MET2
MANUFACTURING ENGINEERING TRIPOS PART IIA

Tuesday 4 May 2021 9.00 to 12.10

Paper 3

MODULE 3P4: OPERATIONS MANAGEMENT
(SECTION A)

MODULE 3P5: INDUSTRIAL ENGINEERING
(SECTION B)

Answer all questions from sections ***A*** *and* ***B****.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Write on single sided paper.
You may type your answers.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
You are allowed access to the electronic version of the Engineering Data Books.
3P5 Data Sheet

10 minutes reading time is allowed for this paper at the start of the exam.
The time taken for scanning/uploading answers is 30 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

SECTION A

1 (a) Consider the following modification of the EOQ problem. A company orders 2 items from their supplier. Let D_i be the annual demand, K_i the fixed ordering cost, and h_i inventory holding cost per unit per time for item i=1,2. It is more convenient to order both items at the same time.

(i) What is the total cost function for the company? [20%]

(ii) Find the optimal order quantity and discuss how it differs from the values calculated with the EOQ formula. [20%]

(b) EOQ comes with very rigid assumptions. Discuss how one can update the inventory policy when we relax some of the assumptions as follows:

(i) demand is stochastic; [20%]

(ii) backorders are allowed; [20%]

(iii) costs are not known for sure. [20%]

2 (a) A hard-drive repair facility is organised such that each incoming hard-drive undergoes an initial registration process. Each fault is then diagnosed by a technician, and is given either a software upgrade, or a complete refurbishment. Currently, 50 hard-drives per hour arrive at the facility, 10% of which are sent for refurbishment. On average, 30 hard-drives are waiting to be registered and 40 are registered and waiting to be seen by the repair technician. The registration process takes, on average, 2 minutes per hard-drive. The repair technician spends, on average, 5 minutes for hard-drives which receive software upgrades and 30 minutes for those that require refurbishment.

(i) On average, how long does a hard-drive stay in the repair facility? [15%]

(ii) On average, how many hard-drives are being examined by the technician at any given time? [10%]

(iii) On average, how many hard-drives are in the repair facility? [15%]

(b) Discuss the advantages and disadvantages of using an MRP system. [30%]

(c) Is *just-in-time* (JIT) applicable to services? If not, why not? If so, how? [30%]

SECTION B

3 (a) A worker operates a machine in an environment with a background noise of 83 dBA. The machine produces an additional noise, also of 83 dBA.

(i) What is the maximum length of time the worker could operate the machine to ensure the daily personal noise exposure level does not exceed 85 dBA? [20%]

(ii) Describe and justify three approaches that could be taken to allow the worker to operate the machine for a full 8 hour shift. [20%]

(b) A production machine is scheduled to run for 10 shifts per week. Each shift lasts for 7.5 hours. The standard cycle time for producing a component on this machine is 5 minutes. Calculate the OEE for this machine for a given week, considering the following information:

- due to a breakdown, the machine was not operational for 3 hours;
- the machine operator was not available on one of the ten scheduled shifts;
- the average scrap rate was 2%;
- whilst running, the machine was producing 10 components an hour. [20%]

State any assumptions you make.

(c) A direct time study was performed for a task. The regular cycle included four elements, *a*, *b*, *c* and *d*. Elements *e* and *f* are irregular elements performed every six and four cycles respectively. The performance rating for each element is shown in Table. 1.

Table 1

Work element	*a*	*b*	*c*	*d*	*e*	*f*
Observed time (min)	0.50	0.25	0.58	0.72	0.85	1.01
Performance rating	80%	100%	95%	100%	100%	95%

(i) Calculate the basic time and the standard time for the cycle using an allowance factor of 17%. [20%]

 (cont.

(ii) It is planned that element *e* of this task will be replaced by element *g*. The observed time of element *g* is estimated to be 0.32 min and the performance rating will be 96%. Calculate the basic time and the standard time for the cycle using the same allowance factor of 17%. [20%]

4 (a) A workshop is lit by 50 lights, positioned at a height of 5 metres. Calculate how many lights can be removed, while maintaining the same level of luminance, if lights are repositioned at a height of 3 meters instead? [20%]

(b) You have just taken over as the cell leader for a poorly performing production cell. Scrap levels are high, mainly due to careless mistakes by operators. Efficiency is low, due to high product variation, high work-in-progress and poor raw materials control. The factory manager has instructed you to carry out a *5S* implementation and to develop a six-month improvement plan based on the principles of the *Toyota Production System*.

