Version KS/5

EGT1
ENGINEERING TRIPOS PART IB

Friday 6 June 2014 9 to 11.30
9 to 10.30 Foreign Language Option

Paper 8

SELECTED TOPICS

Answer ***one*** *question from Section A. In addition:*

> *If you are not taking the Foreign Language option, answer* ***four*** *questions, taken from only two of Sections B – H. Not more than* ***two*** *questions from each section may be answered.*
>
> *If you are taking the Foreign Language option, answer* ***two*** *questions from* ***one*** *of Sections B – H.*

All questions carry the same number of marks.

The ***approximate*** *number of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

Answers to questions in each section should be tied together and handed in separately.

Section A (Introductory Business Economics)	2
Section B (Civil and Structural Engineering)	3
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STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachments: Data Sheets for Section B (6 pages) and for Section E (2 pages)
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A *Introductory Business Economics*

*Answer not more than **one** question from this section.*

1 (a) Use an appropriate diagram or diagrams to represent the industry demand and supply curves, and the individual firm's demand and cost curves in a perfectly competitive market. On this basis:

(i) Identify the point at which the firm's profits are maximised and explain your answer; [5]

(ii) Define the firm's shut-down condition in the short and in the long run. [5]

(b) Under what circumstances may a monopoly be more desirable for consumers than perfect competition? [5]

(c) Compare and contrast the concepts of 'comparative' and 'competitive' advantage in international trade. [10]

2 (a) What factors can determine the degree of market power of a firm? [5]

(b) With reference to the concept of price discrimination:

(i) Describe the different types of price discrimination; [5]

(ii) Explain the conditions that make it possible for a firm to apply price discrimination. [5]

(c) Illustrate the idea of the 'circular flow' of the macroeconomy and describe the relationship between injections and withdrawals at equilibrium. [10]

SECTION B *Civil and Structural Engineering*

*Answer not more than **two** questions from this section.*

Note Data Sheets at end of the paper.

3 (a) A railway tunnel is to be constructed at a depth of 25 m in a city with varying geology. In zone A, the soil is a stiff clay with undrained shear strength s_u = 200 kN m^{-2}. In zone B, the soil is a soft clay with undrained shear strength s_u = 50 kN m^{-2}. In zone C, the soils are sands and gravels, with the water table close to the ground surface. The unit weight of all the soil types in all three zones can be assumed to be 20 kN m^{-3}.

(i) Define stability ratio and use it to determine the stability of the tunnel if it were to be constructed using open-face tunnelling in zones A and B. [6]

(ii) Explain why open-face tunnelling is likely to be a problem in zone C, with reference to the permeability of the soils. Give two examples of techniques that could be used to enable open-face tunnelling in zone C. [5]

(b) Briefly describe the principles of closed-face tunnelling machines, distinguishing between slurry machines and earth pressure balance machines. [4]

(c) Outline two different methods of lining a tunnel, giving advantages and disadvantages of each. [5]

(d) Explain why masonry buildings are susceptible to tunnelling, with reference to differential settlements, and how compensation grouting can be used to control damage to buildings. What is the role of instrumentation in the process? [5]

4 An 8 m deep excavation is to be made to create a double basement for a new building. The excavation is supported by a smooth diaphragm wall, as shown in Fig. 1. An adjacent small building exists at one side of the wall and a surcharge of 30 kN m^{-2} is applied at the ground surface to represent the load from the building. The site investigation revealed 8 m of dense sand underlain by stiff clay. The wall penetrates a distance of 4 m into the stiff clay and is propped by a slab at the level of the original ground surface. The water table (wt in Fig. 1) is at 2 m below the ground surface.

The unit weight of the saturated sand is $\gamma = 18$ kN m^{-3}, and the unit weight of the sand above the water table is $\gamma = 16$ kN m^{-3}. The critical state angle of friction of the sand is $\phi' = 35$ degrees. The unit weight of the clay is $\gamma = 17$ kN m^{-3}, the undrained shear strength is $s_u = 75$ kN m^{-2} and the critical state angle of friction of the clay is $\phi' = 25$ degrees.

(a) Describe why a diaphragm wall is suitable for this case and how it is constructed. [5]

(b) Assume that the wall moves sufficiently for active pressures to be mobilised behind it and passive pressures in front of it, and that the excavation occurs rapidly so that in the short term the clay remains undrained. Calculate the pressures acting on the wall, and sketch the pressure distribution on both sides of the wall immediately after the excavation. [7]

(c) Calculate the short term factor of safety against rotation about the prop. [6]

(d) Completion of the project is unexpectedly delayed. Sand layers in the clay on the passive side of the wall cause the clay to reach a drained condition with the water pressure hydrostatic from the bottom of the excavation. The clay on the active side remains undrained. Calculate the new factor of safety against rotation about the prop. [7]

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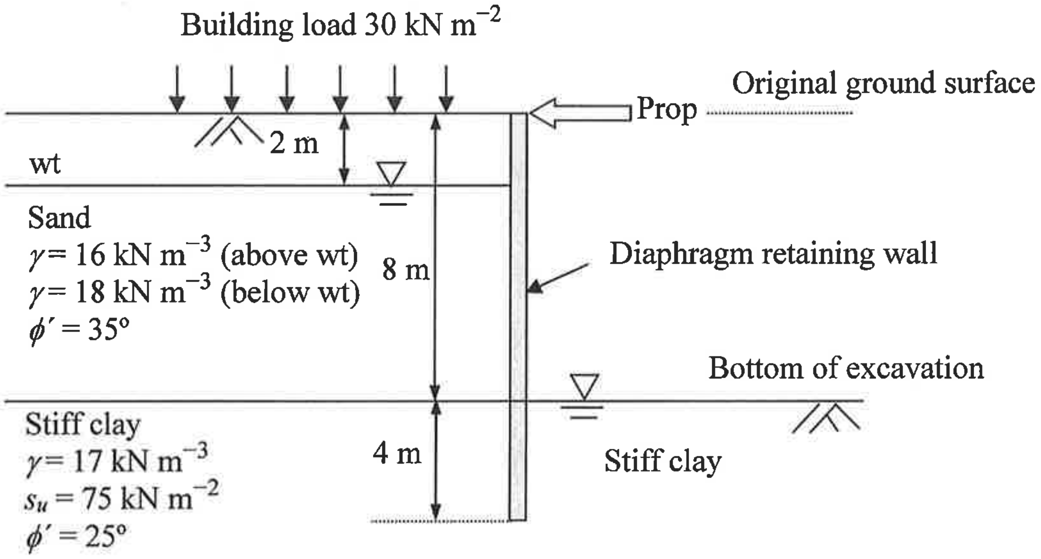
Building load 30 kN m^{-2}
Original ground surface
Prop
2 m
wt
Sand
γ= 16 kN m^{-3} (above wt)
γ= 18 kN m^{-3} (below wt)
ϕ' = 35°
8 m
Diaphragm retaining wall
Bottom of excavation
Stiff clay
γ= 17 kN m^{-3}
s_u = 75 kN m^{-2}
ϕ' = 25°
4 m
Stiff clay

