

Question 1

- a)
- i) Describe the main factors in machine tool design and operation that can lead to dimensional error in part machining.
  - ii) What are the consequences of designing a machine tool with low stiffness and how can these be addressed?
- [25%]
- b)
- i) What properties of the work material have a significant influence on the success of the machining operation?
  - ii) What criteria would you use to assess the success or otherwise of a proposed work material for a particular machining operation?
- [20%]
- c) Titanium is commonly used in aerospace applications.
- i) Discuss the characteristics of titanium that make it difficult to machine.
  - ii) Describe the composition and physical attributes of a cutting tool material that would be most suitable for machining titanium.
- [20%]
- d) Consider the arrangement shown in Figure 1 in which a long section of a titanium bar,  $L_w$ , is being reduced from diameter  $D_1$  to  $D_2$  by turning on a lathe in a number of passes.
- i) Calculate the total machining time, material removal rate, and the power required where  $D_1 = 100$  mm,  $D_2 = 94$  mm,  $O = A = 5$  mm,  $L_w = 80$  mm, cutting velocity  $V_c = 100$  mm  $s^{-1}$ , feed speed  $S_o = 1$  mm  $rev^{-1}$ , and depth of cut per pass  $t = 2$  mm. Assume the specific energy for cutting titanium alloy is  $3.5$  J  $mm^{-3}$ .
  - ii) How might you improve the accuracy of your approach?
- [35%]

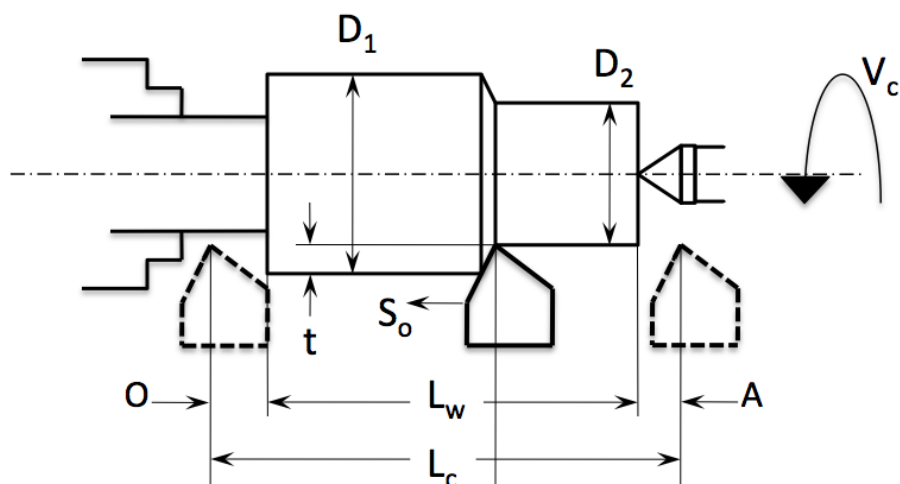
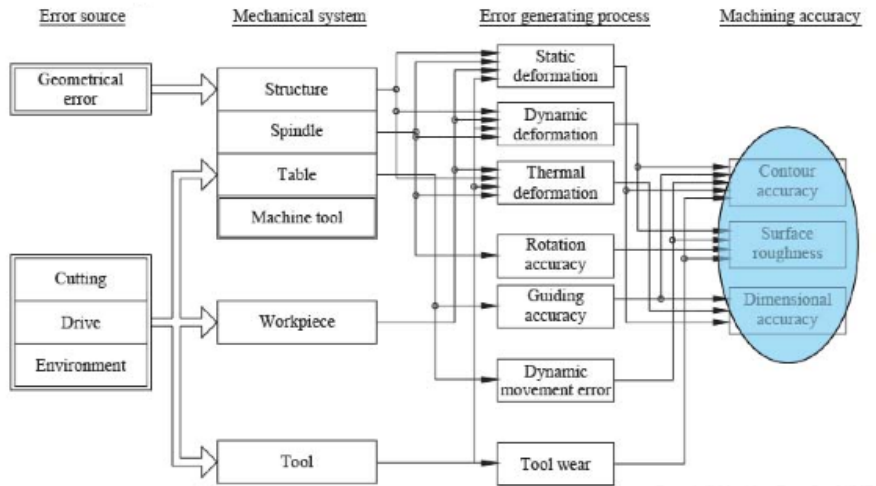


Figure.1

**Question - 1 CRIB**

a)

i) *The factors that affect work piece accuracy stem from the machine tool itself, the tool actions and the work piece material. Sources of error are those from geometrical errors, cutting processes, drive systems, or the environment. These can all be linked to the mechanical system components and error generating processes, which result in machining accuracy limitations. A good answer will sketch out these dependencies as shown below.*