(i) Explain the general set of activities that are involved in your 5S implementation for this production cell. [40%]

(ii) Describe in detail the three initiatives that would underpin your six-month Toyota Production System improvement plan. [40%]

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MET2A 3P5 Data Sheet

Energy expenditure

Table 1: Recommended Energy Expenditure

Physiological measure	Male worker	Female worker
Energy expenditure rate of the physical activity (maximum time-weighted average during shift) $\overline{ER}_m$	5.0 kcal/min	4.0 kcal/min
Energy expenditure of the physical activity for the entire 8 hr shift ER_{8h}	2400 kcal	1920 kcal
Heart rate (maximum time-weighted average during shift) $\overline{HR}_m$	120 beats/min	110 beats/min

Table 2: Typical energy expenditure for a person weighing 72Kg

Activity	Energy Expenditure (ER)[1]
Sleeping	BMR_m
Resting Seated	1.5 kcal/min
Standing (not walking)	2.2 kcal/min
Walking at 4.5 km/hr	4.0 kcal/min
Jogging at 7.2 km/hr	7.5 kcal/min
Mowing lawn (push mower)	8.3 kcal/min
Office work seated	1.6 kcal/min
Light assembly work seated	2.2 kcal/min
Bricklaying	4.0 kcal/min
Shoveling coal	8.5 kcal/min

[1] If a person's weight is W, multiply ER by $W/72$

Noise

Sound intensity is measured relative to a reference pressure and converted to logarithmic scale called sound pressure level (SPL) with units of decibel (dB):

$$SPL = 10\log_{10}\frac{p_s^2}{p_r^2} = 20\log_{10}\frac{p_s}{p_r}$$

where p_s = sound pressure from source, N/m^2, and p_r = reference sound pressure, N/m^2 (the usual reference pressure is 0.00002 N/m^2)

The noise regulations are based on an average SPL over an 8-hour shift.

For SPL above 80dBA, hearing protection must be available on request, above 85dBA it must be used.

Light

Illuminance - luminous flux shining per unit area on a surface, measured in lux.

$$E = \frac{l}{d^2}$$

where E = illuminance, l = luminous intensity, and d = distance from the light source.

When incident light is at an angle

$$E = \frac{l}{d^2}\cos\theta$$

Visual acuity (VA) – capability to discriminate small objects or fine details.

$VA = 1/\alpha_v$ where α_v is measured in arc min.

Normal Vision VA=1

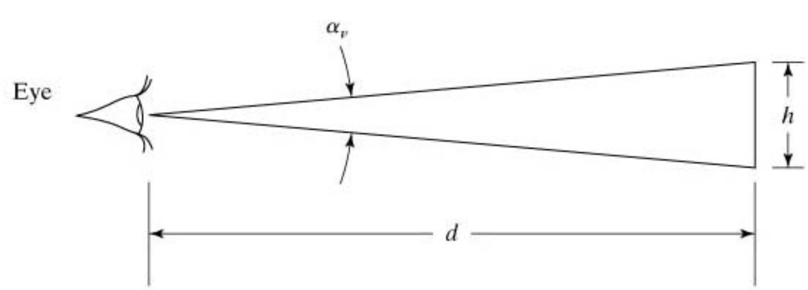

Table 9.2 Illuminance recommended by the IES for industrial tasks (adapted from Kaufman and Christensen, 1984)

Type of task	Range* of illuminance (lux)
Workplaces where visual tasks are only occasionally performed	100–200
Visual tasks of high contrast or large size: printed material, rough bench and machine work, ordinary inspection	200–500
Work at visual display terminals for exended periods of time†	300–500
Visual tasks of medium contrast or small size, e.g. pencilled handwriting, difficult inspection, medium assembly	500–1000
Visual tasks of low contrast or very small size, e.g. handwriting in hard pencil on poor-quality paper, very difficult inspection	1000–2000
Visual tasks of low contrast and very small size over a prolonged period, e.g. fine assembly, highly difficult inspection	2000–5000
Very prolonged and exacting visual tasks, e.g. extra-fine assembly, the most difficult visual inspection	5000–10 000

*The upper values in the range are for individuals aged over 55 years and the lower values are for individuals younger than 40 years.

†This recommendation is from ANSI/HFS 100 (Human Factors Society, 1988).

