EGT1
ENGINEERING TRIPOS PART IB

Thursday 7 June 2018 2 to 4.10

Paper 6

INFORMATION ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from each section.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

Answer not more than ***two*** *questions from this section.*

1 An industrial process is modelled as a simple integrator,

$$y(t) = g \int_{-\infty}^{t} u(t')dt'$$

where $u(t')$ is some input signal, $y(t)$ is the output of the plant, and g is some gain that is not known exactly, but lies between 1 and 100.

(a) What is the transfer function $G(s)$ from $\bar{u}(s)$ to $\bar{y}(s)$? [3]

(b) The output $y(t)$ is not measurable directly. However, a sensor is available that provides some estimate $y_{est}(t)$. The sensor's specifications state that "This device behaves approximately as a linear, first order lag with characteristic time constant $\tau = 0.2$s and unit steady-state gain". Write down the transfer function, $T(s)$, from $\bar{y}(s)$ to $\bar{y}_{est}(s)$. [3]

(c) Someone proposes to use a simple *proportional controller*, $K(s) = k_p$, to make the plant output, $y(t)$, track some arbitrary reference trajectory, $r(t)$. Draw a block diagram of the proposed scheme. [2]

(d) Assuming that the closed loop is stable, what can be said about the steady-state error, and why? [3]

(e) Assume $k_p = 1$. In Fig. 1, someone has already drawn the phase part of the Bode diagram of the open loop for $g = 1$. On the additional copy of Fig. 1 provided at the end of this paper, complete the Bode diagrams (asymptotes will suffice) for $g = 1$ and $g = 100$. Do not forget to add labels to the (largest) ticks on the x and y axes, that have been intentionally ommitted. Annotate your diagram to indicate the phase margin in each case (there is no need to calculate its value). How oscillatory are the corresponding step responses in the closed loop? [8]

(f) By taking into account the interval uncertainty on g, and using the Bode diagrams drawn previously, calculate the range of values that k_p is allowed to take if the phase margin is to remain above 20°. [You are not required to add anything to the additional copy of Fig. 1] [6]

