

Answers to IB Mathematical Methods, 2014

Section A

1. *Divergence and Gauss*

- (a) πI_0
- (b) $\pi I_0 \frac{R^2}{R^2 + H^2}$
- (c) $\pi I_0 \frac{R^2}{R^2 + H^2}$ (same as (b))

2. *Curl, div and grad*

- (a) 0
- (b)(i) 0
- (b)(ii) Γ
- (c) $\nabla \times \underline{V} = \Gamma \delta(\underline{r})$
- (d) $Q \delta(\underline{r})$
- (e) $2\pi\phi = -Q \ln r + \Gamma\theta$
- (f) -

3. *PDE separation of variables*

- (a) -
- (b)

$$q_x = -\lambda_x \frac{\partial T}{\partial x} = A\beta\pi/L \sinh(\beta\pi x/L) \sin(\pi y/L)$$

$$q_y = -\lambda_y \frac{\partial T}{\partial y} = A\frac{\pi}{L} \sinh(\beta\pi x/L) \sin(\pi y/L)$$

- (c) 0
- (d) -

Section B

4. Probability

- (a) -
- (b) 0.37591
- (c)(i) 0.23975
- (c)(ii) 0.27671
- (c)(iii) -

5. Subspaces and Inversion

- (a) its determinant must be non-zero (b) -, $k\sqrt{5}$.
- (c)(i) 2
- (c)(ii)

$$\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$

- (c)(iii)

$$\mathbf{x} = \begin{bmatrix} 2 - 4/17 \\ 1 + 4/17 \\ -6/17 \end{bmatrix}$$

The cost of this design is $2.1828k$.

6. Eigenvectors and Eigenvalues

- (a)

$$\mathbf{Ax} = \lambda\mathbf{x}$$

- (b)(i) It must be an eigenvalue of \mathbf{A}_{22} .
- (b)(ii) It must be an eigenvalue of \mathbf{A}_{11} .

Eigenvalues of \mathbf{A} must be eigenvalues of either \mathbf{A}_{11} or \mathbf{A}_{22} . Any eigenvector unique to \mathbf{A}_{11} must have $\mathbf{x}_2 = \mathbf{0}$.

- (c)(i) 3 and 1.
- (c)(ii)

$$\mathbf{x} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$