Luminance (L) measures the amount of light reflected from a surface

$$L = \frac{E \cdot R}{\pi}$$

where E = Illuminance, R = Reflectance

Weber Contrast is expressed by the ratio:

$$\frac{L_{object} - L_{background}}{L_{background}}$$

Object	Typical Reflectance
Mirrored glass	0.85
White matt paint	0.82
Light green paint	0.65
Aluminium paint	0.65
Medium blue paint	0.35
Dark blue paint	0.08
Black paint	0.04

Learning Curves

Learning Curves have the form $y = kx^m$

$y =$ time/unit, $k =$ constant representing the value of the time for the first work cycle,

$x =$ number of work units completed, $m =$ a constant related to the rate of learning. (Note m is negative)

Reliability

Let T be a random variable defining the lifetime of a component with distribution function $F(t)$, and let $f(t)$ be its probability density function (PDF).

The reliability function is given by

$$R(t) = P(T > t) = 1 - F(t) = 1 - \int_0^t f(t')dt'$$

Failure rate or hazard rate:

$$h(t) = \frac{f(t)}{R(t)}$$

Mean Time to Failure:

$$MTTF = \int_0^\infty t \cdot f(t)dt$$

Maintenance

Dobson's table for age-based preventive replacement policy assuming a Weibull lifetime distribution with shape and scale parameters β and η respectively.

Optimal interval $= m \cdot \eta$

Table 1. Values of *m*

	β							
C_f/C_p	**1.5**	**2.0**	**2.5**	**3.0**	**4.0**	**5.0**	**7.0**	**10.0**
2.0	2.229	1.091	0.883	0.810	0.766	0.761	0.775	0.803
2.2	1.830	0.981	0.816	0.760	0.731	0.733	0.755	0.788
2.4	1.579	0.899	0.764	0.720	0.702	0.711	0.738	0.777
2.6	1.401	0.834	0.722	0.688	0.679	0.692	0.725	0.766
2.8	1.265	0.782	0.687	0.660	0.659	0.675	0.713	0.758
3.0	1.158	0.738	0.657	0.637	0.642	0.661	0.702	0.749
3.3	1.033	0.684	0.620	0.607	0.619	0.642	0.687	0.739
3.6	0.937	0.641	0.589	0.582	0.600	0.627	0.676	0.730
4.0	0.839	0.594	0.555	0.554	0.579	0.609	0.662	0.719
4.5	0.746	0.547	0.521	0.526	0.557	0.591	0.648	0.708
5	0.676	0.511	0.493	0.503	0.538	0.575	0.635	0.699
6	0.574	0.455	0.450	0.466	0.509	0.550	0.615	0.683
7	0.503	0.414	0.418	0.438	0.486	0.530	0.600	0.671
8	0.451	0.382	0.392	0.416	0.468	0.514	0.587	0.661
9	0.411	0.358	0.372	0.398	0.452	0.500	0.575	0.652
10	0.378	0.337	0.355	0.382	0.439	0.488	0.566	0.645
12	0.329	0.304	0.327	0.357	0.417	0.469	0.550	0.632
14	0.293	0.279	0.306	0.338	0.400	0.454	0.537	0.621
16	0.266	0.260	0.288	0.323	0.386	0.441	0.526	0.613
18	0.244	0.244	0.274	0.309	0.374	0.430	0.517	0.605
20	0.226	0.230	0.263	0.298	0.364	0.421	0.508	0.598
25	0.193	0.205	0.239	0.275	0.343	0.402	0.492	0.584
30	0.170	0.186	0.222	0.258	0.328	0.387	0.478	0.573
35	0.152	0.172	0.207	0.245	0.315	0.374	0.468	0.564
40	0.139	0.160	0.197	0.234	0.304	0.364	0.459	0.557
45	0.128	0.151	0.187	0.225	0.295	0.356	0.451	0.550
50	0.119	0.143	0.179	0.217	0.288	0.348	0.444	0.544
60	0.105	0.130	0.167	0.204	0.274	0.335	0.432	0.534
70	0.095	0.120	0.157	0.193	0.264	0.325	0.422	0.526
80	0.087	0.112	0.148	0.185	0.255	0.316	0.415	0.518
90	0.080	0.106	0.141	0.177	0.248	0.309	0.407	0.513
100	0.074	0.101	0.135	0.172	0.241	0.303	0.402	0.507
150	0.057	0.082	0.115	0.150	0.217	0.278	0.379	0.487
200	0.047	0.071	0.103	0.136	0.203	0.263	0.363	0.472
300	0.035	0.058	0.087	0.119	0.182	0.243	0.343	0.454
500	0.025	0.045	0.071	0.100	0.161	0.219	0.319	0.431
1000	0.016	0.032	0.054	0.079	0.135	0.109	0.288	0.403

