 \text{ A}$$

5.
(a)



R_1 = winding resistance in the stator

χ_1 = Inductance of stator – this represents the flux lost between the stator and rotor.

R_0 = Iron-loss resistance – power lost due to eddy-current heating of the core.

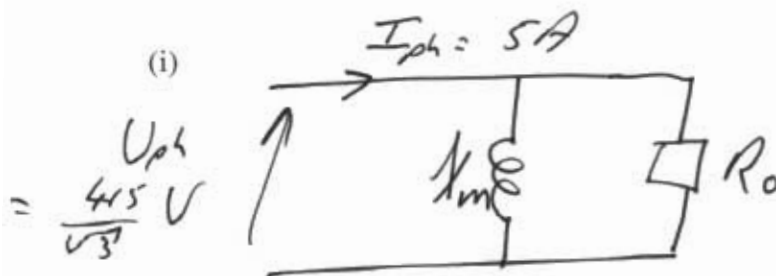
χ_m = Hysteresis losses in the core

R_2'/s is the *effective* resistance of the rotor, and comprises R_2' where eddy currents give rise to losses in the rotor, and $R_2'((1-s)/s)$ which represents the conversion of electrical to mechanical power.

$R_1, \chi_1, R_0, \chi_m, \chi_2'$ and R_2' are the same as for a transformer, and R_2'/s takes in to account that the currents flowing in the rotor depend on the relative speed of the rotor to the stator field (i.e. the slip).

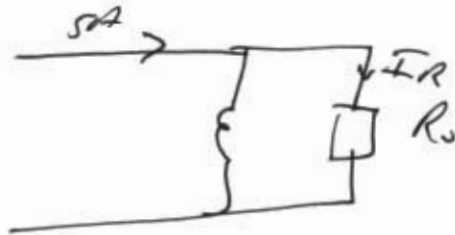
(b)

For the no-load test, the slip, $s = 0 \Rightarrow R_2'/s \rightarrow \infty$, so the equivalent circuit becomes:



$$\text{Power per phase} = \frac{1250 W}{3} = 416.67 W = \frac{V_{ph}^2}{R_0} \Rightarrow R_0 = \frac{\left(\frac{415}{\sqrt{3}}\right)^2}{416.67} = 137.8 \Omega.$$

(ii) Now, the real power, P , is due to a current I_R flowing through R_0 :



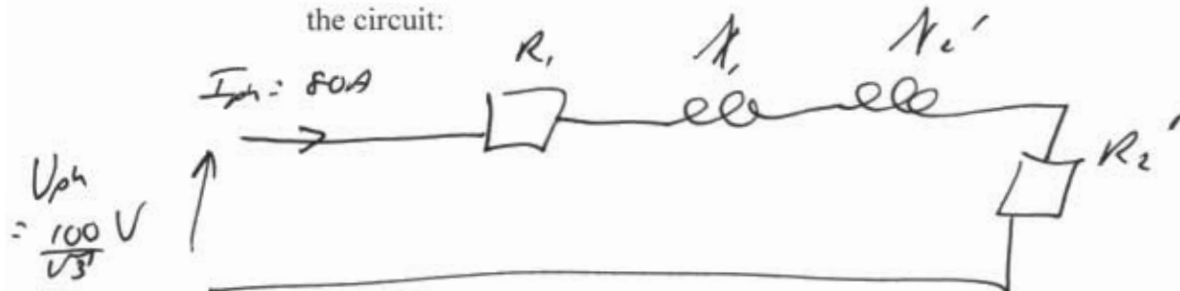
i.e. the power per phase is $I_R^2 R_0$, $\Rightarrow I_R = \sqrt{\frac{416.67}{137.8}} = 1.74 A$

Therefore, the current flowing through the reactance χ_m is

$$I_\chi = \sqrt{5^2 - 1.74^2} = 4.69 A$$

$$\Rightarrow V_{ph} = I_\chi \times \chi_m = \frac{415}{\sqrt{3}} \Rightarrow \chi_m = \frac{V_{ph}}{I_\chi} = \frac{415}{\sqrt{3} \times 4.69} = 51.1 \Omega$$