*Some answers may offer further detail in terms of static and dynamic effects*

	<b>Static Effects</b>	<b>Dynamic Effects</b>
<i>Tool Design</i>	<i>poor tolerancing material flow problems set-up shrinkage</i>	<i>wear (cutting tools) non-linear coupling (chatter) degradation (wear/erosion</i>
<i>Machine design</i>	<i>precision materials environment</i>	<i>dynamic vibration thermal stability environment</i>
<i>Installation</i>	<i>environment location</i>	<i>dynamics thermal operators</i>
<i>Material</i>	<i>performance</i>	<i>poor fixture design</i>

ii) *Low stiffness leads to a collection of vibrations in the machining system as a whole. There are two causes of this.*

Forced Vibrations.

*Caused by some periodic force present in the machine tool, such as from gear drives, imbalance of the machine tool components, misalignment, or motors and pumps.*

Self excited vibration or 'chatter'.

*In machining, chatter is the vibration that feeds on itself as the tool moves across the part.*

*The tool, tool-holder and spindle together will vibrate at some natural frequency—a frequency at which this assembly "naturally" wants to vibrate. In fact, the assembly is likely to vibrate at more than one such natural frequency at the same time. At the tool tip, this vibration leaves waves in the machined surface. The waviness can cause the next cutting edge to experience a variable load. When that*

*happens, this variable load feeds the vibration that already exists, making it worse. "Self-excited vibration" is one term for this phenomenon. "Regenerative chatter" is another.*

#### Adverse Effects

- *Poor surface finish*
- *Loss of dimensional accuracy*
- *Premature wear, chipping, and failure of the cutting tool*
- *Damage to the machine tool components from excessive vibration*
- *Objectionable acoustic emission, such as high frequency squeal*
- *A serious consequence relates to efficiency. Most shops deal with chatter by setting their machining parameters low. Therefore, instead of tool strength and spindle horsepower defining the metal removal rate, chatter becomes the limiting factor that keeps the process from reaching its potential.*

#### How can these be addressed?

*The basic solution to forced vibrations is to isolate or remove the forcing element. If this is not possible, the amplitude of the vibration can be reduced by increasing the stiffness or damping of the system.*

*For self excited vibrations, Increasing rigidity is one option. Use a shorter tool or tool-holder, or switch to a tool-holder that clamps the tool more rigidly—these are examples of changes that might make the process less apt to vibrate. When milling at high spindle speeds, there is a potentially more promising option. Certain limited ranges of spindle speed may be stable zones. Within these ranges, the rate of cutting edge impacts synchronizes with a natural frequency of the system. The chip load becomes level, so the cut is smooth. The depth of cut can therefore be increased, and sometimes it can be dramatically increased. A stable value of spindle rpm applies only to a particular combination of spindle, tool, and tool-holder. These three factors make up a complete assembly. Every different assembly has to be evaluated separately; evaluating just the spindle is not enough. However, any particular spindle and tooling assembly is likely to have more than one stable zone. Therefore, finding just one stable speed zone might not be enough. Another, much faster speed may also be stable.*

*b)*

*i) Material properties that have an influence on the ability of a material to be machined are: properties such as density, hardness, toughness, thermal conductivity, yield stress, chemical affinity to the cutting tool material, coefficient of friction with the cutting tool material.*

*ii) Various criteria are used to assess the success of a machining operation. The most important of which is (1) tool life because of its economic significance in a machining operation. Other criteria include (2) cutting forces (which determines the power of the machine), (3) accuracy and surface finish, and (5) ease of chip disposal. It should be recognized that machining performance depends on more than just material. The type of machining operation, tooling, and cutting conditions are also important factors. One work material may yield a longer tool life, whereas another material provides a better surface finish. All of these factors make evaluation of a material's suitability for machining a complex*

*c)*

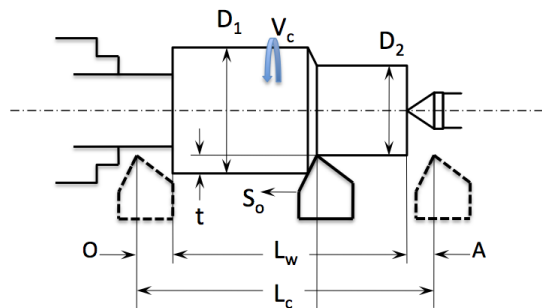
*i) Titanium is a difficult material to machine and delivers relatively poor tool life, even at low cutting speeds. Answers to this question should discuss the following material aspects*

- *High chemical reactivity causes chips to gall and weld to cutting edges.*
- *Low thermal conductivity increases cutting temperatures.*
- *Usually produces abrasive, tough, and stringy chips.*
- *Precautionary measures are needed since it is a reactive (combustible) metal.*