Fig. 1

5 A section of a road is designed to run below the ground surface in a concrete trough, as shown in Fig. 2. It has been determined that the critical loading case for the slab AB is when the hydrostatic pressure is fully active beneath the slab and the piles at A and B are in tension. The degree of fixity between the slab and the walls cannot be guaranteed, so the designer must ensure that the slab is adequate if there is full fixity at A and B, or if the slab is simply supported at those points. The design upward pressure on the slab, which makes an allowance for the weight of the concrete slab and the partial safety factors on load, is 180 kN m^{-2}. The bending moment at the end of a fully fixed beam of length L under a distributed load of w per unit length may be taken as: $wL^2/12$.

(a) Draw the bending moment diagrams for the two limiting conditions of full fixity and simply supported conditions at points A and B respectively, and hence determine the design values of the bending moment at point A and at point C (at the mid span). [8]

(b) Determine the minimum effective depth of the slab, if it is to be singly reinforced. Take $f_y = 460$ N mm^{-2} and $f_{cu} = 40$ N mm^{-2}. [5]

(c) On the assumption that the slab provided is 1800 mm thick, design suitable reinforcement at A and C. [5]

(d) Suggest a reinforcement layout that should be provided in the slab, and in the junction to the walls. Take due account of the fact that the maximum length for a reinforcing bar is 12 m and give details of how you would lay out the reinforcement in the area of any laps. [5]

(e) Comment on the factors that would cause the designer to choose a slab thickness larger than the value you calculated in Part (b) above. [2]

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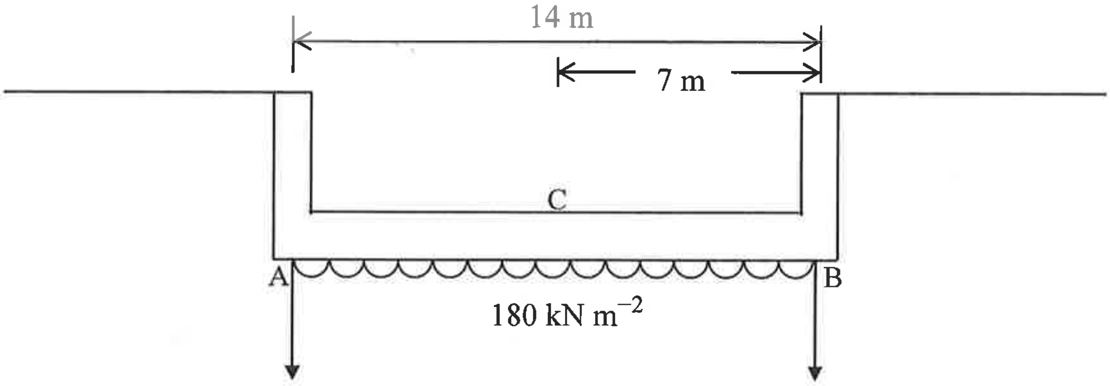


Fig. 2

SECTION C *Mechanics, Materials and Design*

*Answer not more than **two** questions from this section.*

6 (a) Outline the factors likely to be important in determining the fatigue life of the joint between a wind turbine blade and hub. [5]

(b) Describe a methodology to analyse the torsional vibration of the assembly of a wind turbine hub and blades. What modes of vibration would be expected? [5]

(c) The fatigue lifetime of a bolt used in joining a wind turbine blade and hub is governed by the relationship

$$N = \left(\frac{S}{S_0} \right)^{-M}$$

where N is the number of cyclic loads of stress range S, and S_0 and M are material properties. The turbine operation results in 10^7 cycles of loading per year with a mean stress range $\overline{S}$, following a probability distribution function ϕ given by

$$\phi(S) = \frac{1}{\overline{S}} \exp\left(\frac{-S}{\overline{S}} \right)$$

Estimate the expected lifetime in years of the bolt for $M = 10$ and $S_0 = 30\overline{S}$, stating any assumptions that you make.

Note that $\int_0^{\infty} t^z e^{-t} dt = z!$ for integer z. [10]

(d) Outline how the parameters describing the stress probability distribution function detailed in Part (c) might be estimated for a proposed wind turbine design and installation. [5]

7 (a) What are the main factors likely to determine changes in installed wind turbine power capacity in the UK over the next 10 years? What might be the effect on changes in wind turbine installation size and location? [5]

(b) Define the term ‘capacity factor’ in the context of wind turbines, and explain how it relates to the economic viability of proposed wind turbine installations. [3]

(c) A wind turbine of blade diameter 80 m has an optimum tip-speed ratio of 8 at which its power coefficient C_p is 0.35. It is part of a proposed variable-speed wind turbine generator system, in which the turbine is controlled so it operates at optimum tip-speed ratio between cut-in and rated wind speeds. Simplified wind speed data at the site is shown in the table below.

Wind speed, v (m s^{-1})	No. of days per year
< 3	20
8	200
12	90
16	40
> 20	15

Engineers are considering two options for the system: both have cut-in and stall wind speeds of 3 m s^{-1} and 20 m s^{-1} respectively, but Option 1 has a rated wind speed of 12 m s^{-1} whereas Option 2 has a rated wind speed of 16 m s^{-1}. Determine:

(i) the rated power of the two systems and the rotational speed of the turbines at their rated wind speeds;

(ii) the annual energy production of the two systems in MW hr;

(iii) the capacity factor of the two systems.

Comment on the likely economic viability of the two options. [14]

(d) Give three reasons why doubly-fed induction generators are the preferred technology for the generator in modern wind turbine generator systems. [3]

The power P produced by a wind turbine is given by $P = 0.5C_p\rho Av^3$ and the density of air ρ should be taken to be 1.23 kg m^{-3}.

8 (a) The mechanical power P generated by a horizontal axis wind turbine may be obtained from $P = \frac{1}{2}\rho A V_0^3 C_p$ where the power coefficient $C_p = 4a(1-a)^2$ and a is the axial induction factor.