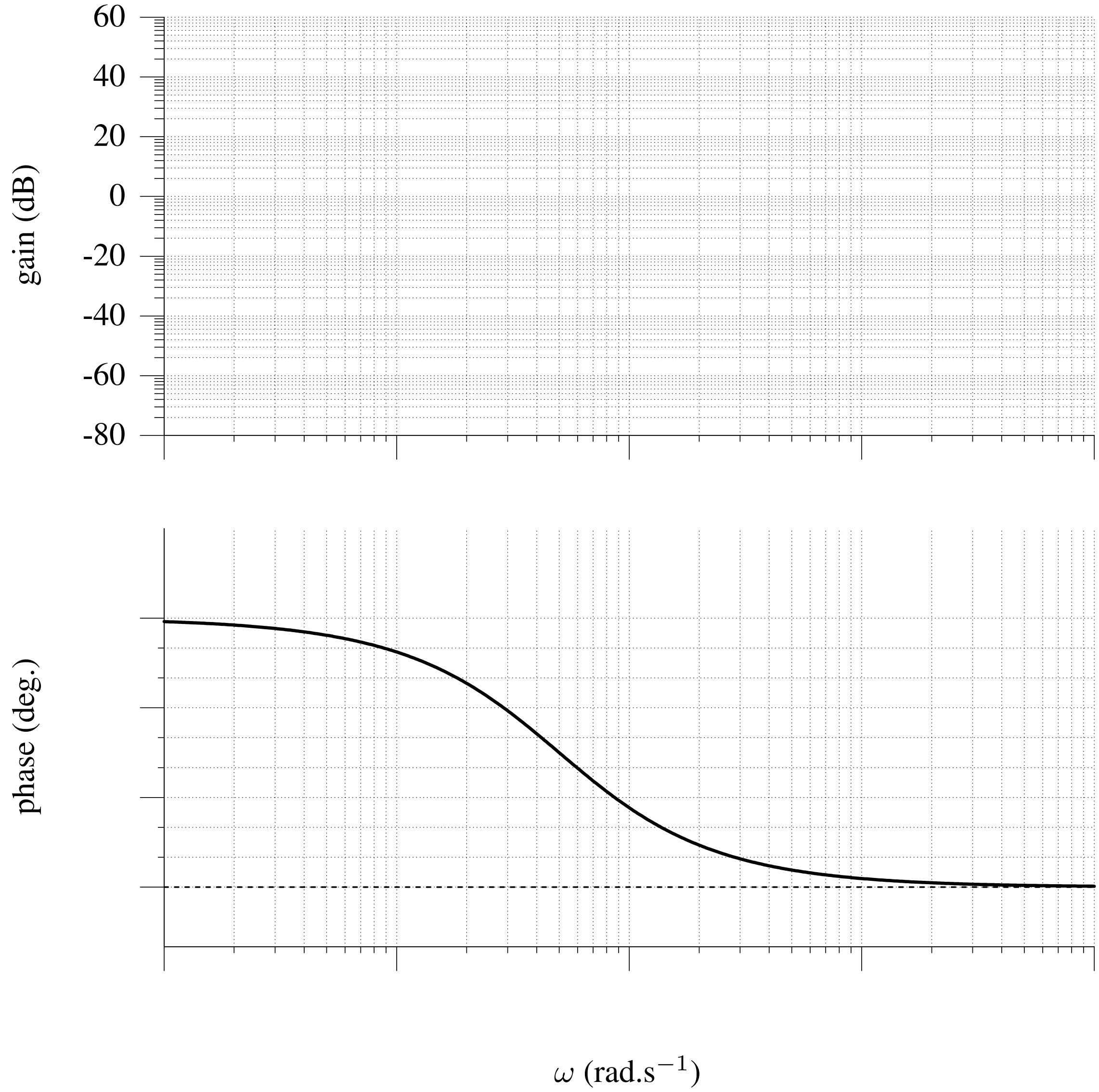

Fig. 1

Note: an additional copy of this figure is attached at the end of this paper. This additional copy should be annotated with your constructions and handed in with your answer to this question.

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2 Consider the negative feedback configuration shown in Fig. 2:

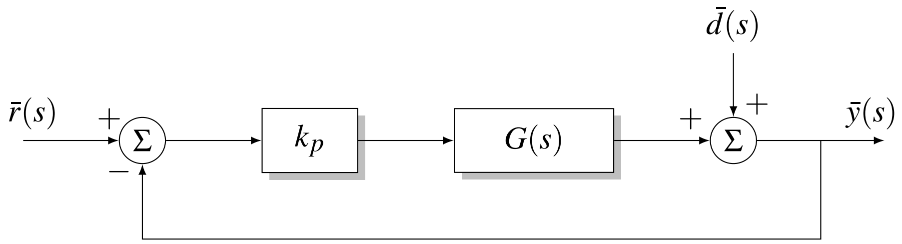

Fig. 2

A part of the Nyquist diagram for this system is shown in Fig. 3 for $k_p = 1$ (with frequency labels given in rad s^{-1}).

(a) Identify the right expression for $G(s)$ among the three transfer functions below; give qualitative justifications that do *not* rely on evaluating those transfer functions at specific values of $s = j\omega$:

$$G_1(s) = \frac{1}{s(s+1.8)(s+3)} \qquad G_2(s) = \frac{4}{s(s^2+0.6s+9)} \qquad G_3(s) = \frac{2}{s(s+1.8)}$$

[5]

(b) Estimate the gain margin of this system for $k_p = 1$. [4]

(c) The loop is closed with $k_p = 1.2$ and $d(t) = 0$. An input command is given of the form, $r(t) = \cos(3t)$. Using the Nyquist diagram in Fig. 3, estimate and write down an expression for the response, $y(t)$, of the closed-loop system, after any transients have died away. [6]

(d) Could you have anticipated the result in part (c) by looking at the phase margin? Explain your reasoning. [4]

(e) Set $r(t) = 1$ and $k_p = 1.2$. Derive an expression for the steady state response, $y(t)$, of the closed-loop system under an oscillatory output disturbance $d(t) = 0.2\cos(1.8t)$. [6]