- Low elastic modulus easily causes deflection of work piece.
- Easy work hardening.

ii) An ideal cutting tool material for titanium would be a cemented carbide such as WC-Co, also useful for other nonferrous metals; In the non-steel-cutting grades, grain size and cobalt content are the factors that influence properties of the cemented carbide material. The typical grain size found in conventional cemented carbides ranges between 0.5 and 5  $\mu$ m. As grain size is increased, hardness and hot hardness decrease, but transverse rupture strength increases. The typical cobalt content in cemented carbides used for cutting tools is 3% to 12%. As cobalt content increases, transverse rupture strength (TRS) improves at the expense of hardness and wear resistance. Cemented carbides with low percentages of cobalt content (3% to 6%) have high hardness and low TRS, whereas carbides with high Co (6% to 12%) have high TRS but lower hardness. Accordingly, cemented carbides with higher cobalt are used for roughing operations and interrupted cuts (such as milling), while carbides with lower cobalt (therefore, higher hardness and wear resistance) are used in finishing cuts.

d) From the information given we can develop the following considerations



$T_c$  = actual cutting time per piece, and can be given as follows

$$T_c = \frac{L_c}{N S_o} n_p$$

where  $L_c$  = actual length cut =  $L_w + A + O$  and

$A, O$  = approach and over run as shown

$N$  = spindle speed, rpm

$S_o$  = feed (tool), mm/rev

$n_p$  = number of passes required

Speed,  $N$ , is determined from cutting velocity,  $V_c$

$$V_c = \rho D_1 N$$

Giving

$$N = \frac{V_c}{\rho D_1}$$

The number of passes,  $n_p$  is determined from,

$$n_p = \frac{D_1 - D_2}{2t}$$

where,  $t$  = the depth of cut per pass. Combining the above gives

$$T_c = \frac{\rho D_1 L_c (D_1 - D_2)}{2 V_c S_o t}$$

Material removal rate is the volume of material removed per unit time and is given by

$$MRR = \frac{L_w \rho (D_1^2 - D_2^2)}{4T_c}$$

The power required for cutting,  $P$ , is given by

$$P = MRR U_s$$

where  $U_s$  is the specific energy for cutting titanium.

Computation of the above gives:-

$$T_c = 424 \text{ s}$$

$$MRR = 731.4 \text{ mm}^3\text{s}^{-1}$$

$$P = 2.56 \text{ kW}$$

*This approach has limited accuracy due to the fact that the calculation of tangential cutting velocity  $V_c$ , is based on average  $D_{avg}$ , or  $(D_1+D_2)/2$ . The error associated with this approach scales with the value of  $D_1-D_2$ . This is not a problem for finishing cuts, although large scale roughing cuts will see variations in machining times from those calculated as the tangential velocity reduces as  $D_2$  reduces, serving to increase machining times. In order to correct this it would be necessary to develop an equation for  $V_c$  as a function of radius (non-trivial), or to segment the calculations by developing solutions across a number of discrete steps when turning, thereby minimising the error in cutting velocity. In reality however, as the tangential velocity reduces, so does the work piece diameter and the cut path length. This has the effect of offsetting the error in  $V_c$ .*

#### Markers Comments

Although there were some detailed answers in part a), many candidates failed to discuss the main sources of error and provided limited descriptions, often only citing thermal errors or those attributed to the machine tool construction materials. Low scoring candidates in part ii) gave answers that simply cited deflection of the cutting tool, rather than developing the discussion to include the adverse effects of environment, tool materials, and cutting parameters. There was a mixed response to part b), with high scoring candidates giving a comprehensive discussion of work-piece properties, and tool-material interactions, in relation to the success of machining operations. Whilst low scoring candidates offered limited discussion of material properties such as hardness as being the principal influence of machining performance. In part b-ii) high scoring candidates were able to give a comprehensive discussion on the criteria for successful machining operations. Part c) challenged most candidates, and many were unable to answer this section in sufficient detail to obtain high marks. Answers were lacking in knowledge of Ti and its physical properties that can lead to difficulties in machining it. Similarly, knowledge of cutting tool properties and the names of specific cutting tools materials were often not given. Part d, despite being a rather simple question, few candidates were able to develop the analysis to a satisfactory level. Mistakes were made in determining the number of passes necessary to reach the final dimension, and many forgot to include the indexing travel as part of their machining time calculations. Candidates often chose to use the average work piece diameter, although they were not penalised since the approach was valid.

Question-2

a)

i) With the aid of an appropriate diagram identify the principal forces acting on the work piece during an orthogonal machining operation and describe what is meant by each force. (Define the cutting ratio,  $r$ , shear plane angle,  $\phi$ , and rake angle,  $\alpha$ .)

ii) Which of the principal forces can be measured and how you would measure them?

iii) Why is it important to have knowledge of the force components in machining operations?