(i) What are the terms ρ, A and V_0 in the above expression for P? [2]

(ii) The axial force N acting on the turbine rotor may be given by $N = \frac{1}{2}\rho A V_0^2 C_{NA}$ where C_{NA} is the thrust coefficient. Find an expression for C_{NA} as a function of a. [6]

(b) A wind-powered car consists of a wind turbine mounted on a wheeled platform as shown in Fig. 3. The turbine is mechanically geared to the car wheels so that the car is driven forwards into the wind. Find an expression for the ground speed of the car V_{car} as a function of the free-stream air speed V_{wind} and the induction factor a. You should assume that there are no resistance forces on the car other than the force on the rotor and that there are no losses in the transmission. [6]

(c) If the induction factor $a = 1/3$, the turbine rotor is operating at a tip speed ratio $\lambda = 8$, and the ratio of the tip radius of the turbine blades to the wheel radius $R/r = 3$, find:

(i) the required gear ratio between the turbine and driven wheels; [5]

(ii) the ratio of teeth on the annulus and sun gears if this gear ratio is to be achieved through an epicyclic gear box where the turbine drives the sun gear, the car wheels are driven by the planet carrier and the annulus gear does not rotate. [6]

(cont.

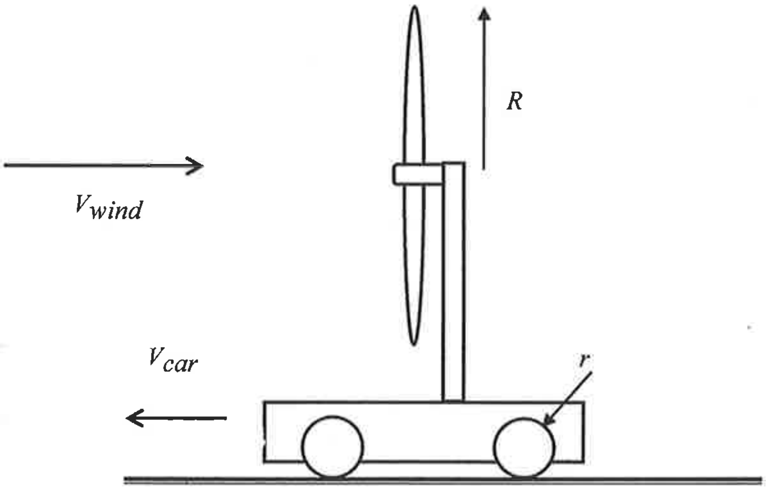


Fig. 3

SECTION D *Aerothermal Engineering*

*Answer not more than **two** questions from this section.*

9 (a) Sketch the temperature-entropy diagram for the core stream of a 2-shaft turbofan engine and indicate on this the effects of irreversibility in the turbomachinery components. Consider the design of a turbofan with a fixed core mass flow, turbine entry temperature and overall pressure ratio. Assume that the core and bypass jet velocities are also fixed. Describe qualitatively how improvements in the compressor and turbine efficiency would affect the design bypass ratio, engine fan diameter, specific fuel consumption and net thrust. [8]

(b) A 2-shaft turbofan powers an aircraft at cruise. After passing through the fan the core mass flow enters the high pressure (HP) compressor with a stagnation temperature of 297 K and a stagnation pressure of 68 kPa. The HP compressor has 10 stages, a stagnation pressure ratio of 28, and an isentropic efficiency of 89 %. The mean blade radius is 0.35 m and the rotational speed of the HP shaft is 10,000 rpm. If the compressor work is divided equally between the stages, determine the HP compressor work coefficient (stage loading). [4]

(c) The 2-stage HP turbine that drives the HP compressor has a mean blade radius of 0.38 m, an isentropic efficiency of 85 % and an inlet stagnation temperature of 1,700 K. Calculate the average HP turbine work coefficient and the stagnation pressure at entry to the low pressure turbine. [5]

(d) The low pressure turbine has an isentropic efficiency of 90 %. At inlet to the engine the stagnation temperature is 255 K, atmospheric pressure is 26.4 kPa, and the stagnation temperature rise through the fan for the bypass stream is 45 K. Find the core jet velocity if the engine has a bypass ratio of 10. [8]

Neglect the mass flow rate of the fuel. Assume that the combustion products behave as a perfect gas with the same properties as air. Neglect any losses in the ductwork and the propelling nozzles. Take $\gamma = 1.4$, $R = 287 \text{ J kg}^{-1} \text{ K}^{-1}$ and $c_p = 1{,}010 \text{ J kg}^{-1} \text{ K}^{-1}$ for air.

10 (a) Define the lift coefficient of an aircraft. Briefly explain why an aircraft has both an optimum cruise lift coefficient and an optimum flight Mach number. [3]

(b) A passenger aircraft has a wing area of 300 m^2, and a total mass of 160 tonnes at the start of cruise. The aircraft lift-to-drag ratio *L/D* can be approximated by the following equation,

$$L/D = \frac{C_L}{0.014 + 0.046 C_L^2} \quad \text{for} \quad 0.68 \le M \le 0.80$$

where C_L is the aircraft lift coefficient and M is the flight Mach number. The aircraft start of cruise altitude is 9.5 km, where the ambient pressure and temperature are 28.7 kPa and 226.7 K. If the Mach number is 0.8, determine the aircraft lift-to-drag ratio. [4]

(c) It is proposed that the cruise Mach number of the aircraft should be reduced such that *L/D* is maximised, whilst flying at the same altitude with the same start of cruise mass. Find the Mach number required and the resulting aircraft lift-to-drag ratio. [6]

(d) For the aircraft cruising a distance of 7,000 km, determine the percentage change in fuel consumption between flying at a Mach number of 0.8 and flying at the Mach number giving maximum *L/D*. Assume that in both cases the lift coefficient remains constant throughout cruise and that the engine thrust specific fuel consumption sfc is 0.016 kg s^{-1} kN^{-1}. You may use without proof the Breguet range equation,

$$s = \frac{V \; L/D}{g \; sfc} \ln\left(\frac{W_{start}}{W_{end}}\right)$$

where V is the flight speed, g is the acceleration due to gravity and W_{start} and W_{end} are the total aircraft weights at the start and end of cruise respectively. [8]

(e) Explain why cruising at maximum *L/D* with reduced M does not lead to a reduction in fuel consumption and suggest an alternative approach that would minimise the cruise fuel burn. [4]

Take $\gamma = 1.4$ and $\mathrm{R} = 287\ \mathrm{J\ kg^{-1}\,K^{-1}}$ for air.