Source: BRYAN DODSON (1994) DETERMINING THE OPTIMUM SCHEDULE FOR PREVENTIVE MAINTENANCE, Quality Engineering, 6:4, 667-679

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \leq u \leq 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot 9^{2}0097$	$\cdot 9^{2}0358$	$\cdot 9^{2}0613$	$\cdot 9^{2}0863$	$\cdot 9^{2}1106$	$\cdot 9^{2}1344$	$\cdot 9^{2}1576$
2·4	$\cdot 9^{2}1802$	$\cdot 9^{2}2024$	$\cdot 9^{2}2240$	$\cdot 9^{2}2451$	$\cdot 9^{2}2656$	$\cdot 9^{2}2857$	$\cdot 9^{2}3053$	$\cdot 9^{2}3244$	$\cdot 9^{2}3431$	$\cdot 9^{2}3613$
2·5	$\cdot 9^{2}3790$	$\cdot 9^{2}3963$	$\cdot 9^{2}4132$	$\cdot 9^{2}4297$	$\cdot 9^{2}4457$	$\cdot 9^{2}4614$	$\cdot 9^{2}4766$	$\cdot 9^{2}4915$	$\cdot 9^{2}5060$	$\cdot 9^{2}5201$
2·6	$\cdot 9^{2}5339$	$\cdot 9^{2}5473$	$\cdot 9^{2}5604$	$\cdot 9^{2}5731$	$\cdot 9^{2}5855$	$\cdot 9^{2}5975$	$\cdot 9^{2}6093$	$\cdot 9^{2}6207$	$\cdot 9^{2}6319$	$\cdot 9^{2}6427$
2·7	$\cdot 9^{2}6533$	$\cdot 9^{2}6636$	$\cdot 9^{2}6736$	$\cdot 9^{2}6833$	$\cdot 9^{2}6928$	$\cdot 9^{2}7020$	$\cdot 9^{2}7110$	$\cdot 9^{2}7197$	$\cdot 9^{2}7282$	$\cdot 9^{2}7365$
2·8	$\cdot 9^{2}7445$	$\cdot 9^{2}7523$	$\cdot 9^{2}7599$	$\cdot 9^{2}7673$	$\cdot 9^{2}7744$	$\cdot 9^{2}7814$	$\cdot 9^{2}7882$	$\cdot 9^{2}7948$	$\cdot 9^{2}8012$	$\cdot 9^{2}8074$
2·9	$\cdot 9^{2}8134$	$\cdot 9^{2}8193$	$\cdot 9^{2}8250$	$\cdot 9^{2}8305$	$\cdot 9^{2}8359$	$\cdot 9^{2}8411$	$\cdot 9^{2}8462$	$\cdot 9^{2}8511$	$\cdot 9^{2}8559$	$\cdot 9^{2}8605$
3·0	$\cdot 9^{2}8650$	$\cdot 9^{2}8694$	$\cdot 9^{2}8736$	$\cdot 9^{2}8777$	$\cdot 9^{2}8817$	$\cdot 9^{2}8856$	$\cdot 9^{2}8893$	$\cdot 9^{2}8930$	$\cdot 9^{2}8965$	$\cdot 9^{2}8999$
3·1	$\cdot 9^{3}0324$	$\cdot 9^{3}0646$	$\cdot 9^{3}0957$	$\cdot 9^{3}1260$	$\cdot 9^{3}1553$	$\cdot 9^{3}1836$	$\cdot 9^{3}2112$	$\cdot 9^{3}2378$	$\cdot 9^{3}2636$	$\cdot 9^{3}2886$
3·2	$\cdot 9^{3}3129$	$\cdot 9^{3}3363$	$\cdot 9^{3}3590$	$\cdot 9^{3}3810$	$\cdot 9^{3}4024$	$\cdot 9^{3}4230$	$\cdot 9^{3}4429$	$\cdot 9^{3}4623$	$\cdot 9^{3}4810$	$\cdot 9^{3}4991$
3·3	$\cdot 9^{3}5166$	$\cdot 9^{3}5335$	$\cdot 9^{3}5499$	$\cdot 9^{3}5658$	$\cdot 9^{3}5811$	$\cdot 9^{3}5959$	$\cdot 9^{3}6103$	$\cdot 9^{3}6242$	$\cdot 9^{3}6376$	$\cdot 9^{3}6505$
3·4	$\cdot 9^{3}6631$	$\cdot 9^{3}6752$	$\cdot 9^{3}6869$	$\cdot 9^{3}6982$	$\cdot 9^{3}7091$	$\cdot 9^{3}7197$	$\cdot 9^{3}7299$	$\cdot 9^{3}7398$	$\cdot 9^{3}7493$	$\cdot 9^{3}7585$
3·5	$\cdot 9^{3}7674$	$\cdot 9^{3}7759$	$\cdot 9^{3}7842$	$\cdot 9^{3}7922$	$\cdot 9^{3}7999$	$\cdot 9^{3}8074$	$\cdot 9^{3}8146$	$\cdot 9^{3}8215$	$\cdot 9^{3}8282$	$\cdot 9^{3}8347$