[35%]

b) Using your knowledge of the orthogonal cutting process, derive the following expression:

$$r = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

where  $r$  is the cutting ratio,  $\phi$  is the shear plane angle, and  $\alpha$  is the rake angle.

[15%]

c) In two tests on the turning of Al 7050, a number of process parameters were recorded as shown in Table 1. In both tests the rake angle was set at  $8^\circ$ , the cutting velocity was  $1500 \text{ m min}^{-1}$ , and the width of the cut was 2 mm. The density of the material is  $0.0028 \text{ g mm}^{-3}$ .

(i) For each test, calculate cutting ratio  $r$ , the shear plane angle  $\phi$ , the shear force  $F_s$ , and the power consumed  $W$ .

(ii) Comment on the sensitivity of the power requirements to the parameters,  $r$ , and  $\phi$ .

[50%]

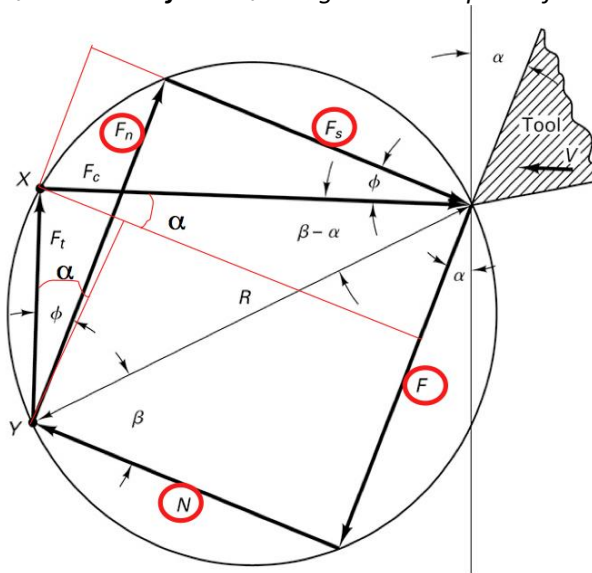
Table.1 Cutting conditions for orthogonal cutting of Al 7050

Test No:	Feed rate (mm/rev)	Chip Length (mm)	Chip Weight (g)	$F_c$ (N)	$F_t$ (N)
1	0.05	87.85	0.04	101	32
2	0.15	168.06	0.18	250	41

Question-2 CRIB

a)

i) The forces acting during an orthogonal machining operation are shown in the figure below. Assume that the **result force R** acting on the back of the chip is equal and opposite to the **resultant force R'** acting on the shear plane. **R** is composed of **friction force F** and **normal force N** acting on tool-chip interface contact area. **R'** is composed of a **shear force F<sub>s</sub>** and **normal force F<sub>n</sub>** acting on the shear plane area A<sub>s</sub>. **R** is also composed of **cutting force F<sub>c</sub>** and **thrust force F<sub>t</sub>** acting on tool-chip interface contact area.



$r$ , is the cutting ratio, or chip thickness compression ratio. It is an important parameter which provides information on the rate of plastic deformation as is given as

$$r = \frac{t_o}{t_c}$$

where  $t_o$  is the undeformed chip thickness, cut depth, or feed, and  $t_c$  is the deformed chip thickness.  $\phi$  is the shear plane angle and is the angle to the horizontal in which the material undergoes shear.  $\alpha$  is the rake angle, and is the angle the cutting tool face makes with the surface normal.

ii) None of the four force components  $F$ ,  $N$ ,  $F_s$ , and  $F_n$  can be directly measured in a machining operation, because the directions in which they are applied vary with different tool geometries and cutting conditions. However, it is possible for the cutting tool to be instrumented using a force measuring device called a **dynamometer**, so that two additional force components acting against the tool can be directly measured: **cutting force**,  $F_c$ , and **thrust force**,  $F_t$ . The cutting force  $F_c$  is in the direction of cutting, the same direction as the cutting speed  $v$ , and the thrust force  $F_t$  is perpendicular to the cutting force and is associated with the chip thickness before the cut  $t_o$ .

iii)

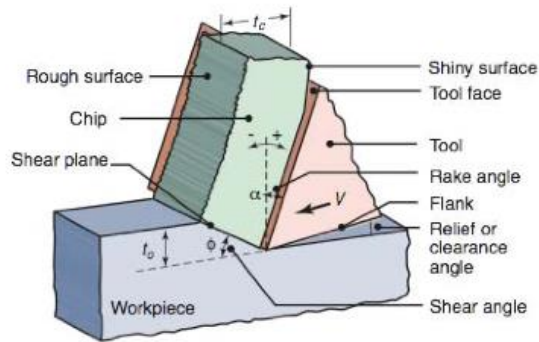
Cutting is a process producing considerable stresses and plastic deformations. The high compressive and frictional contact stresses on the tool face result in a substantial cutting forces. Knowledge of the force components is essential for the following reasons:

1. proper design of the cutting tools
2. proper design of the fixtures used to hold the workpiece and cutting tool
3. calculation of the machine tool power
4. selection of the cutting conditions to avoid an excessive distortion of the workpiece

b)

The chip thickness compression ratio or cutting ratio  $r$  is defined as

$$r = \frac{t_o}{t_c}$$



where  $t_o$  is the undeformed chip thickness, cut depth, or feed, and  $t_c$  is the deformed chip thickness. Let  $l_s$  be the length of the shear plane, and from the fig above, it follows that.

$$t_o = l_s \sin f$$

and

$$t_c = l_s \cos(f - a)$$

Thus,

$$r = \frac{l_s \sin f}{l_s \cos(f - a)} = \frac{\sin f}{\cos(f - a)}$$

c)

i) The question can be answered as follows:

Determine the cutting ratio,  $r$ . Assuming volume is constant during plastic deformation, and chip width is constant, one can determine the average deformed chip thickness,  $t_c$ , from the volume of the chip produced. This can be written as

$$t_c = \frac{W_c}{rbL}$$

The cutting ratio,  $r$ , can be determined from

$$r = \frac{t_o}{t_c}$$

ii) Once,  $r$ , is determined,  $\phi$  can be calculated as follows,

Since

$$\sin f = r \cos(f - a)$$

we wish to establish an expression for the shear angle,  $\phi$ , in terms of  $r$  and  $\alpha$ . The  $\cos(f - a)$  term can be transformed using an angle difference trigonometric identity to,

$$\cos(f - a) = \cos f \cos a + \sin f \sin a$$

Therefore,

$$\sin f = r(\cos f \cos a + \sin f \sin a)$$



dividing through by  $\cos f$  yields

$$\tan f = r(\cos a + \tan f \sin a)$$

grouping the coefficient of  $\tan f$  gives

$$\tan f(1 - r \sin a) = r \cos a$$

$$\tan f = \frac{r \cos a}{1 - r \sin a}$$

iii) From the force diagram it is possible to determine  $F_s$  as follows

$$F_s = F_c \sin f + F_t \cos f$$

iv) The power consumed in cutting can be written as

$$P = P_s + P_f = P_c$$

where  $P_s$  is that consumed in shearing,  $P_f$  is that consumed in friction. This can be lumped into the term  $P_c$  which is given by  $P_c = F_c \times V$ , where  $V$  is the cutting velocity.

Computation of the above gives

Feed rate	Cutting Ratio $r$	Shear Angle $\phi$	Shear Force $F_s$	Power
0.05 mm	0.69	36.8°	87 N	2.5kW
0.15 mm	0.79	40.1°	191 N	6.25kW

Comments.

In trial-1 the feed rate in this case is quite small, which has resulted in a relatively large shear plane angle producing a low shear plane area and therefore low shear force. This conclusion could also be drawn from the fairly large cutting ratio. The consequence of this is the relatively low cutting force, low power consumption of 2.5kW, and a low temperature in the cutting zone, all of which offers easy machining. Things are quite different with the greater feed depth. The shear angle has increased slightly, with much greater shear force due to the larger shear plane area that has resulted in greater cutting force, and more power consumption. This rate of 0.15 mm/rev, which is not large for Al alloys, suggests that a general purpose machine for a wider range of materials would require much greater power.

Markers Comments

The candidates demonstrated a good understanding of the orthogonal cutting model and were well equipped to gain high marks in section a). Very few candidates were able to give an account of the forces that can be measured, and even fewer were able to discuss the use of dynamometers to measure  $F_c$  and  $F_t$ . In part b) the majority of candidates were able to derive the expression for the cutting ratio  $r$ , although those that gave weak accounts of the forces in part i) were often incapable of developing the correct expression. Some candidates chose to use the velocity vector approach instead of the force vector approach described here. Part c) was a challenge for many candidates. Those that were able to spot that determining the cutting ratio,  $r$ , could be achieved through the assumption that volume is constant during plastic deformation, and that chip width was constant, which would allow them to determine the average deformed chip thickness,  $t_c$ , were well on the way to a high scoring mark. Many candidates missed this simple sign-post and failed to score many points as they struggled to find  $r$ , then  $f$ .

## SECTION B

3 (a) The IEC 61131-3 standard specifies five different approaches to PLC programming.