11 (a) Define the *overall efficiency* of a jet engine and show how it is related to the *propulsive efficiency* and *thermal efficiency.* [3]

(b) The cruise condition of a turbofan engine is a flight Mach number of 0.85, at an altitude where the ambient pressure and temperature are 26 kPa and 223 K, respectively. At this condition the engine generates 40 kN of net thrust, has a uniform jet velocity of 360 m s^{-1} and burns fuel at a rate of 0.65 kg s^{-1} . The fuel used has a lower calorific value LCV of 43 MJ kg^{-1} . Determine the cruise propulsive efficiency, thermal efficiency and overall efficiency. [4]

(c) The non-dimensional air mass flow $\tilde{\dot{m}}_a$ and the non-dimensional fuel mass flow $\tilde{\dot{m}}_f$ for the engine are defined as

$$\tilde{\dot{m}}_a = \frac{\dot{m}_a\sqrt{c_p T_{02}}}{p_{02}A_N} \quad \text{and} \quad \tilde{\dot{m}}_f = \frac{\dot{m}_f LCV}{p_{02}A_N\sqrt{c_p T_{02}}}$$

where p_{02} is the inlet stagnation pressure, T_{02} is the inlet stagnation temperature and A_N is the total nozzle exit area, which is 2.6 m^2 . Explain why the mass flow rate of air $\dot{m}_a$ and the mass flow rate of fuel $\dot{m}_f$ are non-dimensionalised in different ways. Calculate the values of p_{02} , T_{02} and $\dot{m}_a$ for the engine at cruise and hence determine the corresponding values of $\tilde{\dot{m}}_a$ and $\tilde{\dot{m}}_f$. [8]

(d) The takeoff condition of the engine is a flight Mach number of 0.23, at sea level, where the ambient pressure and temperature are 101 kPa and 288 K, respectively. At this condition, the engine generates 180 kN of net thrust. Assuming that the non-dimensional operating point of the engine is the same as at cruise, determine the takeoff propulsive efficiency, thermal efficiency and overall efficiency. Comment on the differences between the efficiency values at cruise and takeoff. [10]

Take $\gamma = 1.4$, $R = 287$ J kg^{-1} K^{-1} and $c_p = 1{,}010$ J kg^{-1} K^{-1} for air. It can be assumed throughout that the propelling nozzles of the engine are choked.

SECTION E *Electrical Engineering*

Answer not more than ***two*** *questions from this section.*

Note Data Sheets at end of the paper.

12 (a) Explain why silicon has become the technology of choice for fabrication of (i) integrated circuits (ICs) and (ii) micro-electro-mechanical systems (MEMS). Your answers should be in the form of bullet points and no more than three for each part. [6]

(b) State two thermal oxidation schemes used in IC technology and how they are applied in fabrication of the MOSFET. In less than five lines, explain why the oxidation rate of (111)-oriented silicon is higher than that in (100)-oriented silicon? [6]

(c) Consider a <100> Si substrate passivated with a silicon dioxide masking layer as shown in Fig. 4 below. What are the resulting etch profiles if the wafer is subject to a wet anisotropic etchant with fast convex corner etch rates? Show the cross sectional view at A-A′ along with the orientation of the resulting crystal planes. [6]

(d) Diffused resistors are widely used in MEMS for signal readout in pressure sensors and air-bag accelerometers. A typical sheet resistance of a diffused layer is 200 $\Omega/\square$.

(i) What should be the aspect ratio of a 10 kΩ resistor using this diffusion?

(ii) Sketch a layout for this resistor, which uses minimal area for a width of 5 μm. [7]

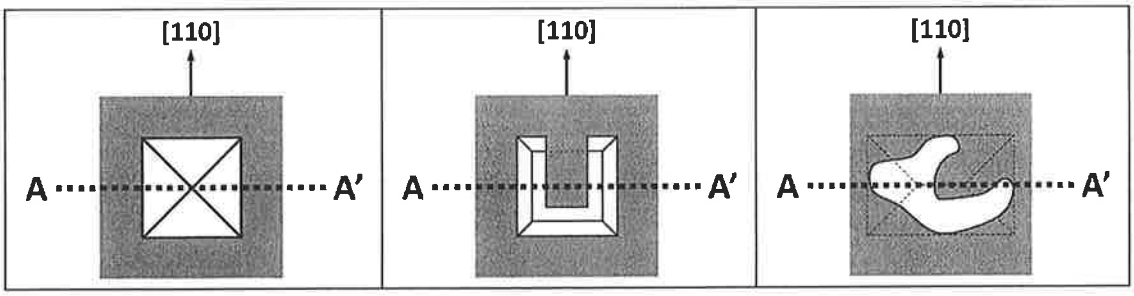


Fig. 4 Anisotropic etching in (100) silicon. White regions indicate mask opening.

13 (a) Explain with the aid of diagrams how the gate voltage controls the source-drain current in an FET. [4]

(b) Explain the difference between an accumulation and depletion mode FET. [4]

(c) What is silicon dioxide used for in an FET and compare the advantages and disadvantages of Si and GaAs based FETs, using device diagrams to illustrate your answer. [6]

(d) Derive the gate threshold voltage to switch off a Si depletion mode FET from Gauss' Law. [6]

(e) Calculate the maximum gate threshold voltage if the dopant density is 10^{19} cm^{-3} so that the gate field does not exceed the breakdown field of Si of 2×10^{7} V m^{-1}. The dielectric constant of Si is 10^{-10} F m^{-1}. [5]

14 (a) Explain how conduction occurs in a doped semiconductor, with the aid of diagrams. What controls the number of carriers? [4]

(b) In an n-type semiconductor, explain what is the electron mobility, conductivity and effective mass. [4]

(c) What is meant by scaling in electronic device design? How do the number of transistors per chip, gate length, circuit speed and power per chip scale with the scaling parameter? [4]

(d) Derive the transit time for an InAs FET with source-drain distance of 0.1 μm, if the mobility is 3.3 m^{2} V^{-1} s^{-1} and V_{SD} is 0.8 V. Comment on the result. [7]

(e) Optimise the dopant density so that the transistor will operate with a gate voltage swing of 0.1 V. Assume a dielectric constant for InAs of 1.5×10^{-10} F m^{-1} and a channel depth of 0.1 μm. [6]

SECTION F *Information Engineering*

*Answer not more than **two** questions from this section.*

15 Consider a data set of N colour images $S = \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$ where each image is represented as a vector of three real-valued features, corresponding to its overall red, green, and blue levels. That is, $\mathbf{x}_n = (x_{n1}, x_{n2}, x_{n3})$ are the amounts of red, green and blue respectively in image n, and $0 \leq x_{nc} \leq 100$ for all $c \in \{1,2,3\}$.

(a) Write down the probability of the data S under a model which assumes all x_{nc} are independent and uniformly distributed in the interval [0,100]. [7]

(b) Now assume instead a model where each x_{nc} is independent and normally distributed with different mean μ_c and variance σ_c^2 for each colour c. Write down the probability of S under this model, and describe a procedure for fitting the parameters μ_c and σ_c^2 to the data. [10]

(c) What are some relative advantages and disadvantages of the models described above in Parts (a) and (b)? Propose another model which you think might be interesting to use for the data S and describe why you think it might be a better model than the ones in Parts (a) and (b). [8]

16 A photograph of size 2048 × 1536 pixels has been taken with a camera that was accidentally held in a non-horizontal position, rotated 5 degrees clockwise from the true horizon.

