EGT1
ENGINEERING TRIPOS PART IB

Thursday 5 June 2014 2 to 4

Paper 6

INFORMATION ENGINEERING

Answer not more than ***four*** *questions.*

Answer not more than ***two*** *questions from each section.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper, graph paper, semilog graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

*Answer not more than **two** questions from this section.*

1 A *linear* system G with transfer function $G(s)$ is controlled in a unity gain negative feedback system with pre-compensator $K(s)$ as in Fig. 1.

(a) Define *linearity*, as it applies to the system G. [5]

(b) Find, in terms of $G(s)$ and $K(s)$, the closed loop transfer functions relating:

(i) $\bar{y}(s)$ and $\bar{r}(s)$,

(ii) $\bar{y}(s)$ and $\bar{d}(s)$,

(iii) $\bar{u}(s)$ and $\bar{d}(s)$.

Explain the importance of each of these transfer functions for the overall design of $K(s)$. [7]

(c) The transfer functions of $G(s)$ and $K(s)$ are given by

$$G(s) = \frac{1}{(s+1)(s+5)}, \quad K(s) = 3.$$

Find the steady state response of $y(t)$ when:

(i) $r(t) = H(t)$ and $d(t) = 0$,

(ii) $r(t) = 0$ and $d(t) = \cos(\omega_0 t)$,

(iii) $r(t) = 2H(t)$ and $d(t) = \cos(\omega_0 t + \pi/3)$,

where $H(t)$ denotes the unit step function, and $\omega_0 = 2$ rad sec^{-1}. [8]

(d) How should $K(s)$ be modified in order to ensure that the steady-state error between $r(t)$ and $y(t)$ in Part (c)(i) is made 0? Give suitable numerical values for the coefficients of $K(s)$. [5]

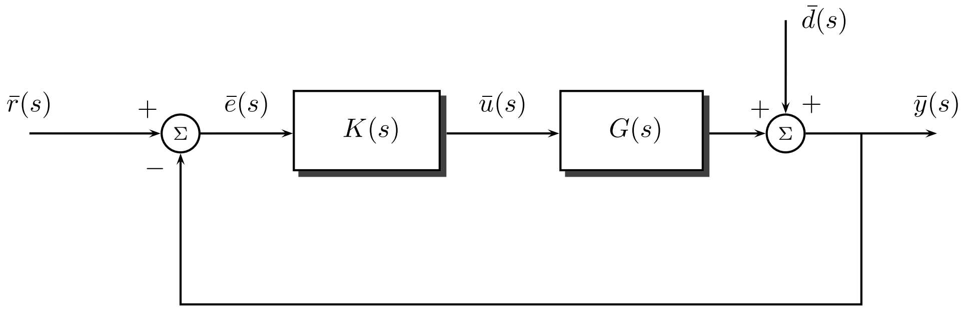

Fig. 1

2 A system G with transfer function

$$G(s) = \frac{10}{s(0.5s+1)^2}$$

is to be controlled in a unity gain negative feedback system with pre-compensator $K(s)$.

(a) Sketch the Bode diagram for the compensated system $G(s)K(s)$ for $K(s) = 1$ and verify that the closed-loop system is stable for all compensators of the form $K(s) = k_p$ if $k_p < 0.4$. [6]

(b) Choose k_p such that, for $K(s) = k_p$, the gain margin of the compensated loop is 2. Estimate the corresponding phase margin. [6]

(c) Consider now a compensator of the form

$$K(s) = 0.8\,\frac{s+1}{s+4}.$$

Add this compensator to your Bode diagram and estimate the new gain and phase margins. [8]

(d) Compare, qualitatively, the designs of Part (b) and Part (c). [5]

3 The Internet's congestion control algorithm (TCP) can be modelled as a feedback system whose return ratio is given by

$$L(s) = \frac{k\,\mathrm{e}^{-s\tau}}{s}$$

i.e. the product of a scaled integrator (representing TCP's response) and a delay (which represents the round trip time).

(a) Calculate the magnitude and phase of the open-loop frequency response $L(j\omega)$ at ω =2, 5, and 10 rad sec^{-1}, for $\tau = 0.25$ sec and $k = 4$. Determine also the asymptotic behaviour of the Nyquist locus as $\omega \to 0$. Hence sketch the Nyquist diagram for these values of k and τ. [10]

(b) If $\tau = 0.25$ sec, find the smallest frequency at which the phase of $L(j\omega)$ is $-\pi$ (rad), and hence determine the gain margin when $k = 4$. [5]

(c) If $k = 4$, calculate the frequency at which the gain of $L(j\omega)$ is unity, and hence determine the smallest time delay τ that will cause instability of the closed loop system. [5]