(i) Why is it important to have standardised approaches to PLC programming? [10%]

(ii) Contrast three of the approaches specified by the IEC 61131-3 standard, identifying the main features and advantages in each case. [15%]

(b) Petri nets and other modelling tools are often used in conjunction with PLC programming approaches. Why is PLC programming on its own generally insufficient for planning and developing a PLC-based control system for a complex manufacturing cell?

[15%]

(c) An automated production cell is being developed to produce a simple gear box. It will comprise two machining stations machining cover parts A and B respectively and an assembly station in which two gears C and D are inserted into cover part A and then cover part B is screwed into place to complete the gear box. There are three buffer storage areas: one for the blanks for parts A and B, a WIP buffer for the machined parts A and B and a further buffer for the complete gearboxes. Each has a limited capacity. A single robot manages the movement of parts between the buffers and the machining and assembly stations.

(i) Describe a Petri Net based modeling approach that you could use to plan the control logic for this system as a precursor to generating a suitable PLC programme. Comment on how your approach applies to the specific features of this production cell. [20%]

(ii) Indicate how you could ensure that the control logic allows no more than five components at any time in the WIP buffer and further ensures that the material handling robot operates without deadlock. [20%]

(iii) What additional factors need to be considered when a Petri Net model of a process is used to generate PLC programme code for automatic control? [20%]



### Solution – Question 3

a)

(i)

Numerous reasons including:

- enables re use of code/code modules/code libraries
- simplifies the task of plc programmers
- enables tools for developing and analysing PLC programmes to be developed
- enables multiple manufacturers PLC devices to be used
- simplifies portability of code

(ii)

- ladder logic: programming as a ladder of logic steps: simple, visual and intuitive
- SFC: programming as a graphical flow chart: visually appealing, follows flow chart logic, allows nesting
- Function block diagram: extending ladder like approach by adding libraries of functions: easy reuse, allows nesting
- Structured text: programming language approach: similar to standard programming languages, easy for programmers to use
- Instruction list: low level assembler like code: efficient coding

b)

- Difficult to fully capture complex logic
- No tools to analyse properties of logic plan [feasibility, deadlock, reachability]
- No systematic approach
- Identify causal/concurrent relationships between operations
- Identify possible conflicts between resource movements
- Identify possible conflicts / priorities for multiple part productions

c)

(i)

In lectures, one specific approach was suggested [below] although it is reasonable for students to suggest variants on this.

#### APPROACH

1. Determine key processing steps/states
2. Determine resources/equipment required
3. For each resource specify: Triggers [control inputs] operations pre req constraints
4. Develop PN model to reflect key processing steps
5. Introduce materials handling steps into PN from 4.
6. Identify allowable states for each resource and develop a PN for that resource

7. Check each PN in 5., 6. to ensure causality, conflict and concurrency conditions and that any upper bounds on capacity are reflected.
8. Combine PNs from 5., 6. to form an overall system PN.

*Some further information on sensing available would be required.*

*Good solutions will*

- a) describe why these different steps might be needed
- b) map each step of this approach onto the specific set of operations to be modeled in this question

*(ii)*

*Buffer limit: include a place to represent "spaces available" in the buffer at any given time. An arc from this place can be used as a limitation on a transition to move more parts to the buffer.*

*Deadlock: ensure that any operation that is "uninterruptible" such as moving an item from A to B is not constrained at its arrival point by any external requirements. These requirements need to be in place before the uninterruptible operation begins.*

*A good solution will include some diagrams to illustrate this*

*(iii)*

*Factors will include issues such as*

- *Checking overall logic of the PN model for issues such as a deadlock, limits, reachability*
- *Interface with the physical environment: sensing and actuation links to the production cell.*
- *Communications enhancement: ensure that handshaking processes are in place for effective confirmation of instructions between PLC and machines.*
- *Ensuring logic is adequate for continuous operation: allowing system to return to initial state*
- *Conversion of PN model to PLC Code*
- *I/O mapping of the physical devices with PLC*

**Examiners Notes:**

In the first part, most students answered the general question on the role of standards in PLC programming but a number had trouble in differentiating between the role and benefits of different standard approaches. The second section, involving a petri net model as the basis for cell automation was generally well attempted. Few students followed the detailed procedure discussed in lectures but most were able to generate a good model of the system and identify inputs and outputs to the physical environment. Issues of deadlock and limited buffer sizes were also covered well.

4. A corrective maintenance strategy is currently used for a particular type of machine tool component in a factory, but the factory manager is interested in adopting a preventative maintenance strategy.

(a) What are the factors that must be taken into account when deciding between preventative maintenance and corrective maintenance strategies?  
[10%]

The factory manager collected historical data on 1500 failures of that type of component. Analysis indicated that the time to failure for the component type was distributed as shown in Table 1.