(a) Outline the basic image processing steps that would be required to create an image rotated back to the true orientation of the scene, giving a formula for the location $[u, v]$ in the input image from which each pixel at $[p, q]$ in the output image should be sampled. Assume $[u, v]$ and $[p, q]$ are both measured from an origin in the centre of the image, and that top-to-bottom and left-to-right are the directions of positive increments in u or p, and v or q, respectively. [7]

(b) Explain why a simple rotation of the image would result in undesirable blank areas in the output image, and calculate how the above pixel-sampling formula should be modified so that a resized output image is generated, cropped so that there are no blank areas visible. You should assume that the resizing is chosen to be the minimum difference from unity, which produces a cropped output image of the same size as the input and with unchanged aspect ratio. [9]

(c) Explain why pixel interpolation will be required for the above derotation process, and discuss the relative merits of bi-cubic versus bi-linear interpolation. You do not need to reproduce the cubic interpolation formula, but you may find it helpful to refer to Fig. 5, which compares the frequency responses of 1-dimensional linear and cubic interpolators. [9]

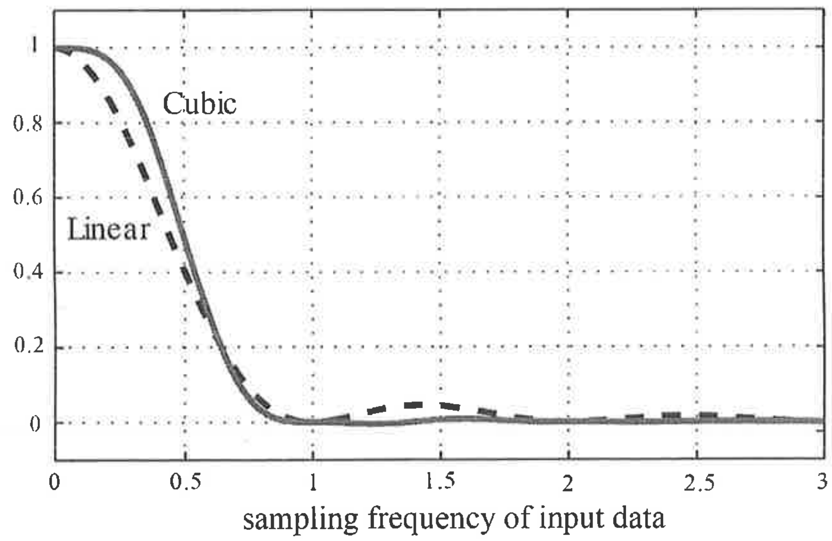


Fig. 5

17 Many feature detection algorithms in computer vision smooth the image $I(x, y)$ using a low pass filter $L(x, y)$ in order to produce smoothed pixel intensities $S(x, y)$.

(a) An algorithm uses a low pass filter of the following form:

$$L(x,y) = \begin{bmatrix} 4 & 6 & 4 \\ 8 & 12 & 8 \\ 4 & 6 & 4 \end{bmatrix}$$

Can the smoothing operation be implemented using discrete 1-dimensional convolutions in this case? Explain your reasoning. [4]

(b) An algorithm uses an isotropic Gaussian filter of size $K \times K$ pixels to smooth an image of size $N \times M$ pixels. The 2-dimensional convolution can be implemented using discrete 1-dimensional convolutions in this case. What is the computational saving of this implementation over a naïve approach? [4]

(c) Describe an algorithm for detecting the position and orientation of edges in an image using spatial derivatives of the smoothed image. [6]

(d) Describe an algorithm for detecting corners in an image using spatial derivatives of the smoothed image. [6]

(e) Discuss whether edges or corners are more appropriate interest points for an application that automatically tracks people in a surveillance video. For low-frame rate cameras and large depth variation in the image, would you expect either interest point to perform well? Explain your reasoning. [5]

SECTION G *Bioengineering*

*Answer not more than **two** questions from this section.*

18 Two thirds of the total focussing power of the eye comes from the cornea.

(a) Describe the composition and structure of the cornea and how it differs from the adjacent sclera. [5]

(b) Explain the three aspects of the mechanical response of a soft tissue such as cornea that differentiate such tissues from traditional engineering materials. Use graphs to illustrate each difference and describe what aspect of the tissue microstructure gives rise to each aspect of the response. [6]

(c) Explain LASIK surgery and describe why LASIK patients are unsuited to be cornea donors. [4]

(d) (i) Describe the transport mechanism in poroelasticity. [3]

(ii) If a cornea has a thickness of 350 μm, an elastic modulus of 0.3 MPa, and an intrinsic permeability of 8.2×10^{-17} m^2, what is the time constant for poroelastic transport through the cornea? Assume that the viscosity of water is 1 mPa s. [3]

(iii) If the cornea thickness decreases to 200 μm, with all other parameters the same as in (ii), what effect does this have on the transport behaviour? [2]

(iv) With the thickness the same as in (iii) and the permeability unchanged, how would the modulus have to change to restore the original transport response in (ii)? [2]

19 (a) Describe, with appropriate diagrams, the key features of a fundus camera that allow a clear image of the retina to be created. [6]

(b) In spectral-domain optical coherence tomography, the interference pattern from the imaged tissue is passed through a diffraction grating and its spectrum is detected by a uniformly spaced linear array of photo-detectors. The inverse Fourier transform of this spectrum gives a measure of tissue reflectivity as a function of depth into the tissue. Assume that the laser light centre wavelength (in air) is 850 nm, the bandwidth is 100 nm and the photo-detectors are arranged to make the best possible use of this spectrum. The refractive index of the tissue is 1.36.

(i) If the diffraction grating spreads the light uniformly over the detector array with frequency, how many photo-detectors are required to image the tissue to a depth of 0.5 cm? [5]

(ii) If the diffraction grating spreads the light uniformly over the detector array with wavelength, what are the minimum and maximum frequency differences between neighbouring photo-detectors, if the same number of photo-detectors are used as in (i)? [3]

(iii) How would you expect the tissue reflectivity measured in (i) to differ from that measured in (ii)? [3]

(c) Data sampled in three dimensions, such as might be acquired from an optical coherence tomography system, is usually visualised on a two-dimensional display.