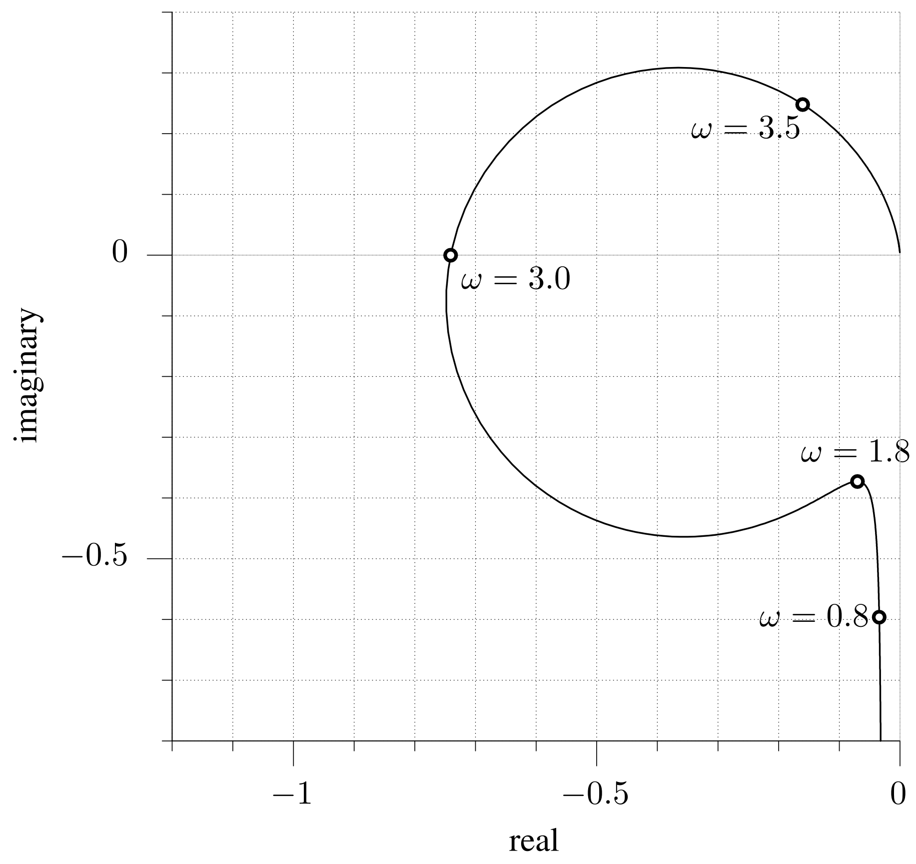

Fig. 3

3 A population of rabbits share their habitat with a population of foxes. Rabbits are fast breeders, their proliferation being limited only by the presence of foxes. Foxes, on the other hand, breed to the extent that they can eat rabbits. Let r and f denote the number of rabbits and foxes (each relative to some baseline). Their interplay is summarised by the following simplified equations:

$$\frac{dr}{dt} = r(t) - \alpha f(t) \qquad\qquad \frac{df}{dt} = -f(t) + \alpha r(t) - u(t)$$

where $\alpha > 0$ and $u(t)$ is some human-controlled stimulus that directly affects the growth rate of the fox population.

(a) By using the Laplace transform, find the transfer function $G(s)$ from $\bar{u}(s)$ to $\bar{r}(s)$. Discuss the stability of these population interactions, including the influence of α. [5]

(b) From now on, assume $\alpha = 2$. Nearby farmers meet to discuss a strategy to control the number of rabbits. They agree on the block diagram below in Fig. 4:

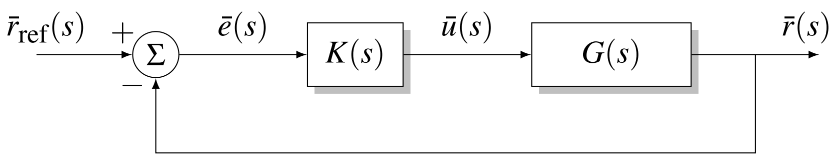

Fig. 4

with a controller $K(s) = k_p + k_d s$ where k_p and k_d are parameters to be tuned.