(iii) In the locked-rotor test, most of the current flows through the rest of the circuit:



Now, the power, $P = I_{ph}^2 \times (R_1 + R_2') \Rightarrow R_1 + R_2' = P / I_{ph}^2$

$$= \frac{6000/3}{80^2} = 0.3125 \Omega$$

Therefore, $R_2' = 0.3125 \Omega - R_1$

$$= 0.3125 \Omega - 0.15 \Omega$$

$$= 0.1625 \Omega$$

(iv) $V_{ph} = I_{ph}Z$ where Z is the impedance of the above circuit. Here, $V_{ph} = \frac{100}{\sqrt{3}}$ V, $I_{ph} = 80$ A.

\Rightarrow The overall impedance, $Z = V_{ph}/I_{ph} = \frac{100}{\sqrt{3} \cdot 80} = 0.722 \Omega$

where $Z = \sqrt{(R_1 + R_2')^2 + (\chi_1 + \chi_2')^2}$

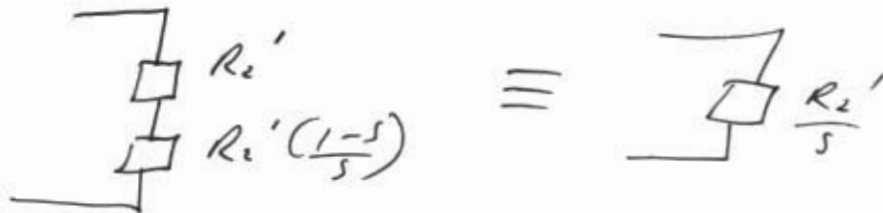
$$= \sqrt{(0.3125)^2 + (\chi_1 + \chi_2')^2} = 0.722$$

$$\Rightarrow (\chi_1 + \chi_2')^2 = 0.722^2 - 0.3125^2 = 0.423$$

$$\Rightarrow \chi_1 + \chi_2' = 0.65$$

$$\Rightarrow \chi_1 = \chi_2' = j0.325 \Omega \text{ at } 50 \text{ Hz}$$

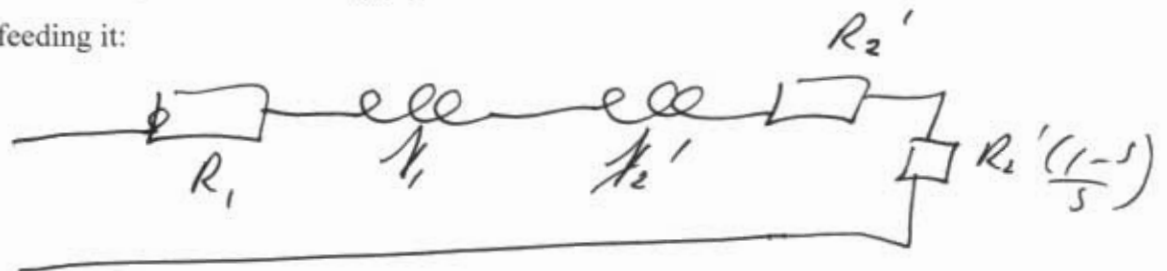
(c) Maximum power: well, R_2'/s can be written as :



The power dissipated in R_2' is lost as heat, so the mechanical power out is related to the power generated in the remaining $R_2' \times \left(\frac{1-s}{s}\right)$.

Therefore, to maximize the power out, use the maximum power transfer theorem:

Using this, the impedance of $R_2' \times \left(\frac{1-s}{s}\right)$ must equal the impedance of the rest of the circuit feeding it:



So, $R_2' \times \left(\frac{1-s}{s}\right) = |R_1 + R_2' + j(\chi_1 + \chi_2')| = 0.722 \Omega$ from earlier, from which we find that $s = 0.185$.