(i) Explain the term *reslicing* and outline the difficulties associated with reslicing sampled optical pulse-echo data. [4]

(ii) Briefly outline two techniques, other than reslicing, which can be used to visualise sampled three-dimensional data on a two-dimensional display. [4]

20 (a) Sketch and describe the receptive fields of simple and complex cells in the primary visual cortex. Also sketch and describe a network architecture that could form simple cell receptive fields from the centre-surround receptive fields found in the lateral geniculate nucleus. [4]

(b) Name four visual cues that can help to determine which of two objects is farther away. [2]

(c) Explain why, after fixating a waterfall for a minute, stationary objects you look at will appear to be moving upwards. [3]

(d) Explain why, after accidents involving damage to the visual system, doctors check if the perceptual defect concerns a visual hemifield or an individual eye. [3]

(e) The following questions are about efficient coding in the (vertebrate) retina.

(i) Mutual information is a measure of the statistical dependency of two random variables. According to the efficient coding hypothesis, the mutual information of two specific random variables is maximized in retinal processing. Name these two variables and provide the formula for the mutual information of two continuous variables. [3]

(ii) Describe the approximations and assumptions underlying the proposition that information maximisation in the retina can be achieved by whitening. [4]

(iii) Describe the shape of the Fourier spectrum of a whitening filter for natural images, and provide an explanation for this shape. [3]

(iv) Explain with reasons why pure whitening in the retina may not be advantageous. [3]

SECTION H *Manufacturing and Management*

*Answer not more than **two** questions from this section.*

21 You have developed a very low-cost, wearable, wirelessly connectable device for monitoring a person's level of alertness.

(a) Discuss how you could identify and assess the scale of potential markets for this concept. [9]

(b) Describe the steps that you could take to understand the requirements of potential customers for such a concept. [8]

(c) Discuss the different business models you could use to capture value from this concept. [8]

22 (a) Describe and give an example for each of the following types of innovation:

(i) product;

(ii) process;

(iii) placement; and

(iv) paradigm. [8]

(b) Discuss why large, long-established firms tend to focus on incremental rather than radical innovations. [9]

(c) Discuss why technology-based start-up firms often form partnerships with larger, more commercially experienced firms. [8]

23 (a) Describe, using examples, what is meant by:

(i) technology push; and

(ii) market pull. [4]

(b) Describe the four tests that an invention must satisfy for it to be patentable. [4]

(c) Discuss the relative advantages and disadvantages of protecting an invention through:

(i) filing a patent; or

(ii) keeping the invention confidential. [8]

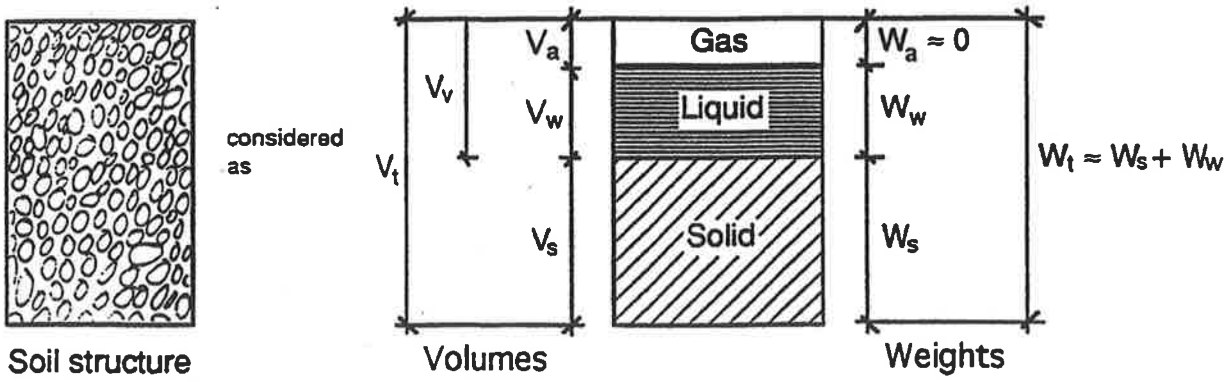
(d) Discuss the challenges that an inventor may face when seeking to commercially exploit potentially disruptive intellectual property in a sector dominated by large companies. [9]

END OF PAPER

Engineering Tripos Part IB Civil Engineering Elective

Data sheet – Soil Mechanics

General definitions



Specific gravity of solid	G_s
Voids ratio	$e = V_v / V_s$
Specific volume	$v = V_t / V_s = 1 + e$
Water content	$w = (W_w / W_s)$
Degree of saturation	$S_r = V_w / V_v = (w G_s / e)$
Unit weight of water	$\gamma_w = 9.81 \text{ kN/m}^3$ (although we assume 10 kN/m^3)
Unit weight of soil	$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e} \right) \gamma_w$
Buoyant (effective or submerged) unit weight	$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w$ (soil saturated)
Unit weight of dry soil	$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e} \right) \gamma_w$
Relative density	$I_d = \frac{(e_{max} - e)}{(e_{max} - e_{min})}$

where e_{max} is the maximum voids ratio achievable in the quick tilt test (for sands), and
e_{min} is the minimum voids ratio achievable by vibratory compaction (for sands).

Classification of particle sizes

Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D	equivalent diameter of soil particle
D_{10}, D_{60} etc	particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains.

Stress components

Principle of effective stress (saturated soil):

$$\begin{array}{lccccc} \text{total stress} & \sigma & = & \text{effective stress } \sigma' + & \text{pore water pressure } u \\ & \tau & = & \tau' + & 0 \end{array}$$

and

σ_v = vertical stress

σ_h = horizontal stresss

τ = shear stress

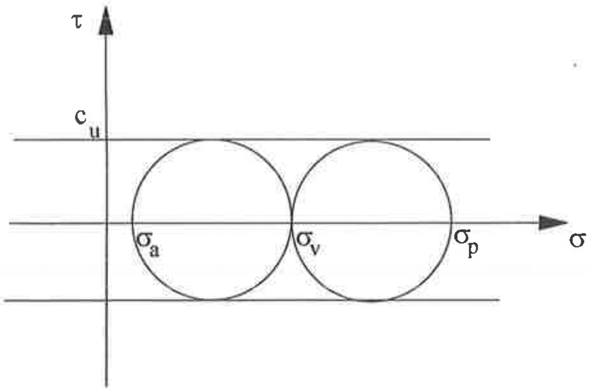
Strength of clays (undrained behaviour only)

Under constant volume (undrained) conditions only, the strength of clays can be characterised by the *undrained shear strength* c_u which is mobilized when the shear stress $\tau = c_u$. This conforms to Tresca's criterion, and the active and passive total horizontal stresses, σ_a and σ_p respectively, are given by

$$\sigma_a = \sigma_v - 2\,c_u$$

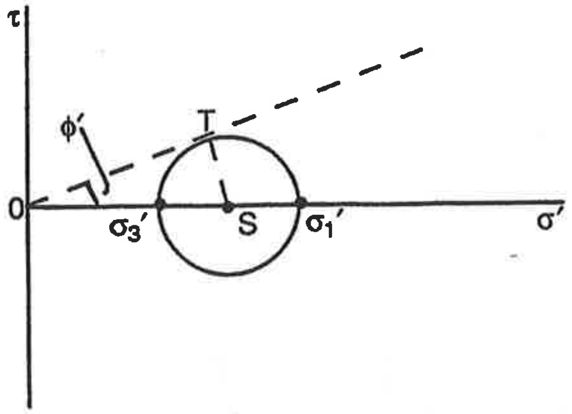
$$\sigma_p = \sigma_v + 2\,c_u$$

where σ_v is the total vertical stress.