Time to failure	0-1 months	1-2 months	2-3 months	3-4 months
No of Failed Components	250	250	500	500

Table 1

The manager also obtained the following cost data. The total cost of preventive replacement of the component,  $c_p$ , is £100. The total cost of failure replacement,  $c_f$ , is £200, which includes a penalty cost.

(b) Assume that the time taken to replace the component is negligible and let  $f(t)$  be the probability density function of the failure times of the component and  $F(t)$  be the cumulative probability function of the failure times of the component.

(i) Clearly explaining your derivations, develop an expression for the total expected replacement cost per unit time,  $c$ , as a function of replacement age,  $t_p$ ,

(ii) Calculate the optimal replacement age,  $t_{po}$ , for this component type. State any assumptions you make [60%]

(c) Assume now that the time taken for preventive replacement of the component is in fact  $T_p$  and the time taken for replacement after failure is  $T_f$ .

(i) Explain how this would affect your answers in b).

(ii) Under what circumstances would you expect replacement times  $T_p$  and  $T_f$  to significantly influence the optimal replacement age for a component? [30%]

**Solution to Question 4**

- a) Corrective or reactive maintenance is where the component is allowed to run until it fails. Preventive maintenance aims to reduce the risk of failure by replacing the component at regular intervals (either based on its age or usage).
- The primary factor that determines the feasibility of preventive maintenance is the tradeoff between the cost of preventive replacement vs. the risk of failure (i.e., probability of failure x cost of failure replacement). The failure replacement cost is often much higher (as can be seen in this case) than preventive replacement since a failure would result in machine downtime, loss of production, inability to meet customer due-dates etc.
  - In addition, if a reactive maintenance strategy is adopted, there is often the need to carry spares of the component to minimise the replacement time when it fails – incurring addition inventory management costs.
  - Furthermore, machine failure can sometimes pose a safety hazard to the operators.
  - Finally, a production line that suffers from a lot of failures means that the production manager will spend more of his time “fire-fighting” rather than improving the efficiency of the line. However, not all components and equipment require a preventive maintenance strategy.
  - The cost and effort required to plan and implement a preventive maintenance programme must be taken into consideration, and compared with the overall risk of machine failure to the business.
- b) The objective here is to determine the optimal replacement age of the component to minimise the total expected cost of replacement per unit time.

$c_p$  is the cost of preventive replacement

$c_f$  is the cost of failure replacement

$f(t)$  is the probability density function of the failure times of the component

$F(t)$  is the cumulative probability function of the failure times of the component

(i) The trade-off between cost of preventive replacement and the risk of component failure (both calculated per unit time) is shown in Figure 1.

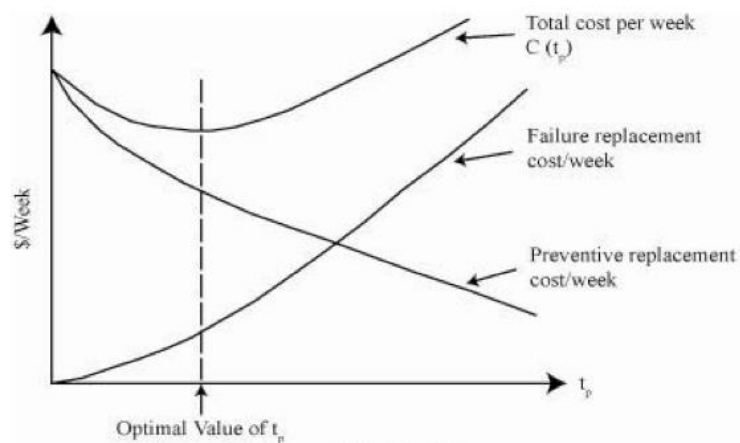


Figure 1

The replacement policy is to perform a preventive replacement when the component has reached a specific age,  $t_p$ , plus failure replacements when necessary. This policy is illustrated in Figure 2.

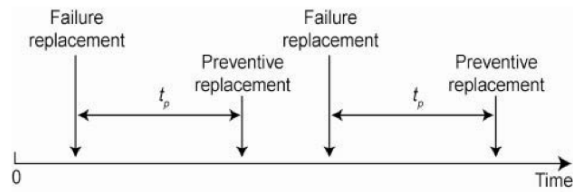


Figure 2

In this problem, there are two possible cycles of operation: one cycle determined by the component reaching its planned replacement age,  $t_p$ , and the other determined by the component failing before the replacement time. The two possible cycles are shown in Figure 3.