(d) For $\tau = 0.25$ sec, find k such that the gain margin is 2. How should k vary as a function of the round trip time τ, if a gain margin of 2 is required for all τ? [5]

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) By direct integration, find the Fourier transform, $F(\omega)$, of the function $f(x)$ which is given by

$$f(x) = \begin{cases} 1 & -a \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

Check that this agrees with the form of the Fourier transform of a pulse given in the Databook. [5]

(b) By evaluating the inverse Fourier transform of $F(\omega)$ in Part (a) at $x = 0$, show that the following expression holds:

$$\int_0^\infty \frac{\cos(\omega)\ \sin(\omega)}{\omega}\, d\omega \ = \ \frac{\pi}{4}$$

[6]

(c) Two functions $f(x)$ and $g(x)$ have Fourier transforms of $F(\omega)$ and $G(\omega)$ respectively. If $h(x)$ is the product of $f(x)$ and $g(x)$, prove that $H(\omega)$ is given by

$$H(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega')G(\omega - \omega')\, d\omega'$$

The proof should be from first principles. [6]

(d) Let $f(x) = \cos x$ and $g(x)$ be a pulse of width π centered on the origin with height 1. Sketch the function $h(x)$ which is the product $f(x)g(x)$. Then use the result in Part (c) to evaluate the Fourier transform, $H(\omega)$, of $h(x)$. Check that your result agrees with the Fourier transform of the appropriate function given in the Databook. [8]

5 (a) The sequence $x_n = \{1, 0, -1, 0\}$ is obtained by sampling the signal $x(t) = \cos(2\pi t)$ at intervals of 0.25 seconds, starting at $t = 0$.

(i) Calculate and sketch the discrete Fourier transform (DFT) of the sequence $\{x_n\}$. What frequencies do the individual terms in the DFT correspond to? [7]

(ii) Explain why there are two non-zero terms in the DFT of $\{x_n\}$ even though $x(t)$ is a pure sinusoid. [5]

(b) Consider the process of digitising audio, which consists of sampling followed by quantisation. It is known that audio signals have a bandwidth of 22 kHz.

(i) What is the minimum sampling rate required so that any audio signal can be perfectly reconstructed from its samples? [3]

(ii) Suppose that high-fidelity representation of audio requires a quantizer SNR of at least 48 dB. What is the minimum number of bits required for quantising each sample of the audio signal? You may assume that audio signals can be approximated as sine waves. [5]

(iii) Suppose that we wish to stream digital audio over a communication link. Assuming that sampling and quantisation have been performed at the rates calculated above, what is the bit-rate that the communication link must carry in order to stream the audio? [5]

6 (a) A carrier is modulated with a continuous time speech waveform $m(t)$ in two different ways to produce signals $s_1(t)$ and $s_2(t)$, respectively. The carrier frequency is f_c, and the two signals are given by:

$$s_1(t) = [10 + m(t)]\cos(2\pi f_c t),$$
$$s_2(t) = m(t)\cos(2\pi f_c t).$$

(i) Express $S_1(f)$ and $S_2(f)$, the spectra of $s_1(t)$ and $s_2(t)$, in terms of $M(f)$, the spectrum of $m(t)$. [4]

(ii) Discuss the advantages/disadvantages of $s_1(t)$ versus $s_2(t)$ for transmitting $m(t)$. [4]

(b) In *Pulse Amplitude Modulation* (PAM), the information symbols $X_1, X_2, \ldots$ modulate a pulse $p(t)$ to produce the baseband waveform

$$X(t) = \sum_k X_k \, p(t - kT)$$

where T is the symbol period. Suppose that each symbol X_k can take one of two values, 0 and A. This is called *on-off* keying. Assume that $p(t)$ is a unit-energy rectangular pulse given by

$$p(t) = \begin{cases} 1/\sqrt{T}, & 0 \leq t < T \\ 0 & \text{otherwise} \end{cases}$$

(i) Sketch the PAM waveform $X(t)$ for the information sequence $\{0, A, A, 0, 0, A\}$. [4]

(ii) What is the spectrum $P(f)$ of the pulse $p(t)$? Sketch the magnitude of $P(f)$, clearly specifying the zero crossings and the main peak. [5]

(iii) The waveform $X(t)$ is transmitted over an additive white Gaussian noise (AWGN) channel, and the discrete-time received sequence is

$$Y_k = X_k + N_k,$$

where N_k is additive Gaussian noise of mean zero and variance σ^2. What is the optimal detection rule, assuming that the two symbols 0 and A are equally likely? Show that the probability of detection error, P_e, is given by

$$P_e = Q\left(\frac{A}{2\sigma}\right)$$

where $Q(y) = 1 - \Phi(y)$, and $\Phi(y)$ is the Gaussian cumulative distribution function. [8]

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