Strength of sands

Mobilised angle of shearing ϕ'
where $\tau = \sigma' \tan \phi'$



$$\sin\phi' = TS/OS$$

$$= \frac{(\sigma_1' - \sigma_3')/2}{(\sigma_1' + \sigma_3')/2}$$

$$\therefore \phi' = \sin^{-1}\left[\frac{\left(\sigma_1'/\sigma_3'\right)-1}{\left(\sigma_1'/\sigma_3'\right)+1}\right]$$

Earth pressure coefficient K: $\sigma_h' = K\sigma_v'$

Active pressure: $\sigma_v' > \sigma_h'$ $\therefore$ $\sigma_1' = \sigma_v'$

$\sigma_3' = \sigma_h'$

$K_a = (1 - \sin\phi'/1 + \sin\phi')$

Passive pressure: $\sigma_h' > \sigma_v'$ $\therefore$ $\sigma_1' = \sigma_h'$

[We assume principal stresses are horizontal and vertical]

$\sigma_3' = \sigma_v'$

$K_p = (1 + \sin\phi')/(1 - \sin\phi') = \dfrac{1}{K_a}$

Angle of shearing resistance:

at peak strength ϕ'_{max} at $\left(\sigma_1'/\sigma_3'\right)_{max}$

at critical state ϕ'_{crit} after large strains.

Sand strength data: friction hypothesis

In any shear test on soil, failure occurs when $\phi' = \phi'_{max}$ and

$$\phi'_{max} = \phi'_{crit} + \phi'_{dilatancy}$$

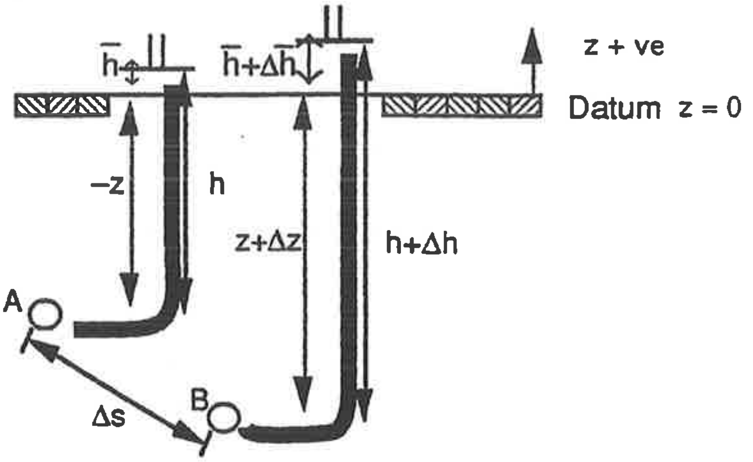
where ϕ'_{crit} is the ultimate angle of shearing resistance of a random aggregate which deforms at constant volume, so the dilatancy, which indicates an increase in volume during shearing, approaches zero $(\phi'_{dilatancy} -> 0)$ as $\phi'_{max} -> \phi'_{crit}$.

ϕ'_{crit} is a function of the mineralogy, size, shape and distribution of particles. For a particular soil it is almost independent of initial conditions. Typical values (±2°):

	ϕ'_{crit}	ϕ'_{max}	
feldspar	40°		
quartz	33°	53°	($I_d = 1$, and mean effective stress OS < 150 kPa)
mica	25°		

Seepage

Excess pore water pressure



Head $h = u/\gamma_w$

Potential $\bar{h} = h + z$

Total pore water pressure head at A: $u = \gamma_w h = \gamma_w(\bar{h} - (-z))$

B: $u + \Delta u = \gamma_w(h + \Delta h) = \gamma_w(\bar{h} + z + \Delta\bar{h} + \Delta z)$

[Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

B: $\bar{u} + \Delta\bar{u} = \gamma_w(\bar{h} + \Delta\bar{h})$]

Hydraulic gradient A–B $i = -\dfrac{\Delta\bar{h}}{\Delta s} = -\dfrac{\Delta\bar{u}}{\gamma_w \Delta s}$

Darcy's law v = ki
v = average or superficial seepage velocity
k = coefficient of permeability

Typical permeabilities

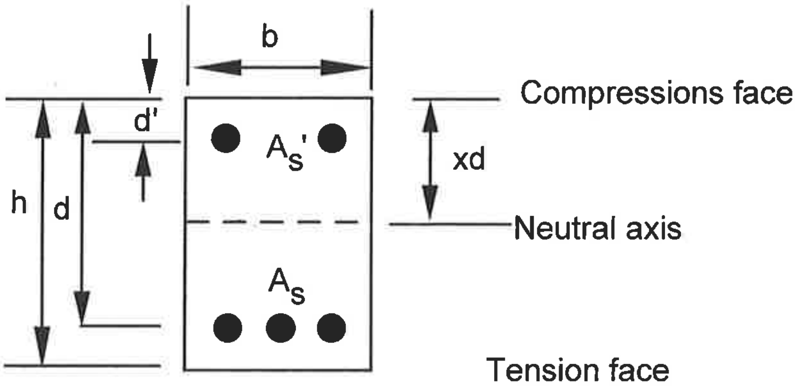
$D_{10} > 10\text{mm}$: non-laminar flow

$10\text{ mm} > D_{10} > 1\mu\text{m}$: $k \cong 0.01(D_{10}\text{in mm})^2$ m/s

clays : $k \cong 10^{-9}$ to 10^{-11} m/s

Design of reinforced concrete

Data sheet for use in Part IB Civil Engineering Elective Course.



Design Stresses

Cube strength for concrete f_{cu}. At failure in bending, stress in concrete $= 0.4f_{cu}$ over whole area of concrete in compression.

Tensile yield stress of steel f_y. At failure in bending, stress in bars in tension $= 0.87f_y$, stress in bars in compression $= 0.75f_y$.