3·6	$\cdot 9^{3}8409$	$\cdot 9^{3}8469$	$\cdot 9^{3}8527$	$\cdot 9^{3}8583$	$\cdot 9^{3}8637$	$\cdot 9^{3}8689$	$\cdot 9^{3}8739$	$\cdot 9^{3}8787$	$\cdot 9^{3}8834$	$\cdot 9^{3}8879$
3·7	$\cdot 9^{3}8922$	$\cdot 9^{3}8964$	$\cdot 9^{4}0039$	$\cdot 9^{4}0426$	$\cdot 9^{4}0799$	$\cdot 9^{4}1158$	$\cdot 9^{4}1504$	$\cdot 9^{4}1838$	$\cdot 9^{4}2159$	$\cdot 9^{4}2468$
3·8	$\cdot 9^{4}2765$	$\cdot 9^{4}3052$	$\cdot 9^{4}3327$	$\cdot 9^{4}3593$	$\cdot 9^{4}3848$	$\cdot 9^{4}4094$	$\cdot 9^{4}4331$	$\cdot 9^{4}4558$	$\cdot 9^{4}4777$	$\cdot 9^{4}4988$
3·9	$\cdot 9^{4}5190$	$\cdot 9^{4}5385$	$\cdot 9^{4}5573$	$\cdot 9^{4}5753$	$\cdot 9^{4}5926$	$\cdot 9^{4}6092$	$\cdot 9^{4}6253$	$\cdot 9^{4}6406$	$\cdot 9^{4}6554$	$\cdot 9^{4}6696$
4·0	$\cdot 9^{4}6833$	$\cdot 9^{4}6964$	$\cdot 9^{4}7090$	$\cdot 9^{4}7211$	$\cdot 9^{4}7327$	$\cdot 9^{4}7439$	$\cdot 9^{4}7546$	$\cdot 9^{4}7649$	$\cdot 9^{4}7748$	$\cdot 9^{4}7843$
4·1	$\cdot 9^{4}7934$	$\cdot 9^{4}8022$	$\cdot 9^{4}8106$	$\cdot 9^{4}8186$	$\cdot 9^{4}8263$	$\cdot 9^{4}8338$	$\cdot 9^{4}8409$	$\cdot 9^{4}8477$	$\cdot 9^{4}8542$	$\cdot 9^{4}8605$
4·2	$\cdot 9^{4}8665$	$\cdot 9^{4}8723$	$\cdot 9^{4}8778$	$\cdot 9^{4}8832$	$\cdot 9^{4}8882$	$\cdot 9^{4}8931$	$\cdot 9^{4}8978$	$\cdot 9^{5}0226$	$\cdot 9^{5}0655$	$\cdot 9^{5}1066$
4·3	$\cdot 9^{5}1460$	$\cdot 9^{5}1837$	$\cdot 9^{5}2199$	$\cdot 9^{5}2545$	$\cdot 9^{5}2876$	$\cdot 9^{5}3193$	$\cdot 9^{5}3497$	$\cdot 9^{5}3788$	$\cdot 9^{5}4066$	$\cdot 9^{5}4332$
4·4	$\cdot 9^{5}4587$	$\cdot 9^{5}4831$	$\cdot 9^{5}5065$	$\cdot 9^{5}5288$	$\cdot 9^{5}5502$	$\cdot 9^{5}5706$	$\cdot 9^{5}5902$	$\cdot 9^{5}6089$	$\cdot 9^{5}6268$	$\cdot 9^{5}6439$
4·5	$\cdot 9^{5}6602$	$\cdot 9^{5}6759$	$\cdot 9^{5}6908$	$\cdot 9^{5}7051$	$\cdot 9^{5}7187$	$\cdot 9^{5}7318$	$\cdot 9^{5}7442$	$\cdot 9^{5}7561$	$\cdot 9^{5}7675$	$\cdot 9^{5}7784$
4·6	$\cdot 9^{5}7888$	$\cdot 9^{5}7987$	$\cdot 9^{5}8081$	$\cdot 9^{5}8172$	$\cdot 9^{5}8258$	$\cdot 9^{5}8340$	$\cdot 9^{5}8419$	$\cdot 9^{5}8494$	$\cdot 9^{5}8566$	$\cdot 9^{5}8634$
4·7	$\cdot 9^{5}8699$	$\cdot 9^{5}8761$	$\cdot 9^{5}8821$	$\cdot 9^{5}8877$	$\cdot 9^{5}8931$	$\cdot 9^{5}8983$	$\cdot 9^{6}0320$	$\cdot 9^{6}0789$	$\cdot 9^{6}1235$	$\cdot 9^{6}1661$
4·8	$\cdot 9^{6}2067$	$\cdot 9^{6}2453$	$\cdot 9^{6}2822$	$\cdot 9^{6}3173$	$\cdot 9^{6}3508$	$\cdot 9^{6}3827$	$\cdot 9^{6}4131$	$\cdot 9^{6}4420$	$\cdot 9^{6}4696$	$\cdot 9^{6}4958$
4·9	$\cdot 9^{6}5208$	$\cdot 9^{6}5446$	$\cdot 9^{6}5673$	$\cdot 9^{6}5889$	$\cdot 9^{6}6094$	$\cdot 9^{6}6289$	$\cdot 9^{6}6475$	$\cdot 9^{6}6652$	$\cdot 9^{6}6821$	$\cdot 9^{6}6981$