For this value of s , the power = $3I_{ph}^2 R_2' \times \left(\frac{1}{s} - 1\right) = 2.16 I_{ph}^2$

$$\text{The current, } I_{ph} = V/Z_{\text{total}} = \frac{\frac{415}{\sqrt{3}}}{\sqrt{\left(R_1 + \frac{R_2'}{s}\right)^2 + (\chi_1 + \chi_2')^2}} = 196.8 \text{ A}$$

$$\Rightarrow P_{\text{max}} = 2.16 \times 196.8^2 = 83.64 \text{ kW}$$

6

(a) The characteristic impedance of a transmission line is the ratio of the forward voltage to the forward current, i.e. voltage to current, in the absence of reflections. It is also defined as the ratio E/H and $\sqrt{\frac{L}{C}}$. It is also worth mentioning that it is real, leading to an undamped wave solution to the telegrapher's equations, in the absence of losses (resistance).

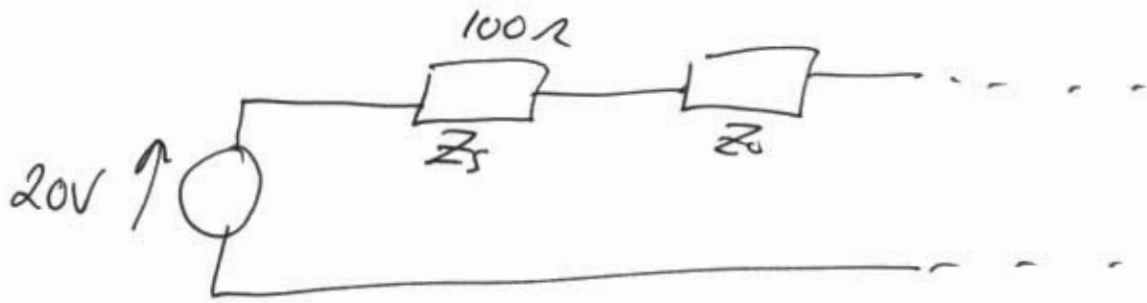
$$(b) Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.25 \times 10^{-6}}{100 \times 10^{-12}}} = 50 \Omega$$

The phase velocity is defined as $\frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.25 \times 10^{-6} \times 100 \times 10^{-12}}} = 2 \times 10^8 \text{ m/s}$.

This is 2/3 of the speed of light, and is the phase velocity, which has no upper bound. The group velocity however ($d\omega/dk$) must be less than or equal to the speed of light, as it is the speed at which information is transferred.

(Examiner's note: A number of candidates incorrectly thought that pF were 10^{-10} rather than 10^{-12} , and some others thought that the values for L and C needed to be scaled by the cable length).

(c) The first wave travelling along the cable does not see the load impedance, so the voltage is split between the internal resistance of the source and the characteristic impedance of the cable, as with a potential divider.



The first wave therefore has a voltage of $20 \times \frac{Z_0}{100 + Z_0} \text{ V} = 20/3 = 6.67 \text{ V}$

(d) From the data book, $Z_{in} = Z_0 \times \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$ where $Z_0 = 50 \Omega$
and $\beta = 2\pi/\lambda$

(i) $Z_L = 0$, i.e. the load is a short-circuit.

In that case, $Z_{in} = Z_0 \times \left(\frac{0 + jZ_0 \tan \beta l}{Z_0 + j0 \tan \beta l} \right) = jZ_0 \tan \beta l = j50 \tan \left(\frac{2\pi l}{\lambda} \right) \Omega$, which varies periodically with cable length and wave frequency.

(ii) $Z_L = \infty$, i.e. the load is an open-circuit.

$Z_{in} = Z_0 \times \left(\frac{\infty + jZ_0 \tan \beta l}{Z_0 + j\infty \tan \beta l} \right) = \frac{Z_0}{j \tan \beta l} = \frac{50}{j \tan \left(\frac{2\pi l}{\lambda} \right)} \Omega$, which varies periodically with cable length and wave frequency.