Design Equations

Moment capacity of singly reinforced beam

$$M \leq 0.15 f_{cu} bd^2$$
$$M = 0.87 f_y A_s d(1 - x/2)$$
$$x = 2.175(f_y/f_{cu}).(A_s/bd) \quad (\leq 0.5 \text{ to avoid over reinforcement})$$

Moment capacity of doubly reinforced beam

$$M = 0.15 f_{cu} bd^2 + 0.75 f_y A'_s (d - d')$$
$$0.87 f_y A_s = 0.75 f_y A'_s + 0.2 f_{cu} bd$$

Shear capacity of all beams

Total shear capacity $V = (v_c + v_s)bd$

Where, $v_c = 0.68(100\,A_s/bd)^{0.33}.(400/d)^{0.25} \qquad (N/mm^2)$

and $v_s = 0.87 f_y\, A_{sq}/(bs)$

in which s = shear link spacing, A_{sq} is total area of all shear bars in a link and A_s is the total area of effective longitudinal *tension* steel at the section.

Standard bar sizes

Diameter (mm)	6	8	10	12	16	20	25	32	40	50
Area (mm^2)	28	50	78	113	201	314	491	804	1256	1963

Available steel types

Deformed high yield steel	$f_y = 460 \quad N/mm^2$
Plain mild steel	$f_y = 250 \quad N/mm^2$

Lap and anchorage lengths 40 bar diameters

Density of reinforced concrete: $24\ kN/m^3$

Reinforcement areas per metre width

	Spacing of bars (mm)									
	75	100	125	150	175	200	225	250	275	300
Bar Dia. (mm)										
6	377	283	226	189	162	142	126	113	103	94.3
8	671	503	402	335	287	252	224	201	183	168
10	1050	785	628	523	449	393	349	314	285	262
12	1510	1130	905	754	646	566	503	452	411	377
16	2680	2010	1610	1340	1150	1010	894	804	731	670
20	4190	3140	2510	2090	1800	1570	1400	1260	1140	1050
25	6550	4910	3930	3270	2810	2450	2180	1960	1790	1640
32	10700	8040	6430	5360	4600	4020	3570	3220	2920	2680
40	16800	12600	10100	8380	7180	6280	5580	5030	4570	4190
50	26200	19600	15700	13100	11200	9820	8730	7850	7140	6540
Areas calculated to 3 significant figures according to B.S.I recommendations										

9 May 2005

Part IB Data Sheet: Electrical Engineering Elective Transistor Design Summary Sheet

Gauss's Theorem

$\varepsilon_0\varepsilon_r E_1 - \varepsilon_0\varepsilon_r E_2$ = charge per unit area enclosed between upper surface 1 and lower surface 2.

FET Design Summary

- $\tau_t \rightarrow$ switching time as 1st approx. (scattering limited transit time).
- $\tau_{eff} = \tau_t + R_{load}C_{eff(output)} \rightarrow$ switching time as 2nd approx.
- $L = v_s\tau_t$ (source-drain spacing).
- $I_{sat} = eNv_s W d_s = eNWLd_s/\tau_t$
- Aspect ratio W/L (technology?).
- $(1/2)eN(d_s)^2/\varepsilon_0\varepsilon_r = $ (Max Gate Voltage)
- $E'_{peak} = eNd_s/\varepsilon_0\varepsilon_r < E_{breakdown}$
- Minimum Drain Source Voltage $\sim E_sL$ (E_s is the field required to reach limiting velocities).

Mutual Conductance

$g_{mo} \sim I_{sat}/V_{gate(max)}$

Mutual conductance reduces with frequency as $g_m(\omega) \approx g_{mo}/(1 + j\omega\tau_t)$;

$$v_{out} = g_m(\omega)\,R(1 + j\omega RC_{eff(out)}) \approx g_{mo}R/[1 + j\omega(\tau_t + RC_{eff(out)})]$$

$$= g_{mo}R/[1 + j\omega\tau_{eff}]$$

Capacitances for FET

Parallel Plate Capacitance: $\varepsilon_0\varepsilon_r$ Area/spacing

Used for rough estimates of parasitic capacitance.

Effective Capacitances for FET

$C_{eff(out)} \rightarrow C_{gate/drain} + C_{drain/source} + C_{load};$

$C_{eff(in)} \rightarrow MC_{gate/drain} + C_{gate/source\,(proximity)} + C_{gate/source\,(electronic)};$

$C_{electronic} = g_{mo}\tau_t \quad ; \quad M = (1 + g_{mo}R_{load}).$

Time Constants for FET

$\mu = e\tau/m^*$ relates mean free time τ and mobility.

Transit time τ_t over distance L and scattering limited velocity v_s are related by $\tau_t = L/v_s$.

$v_{out} \approx g_{mo}R/[1 + j\omega\tau_{eff}] = g_{mo}R/[1 + j\omega/(2\pi f_t)]$

$1/(2\pi f_\tau) = \tau_t + RC_{eff(out)} = \tau_{eff}$ The transition frequency is f_t.

10% to 90% rise time is $T = 2.2\,\tau_{eff} = (2.2/2\pi)(1/f_t) = 0.35/f_t$.

Electrical Engineering Elective:
Tunnel Barrier Design Summary Sheet

Schrodinger's Equation

Complex Wave $\Psi = A\ \exp(-j2\pi f\, t + j2\pi x/\lambda) = A\exp(-j\omega t)\exp(jkx);$
<momentum> $\Psi = p\Psi = (h/\lambda)\Psi = -j(h/2\pi)\partial\Psi/\partial x;$

<Total energy> $\Psi = E\Psi = hf\Psi = j(h/2\pi)\partial\Psi/\partial t$
$(h/2\pi) \rightarrow \hbar;\ \ h = 6.625\times10^{-34}$ J/s.

Schrodinger's equation:-

$$E\Psi = (1/2m)[-j\hbar\partial/\partial x]^2\Psi + e\phi\Psi$$

Tunneling (Rectangular barrier $\varepsilon\phi$)

Propagating waves outside barrier with incident kinetic energy $U_{\text{incident}} = (\hbar k_i)^2/2m^*$

Evanescent waves inside barrier: $-(\hbar k)^2/2m^* = (\hbar k_i)^2/2m^* - e\phi$

Technology Design Summary

Diffusion

Constant Surface Concentration:

$$C(x,t) = C_s\ \operatorname{erfc}\left[\frac{x}{2(Dt)^{1/2}}\right]$$

Constant Total Dopant:

$$C(x,t) = \frac{S}{(\pi Dt)^{1/2}}\exp\left[\frac{-x^2}{4Dt}\right]$$

W I Milne
May 2011