Example: $\Phi(3{\cdot}57) = \cdot 9^{3}8215 = 0{\cdot}9998215$.

FRACTILES OF THE t DISTRIBUTION. $t_{1-P} = -t_P$.

P \ f	Probability in per cent									
	60	70	80	90	95	97·5	99	99·5	99·9	99·95
1	·325	·727	1·376	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	·289	·617	1·061	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	·277	·584	·978	1·638	2·353	3·182	4·541	5·841	10·22	12·94
4	·271	·569	·941	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	·267	·559	·920	1·476	2·015	2·571	3·365	4·032	5·893	6·859
6	·265	·553	·906	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	·263	·549	·896	1·415	1·895	2·365	2·998	3·499	4·785	5·405
8	·262	·546	·889	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	·261	·543	·883	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	·260	·542	·879	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	·260	·540	·876	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	·259	·539	·873	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	·259	·538	·870	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	·258	·537	·868	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	·258	·536	·866	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	·258	·535	·865	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	·257	·534	·863	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	·257	·534	·862	1·330	1·734	2·101	2·552	2·878	3·611	3·922
19	·257	·533	·861	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	·257	·533	·860	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	·257	·532	·859	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	·256	·532	·858	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	·256	·532	·858	1·319	1·714	2·069	2·500	2·807	3·485	3·767
24	·256	·531	·857	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	·256	·531	·856	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	·256	·531	·856	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	·256	·531	·855	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	·256	·530	·855	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	·256	·530	·854	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	·256	·530	·854	1·310	1·697	2·042	2·457	2·750	3·385	3·646
40	·255	·529	·851	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	·255	·528	·849	1·298	1·676	2·009	2·403	2·678	3·262	3·495
60	·254	·527	·848	1·296	1·671	2·000	2·390	2·660	3·232	3·460
80	·254	·527	·846	1·292	1·664	1·990	2·374	2·639	3·195	3·415
100	·254	·526	·845	1·290	1·660	1·984	2·365	2·626	3·174	3·389
200	·254	·525	·843	1·286	1·653	1·972	2·345	2·601	3·131	3·339
500	·253	·525	·842	1·283	1·648	1·965	2·334	2·586	3·106	3·310
∞	·253	·524	·842	1·282	1·645	1·960	2·326	2·576	3·090	3·291
$2(1-P)$	80	60	40	20	10	5	2	1	0·2	0·1

Example: $P\{t < 2{\cdot}086\} = 97{\cdot}5\%$ for $f = 20$.

$P\{|t| > t_P\} = 2(1-P)$. $P\{|t| > 2{\cdot}086\} = 5\%$ for $f = 20$.

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