(i) Derive the closed-loop transfer function, expressing it in rational form. [4]

(ii) Assume $k_p > 0$ and $k_d = 0$. Does this controller successfully stabilise the population dynamics? Explain your reasoning. [4]

(iii) Assume $k_p > 0$ and $k_d = 1$. Find the lowest achievable steady-state error (in %) under the constraint that the damping factor, ζ, (defined as in the Databook) of the closed loop must be > 0.5. [6]

(iv) Take $k_p = 1$ and $k_d = 10$. Seasonal rabbit flu periodically kills some of the rabbits, effectively adding an output disturbance to $r(t)$ of the form $d(t) = \sin(\omega_d t)$. For which values of ω_d is this disturbance attenuated by a factor of more than 2 in amplitude? [6]

SECTION B

*Answer not more than **two** questions from this section.*

4 (a) If the function $f(x)$ has Fourier transform, $F(\omega)$, show that $f(x-a)$ has Fourier transform, $e^{-j\omega a}F(\omega)$ and $\frac{df(x)}{dx}$ has Fourier transform, $j\omega F(\omega)$. [4]

(b) Sketch the following function, $f(x)$,

$$f(x) = \begin{cases} 1-x^2 & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that its Fourier transform, $F(\omega)$, takes the form

$$F(\omega) = p(\omega)(\alpha \ \mathrm{sinc}\omega + \beta \cos\omega)$$

Give the values of the constants α and β, and the form of the function $p(\omega)$. [10]

(c) The piecewise function, $g(x)$, is given by

$$g(x) = \begin{cases} x(2+x) & -2 \leq x \leq 0 \\ x(2-x) & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Sketch $g(x)$. Using parts (a) and (b), or otherwise, find the Fourier transform, $G(\omega)$, of $g(x)$. [5]

(d) Evaluate, by direct integration, the Fourier transform, $K(\omega)$, of $k(x)$, where $k(x)$ is given by:

$$k(x) = \begin{cases} -2x & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Sketch $k(x)$. Now find $K(\omega)$ using the results of parts (a) and (b), verifying that the two expressions for $K(\omega)$ are equal. [6]

5 (a) A signal $f(t)$ is bandlimited (only contains frequencies less than f_B) and is sampled every T seconds to give samples $f(nT)$. We can interpolate these samples with a function $g(t)$ to reconstruct the signal for all t from its samples:

$$f(t) = \sum_{-\infty}^{+\infty} f(nT)\, g(t - nT)$$

For perfect reconstruction, what are the restrictions on the sampling rate and what is the function $g(t)$? [5]

(b) The Fourier transform of a finite, sampled signal is given by the *discrete Fourier transform* (DFT). For a sampling interval of T, find the DFT, $\{X_0, X_1, X_2, X_3\}$, of the following set of 4 samples ($n = 0, 1, 2, 3$):

$$\{x_n\} = \{-1,\, 0,\, 1,\, 0\}$$

[4]

(c) If $T = 2$ in part (b), find the frequencies of the DFT components $\{X_0, X_1, X_2, X_3\}$. Comment on possible interpretations of the original signal, $f(t)$, and the spectrum produced by the DFT. [4]

(d) *Modulation* is the process by which some characteristic of a carrier wave is varied by an information-bearing signal.

(i) Describe the processes of *amplitude modulation* (AM) and *frequency modulation* (FM) in an analogue setting. [3]

(ii) Give expressions for the spectra of AM and FM signals and hence calculate the bandwidth and power of each. [6]

(iii) Discuss the relative merits of AM and FM for analogue modulation via an information signal of bandwidth W, with specific reference to the power required and the resulting signal's robustness to noise. [3]

6 (a) To transmit *digital information* over a channel we must transform *bits* into a *waveform*.

(i) Describe the technique of *Pulse Amplitude Modulation* (PAM), outlining the two basic components of such a scheme. [4]

(ii) For 4-ary and 8-ary constellations given respectively by $[-3A,-A,A,3A]$ and $[-7A,-5A,-3A,-A,A,3A,5A,7A]$, give the mappings of the following sequence of bits to constellation symbols:

$$000010100110$$

[4]

(iii) Give two common unit-energy pulse shapes used in PAM, describing the desirable properties that such shapes have. [4]

(b) *Quadrature Amplitude Modulation* differs from PAM in having *complex* constellation values, $\{X_k\}$.

(i) Explain how the values of $\{X_k\}$ vary for *Phase Shift Keying* (PSK). For QPSK and 8-PSK, map the sequence in part (a)(ii) to their relevant constellation symbols (take the magnitude of the symbols to be A). [5]

(ii) For the QPSK and 8-PSK schemes discussed in part (b)(i), sketch the decision regions in the complex plane which are used to assign a given constellation symbol to an observed output. [3]

(iii) For the symbol p_1 in the QPSK constellation (where p_1 is in the first quadrant of the complex plane), obtain an expression for the probability of correctly decoding the symbol from the observation, given p_1 is the true mapping. Assume both real and imaginary components of the noise are independent and Gaussian distributed with zero mean and variance σ^2, and take the symbol magnitude as A. [Hint: use the fact that both the real and the imaginary parts of the observation need to be positive]. [5]

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Candidate Number ....