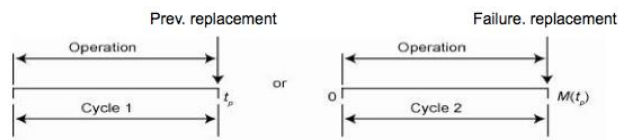


Figure 3

The total expected replacement cost per unit time,  $C(t_p)$ , is

$$C(t_p) = \frac{\text{Total expected replacement cost per cycle}}{\text{Expected cycle length}}$$

Total expected replacement cost per cycle

$$= \text{cost of a preventive cycle} \times \text{probability of a preventive cycle} \\ + \text{cost of a failure cycle} \times \text{probability of a failure cycle} \\ = c_p \times R(t_p) + c_f \times [1 - R(t_p)] \text{ where } R(t) = 1 - F(t)$$

Expected cycle length

$$= \text{length of a preventive cycle} \times \text{probability of a preventive cycle} \\ + \text{expected length of a failure cycle} \times \text{probability of a failure cycle}$$

If we denote the expected length of a failure cycle by  $M(t_p)$ , then we have:

$$\text{Expected cycle length} = t_p \times R(t_p) + M(t_p) \times [1 - R(t_p)]$$

The total expected replacement cost per unit time,  $C(t_p)$ , is then given by:

$$C(t_p) = \frac{c_p \times R(t_p) + c_f \times [1 - R(t_p)]}{t_p \times R(t_p) + M(t_p) \times [1 - R(t_p)]}$$

(ii) For the given problem,  $c_p = £100$ ;  $c_f = £200$ . Calculation of  $M(t_p)$  is often the trickiest part of the problem! A “centre of gravity” approach is the best to take.



i.e.  $M(t) = (f(t_1)*0.5+f(t_2)*1.5+....) / (f(t_1)+f(t_2)+....)$

The table below shows the calculations required to be done:

$t_p$	$f(t_p)$	$F(t_p)$	$R(t_p)$	$M(t_p)$	Expected cost per cycle	Expected cycle length	$C(t_p)$
1 month	0.167	0.167	0.833	0.5 months	£116.67	0.92 months	£126.82
2 months	0.167	0.333	0.667	1 month	£133.33	1.67 months	£79.83
3 months	0.333	0.666	0.333	1.75 months	£166.67	2.17 months	£76.80
4 months	0.333	1	0	2.33 months	£200	2.33 months	£85.83

Hence, the optimal replacement age is 3 months.

c)

(i) The optimal preventive replacement age of the component is again taken as that age which minimises the total expected cost of replacements per unit time. However, the difference here is the calculation of the expected cycle length. The two possible cycles in this case are shown in Figure 4.

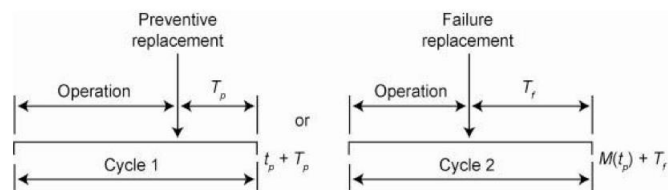


Figure 4

Expected cycle length

= length of a preventive cycle  $\times$  probability of a preventive cycle  
 + expected length of a failure cycle  $\times$  probability of a failure cycle

$$= [t_p + T_p] \times R(t_p) + [M(t_p) + T_r] \times [1 - R(t_p)]$$

The total expected replacement cost per unit time,  $C(t_p)$ , is then given by:

$$C(t_p) = \frac{c_p \times R(t_p) + c_f \times [1 - R(t_p)]}{[t_p + T_p] \times R(t_p) + [M(t_p) + T_r] \times [1 - R(t_p)]}$$

(ii)

- Note that the cost equation derived in part (c)(i) is composed of two components – the preventive cost per unit time and the risk of failure per unit time

- *Refer to figure 1 - the solution is to balance the two components.*
- *The key here is to realise that the time required for replacement will have an influence on the cost of maintenance (planned as well as unplanned).*
- *The time required for failure replacement is normally higher than that required for planned replacement.*
- *The rate of increase cost of unplanned replacement will normally increase with time, as penalties for missing deadlines and cost of lost sales accumulate.*
- *Keeping the preventive maintenance time (and hence cost) constant, if we increase the unplanned maintenance time (and hence cost), it is easy to see that the preventive maintenance cost per unit time will decrease whereas the risk of unplanned maintenance will increase.*
- *It is therefore clear that when  $T_f \gg T_p$  this would significantly affect cost and replacement age and preventive replacement will become increasingly attractive to do at a "younger age".*
- *This also indicates how important it is to carry more spares when a corrective maintenance strategy is adopted in order to minimise the time required for replacement (and hence the cost).*

Examiners Notes:

This question asked the students to contrast reactive and preventive maintenance methods. Responses to the broad comparison of the two approaches were done well. The calculations for an optimal replacement time for a particular component were mixed. Most students managed a significant part of this development although only a few managed to complete the whole question correctly. The final part of the question in which the actual replacement times for the components we included was generally well done and the impact on the result from the previous calculation understood.