(iii) $Z_L = Z_0$, i.e. the load is matched to the cable

$Z_{in} = Z_0 \times \left(\frac{Z_0 + jZ_0 \tan \beta l}{Z_0 + jZ_0 \tan \beta l} \right) = Z_0 = 50 \Omega$, irrespective of cable length or frequency (as long as transmission line effects are noticeable).

(e)

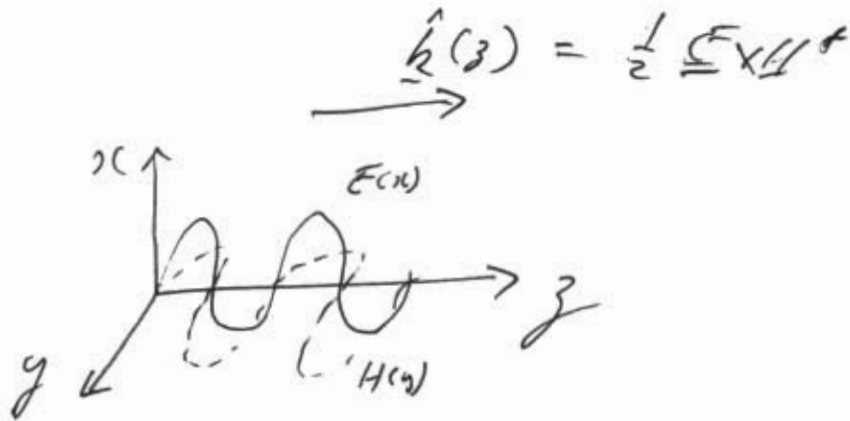
The voltage that travels down the cable towards the load is $20/3 \text{ V}$.

The reflection coefficient at the load is $\rho = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25 - 50}{25 + 50} = -\frac{1}{3}$

\Rightarrow the voltage reflected $= -\frac{20}{3} \times \frac{1}{3} = -\frac{20}{9} \text{ V}$, which is anti-phase with the source.

7.

(a)



(b) The Poynting vector points in the direction of propagation, is perpendicular to both the Electric and Magnetic fields and has the magnitude $\frac{1}{2} \mathbf{E} \times \mathbf{H}$ which is the power per unit area, otherwise called *Intensity*.

$$E = 10^3 \text{ V/m}$$

$$H = E/\eta, \text{ where } \eta \text{ is the impedance of free space} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 376.7 \Omega.$$

$$\Rightarrow \text{the power, } P = \frac{1}{2} \mathbf{E} \times \mathbf{H} = \frac{1}{2} \frac{E^2}{\eta} = \frac{1}{2} \frac{1000^2}{376.7} = 1327 \text{ W m}^{-2}.$$

(c) The intensity = power/area = $\frac{1}{2} \frac{E^2}{\eta} = \frac{P}{\pi r^2} \Rightarrow E^2 = \frac{2\eta P}{\pi r^2}$

$$\Rightarrow E = \sqrt{\frac{2\eta P}{\pi r^2}} = \sqrt{\frac{2 \times 376.7 \times 10^{-3}}{\pi \times (0.3 \times 10^{-6})^2}} = 1.63 \times 10^6 \text{ V/m}$$

If the maximum E is 2×10^6 V/m, then the maximum power is

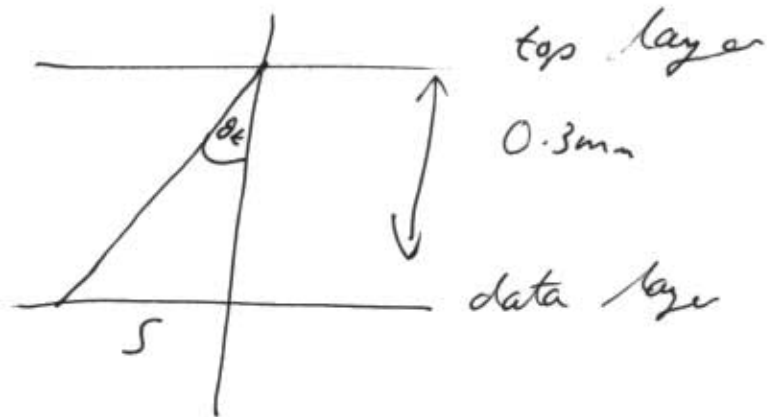
$$(2/1.63)^2 \times 1 \text{ mW} = 1.5 \text{ mW}$$