Question 1

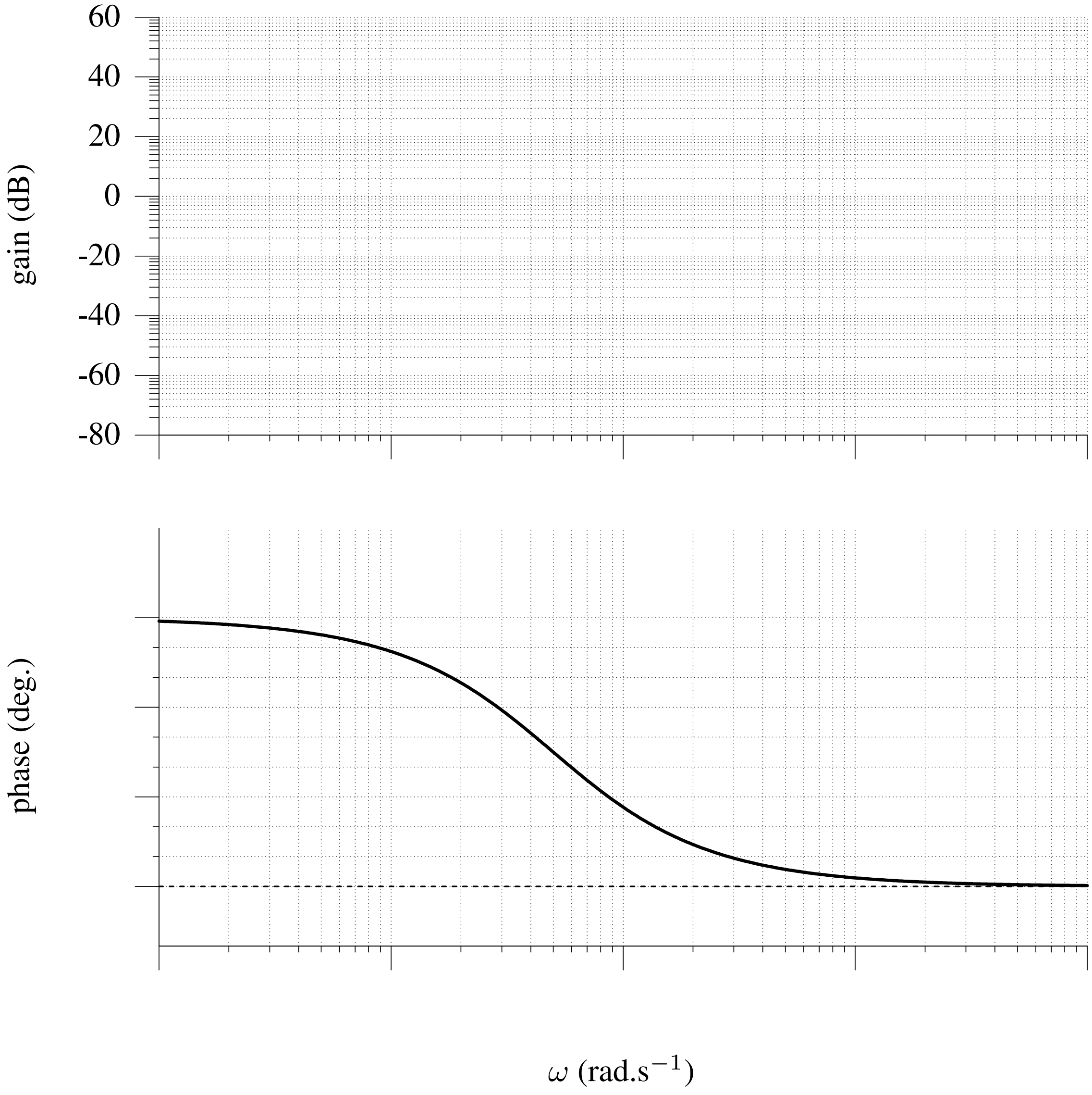

Fig.1

This is an additional copy of Fig. 1 which should be annotated with your constructions and handed in with your answer to this question.

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