(d) (i) Well, from Snell's law,

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{\eta_t}{\eta_i}$$

$$\Rightarrow \sin \theta_t = \sin \theta_i \times \frac{\eta_t}{\eta_i} = 0.5 \times \frac{\sqrt{\mu_0/\epsilon_{r2}}}{\sqrt{\mu_0/\epsilon_r}} = \frac{1}{2} \times \frac{1}{\sqrt{\epsilon_r}} = \frac{1}{2} \times \frac{1}{\sqrt{2.25}} = \frac{1}{3}$$

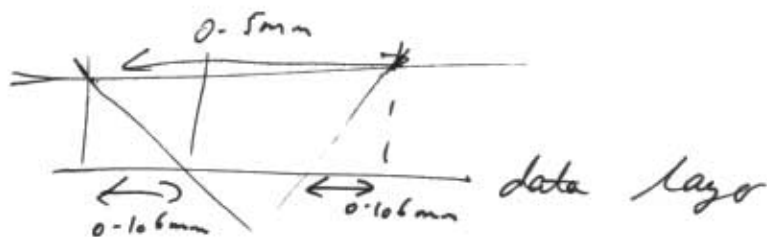
$$\Rightarrow \theta_c = \sin^{-1}\left(\frac{1}{3}\right) = 19.47^\circ$$



$$s = 0.3 \times \tan \theta_c \text{ mm}$$

$$= 0.106 \text{ mm}$$

i.e. edge of spot moves over by 0.106 mm



\Rightarrow Spot size in data layer

$$= 0.5 \text{ mm} - 2 \times (0.106) \text{ mm}$$

$$= 0.288 \text{ mm}$$

ii) As the disc has a different impedance, some of the laser light will be reflected. The transmitted = $1 - \rho^2$ where ρ = reflection coefficient.

$$\rho = \frac{\eta_{\text{trans}} \eta_t - \eta_i}{\eta_t + \eta_i} \quad \eta_t = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}$$

$$\eta_i = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= \frac{\sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} + \sqrt{\frac{\mu_0}{\epsilon_0}}}$$

x by $\sqrt{\frac{\epsilon_0}{\mu_0}}$

$$\Rightarrow \rho = \frac{\frac{1}{\sqrt{\epsilon_r}} - 1}{\frac{1}{\sqrt{\epsilon_r}} + 1} = \frac{1 - \sqrt{\epsilon_r}}{1 + \sqrt{\epsilon_r}}$$

$$= \frac{\frac{1}{1.5} - 1}{\frac{1}{1.5} + 1} = -0.2$$

$$\Rightarrow P_{\text{transmitted}} = P_{\text{incident}} \times (1 - 0.2^2)$$

$$= 0.96 \times P_{\text{incident}}$$

$$= 0.96 \text{ mW}$$

$$\text{Then, Intensity} = \frac{P_{\text{transmitted}}}{\pi \times (\text{spot radius})^2} = \frac{0.96 \text{ mW}}{\pi \times \left(\frac{0.28 \text{ mm}}{2}\right)^2}$$

$$= 14736 \text{ W m}^{-2}$$

iii) Electric field :

$$\text{well, Intensity} = \frac{E^2}{2\eta} = I$$

$$= \frac{E^2}{2\sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}}$$

$$\Rightarrow E^2 = I \times 2\sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}$$

$$= 14736 \times 2 \times \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12} \times 2.25}}$$

$$\Rightarrow E = 2721 \text{ Vm}^{-1}$$