Version SWP/5

MET3
MANUFACTURING ENGINEERING TRIPOS PART IIB

Tuesday 26 April 2022 9 to 12.10

Paper 1

Answer not more than ***four*** *questions.*

Answer each question in a separate booklet

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
8 page answer booklet x 4
Rough work pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Engineering Data Books.
Probability Tables for Question 5

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Elemental carbon exists in several forms, each of which has its own physical characteristics. Examples of this are graphite and carbon nanotubes (CNTs).

(i) Describe the structural differences and the resulting properties of graphite and CNTs. [10%]

(ii) Unlike CNTs, carbon fibres have been employed in engineering applications for many decades. Explain the differences between CNTs and carbon fibres. [10%]

(b) (i) Name three different methods used to manufacture CNTs and describe the advantages and disadvantages of each. [10%]

(ii) Describe the process steps for the manufacturing route most widely used to produce CNTs. [30%]

(c) (i) Highlight four applications of CNTs that make specific use of their unique electrical or mechanical attributes. [20%]

(ii) What are the future challenges that manufacturers face in order to expand the use of CNTs? [20%]

2 (a) (i) Describe what is meant by the term *biomimetics*. [10%]

(ii) Describe one example of a biomimetic material. Include in your description details of the material properties, how these properties are achieved, and why it is considered a biomimetic material. [25%]

(b) (i) Describe what is meant by the term *biopolymer*. [10%]

(ii) Explain the challenges faced when attempting to increase the use of biopolymers globally. [30%]

(c) A firm has decided to invest in building a chemical processing factory to make a specialised polymer for the medical device sector. Describe any 5 steps they will need to complete before they can fully cost a new factory. [25%]

(TURN OVER

3 A small machining company is contracted to machine a pulley component. The pulley component can be seen in Fig. 1a. The pulley is currently machined on a single turning centre at a rate of five parts per hour and is constantly supervised by a single operator. The turning centre is a two axes machine (X-, Z-axes), used for turning profiles on parts, with an additional axis (Y-axis) fitted with a live turret for milling features, none central to the chuck. It is also fitted with an automated door and chuck to allow automated operations. The turning centre can be seen in Fig. 1b. Part handling, material orientations, machining operations and cycle times can be seen in Fig. 2. The machining company wants to improve the production rate of the current solution, and proposes to introduce an additional turning centre and automation to reduce manual part handling tasks. The new turning centre has the same functionality as the existing one, except it is not fitted with a Y axis and live turret. Thus the new turning centre is not capable of performing Operation 3 (see Fig. 2).

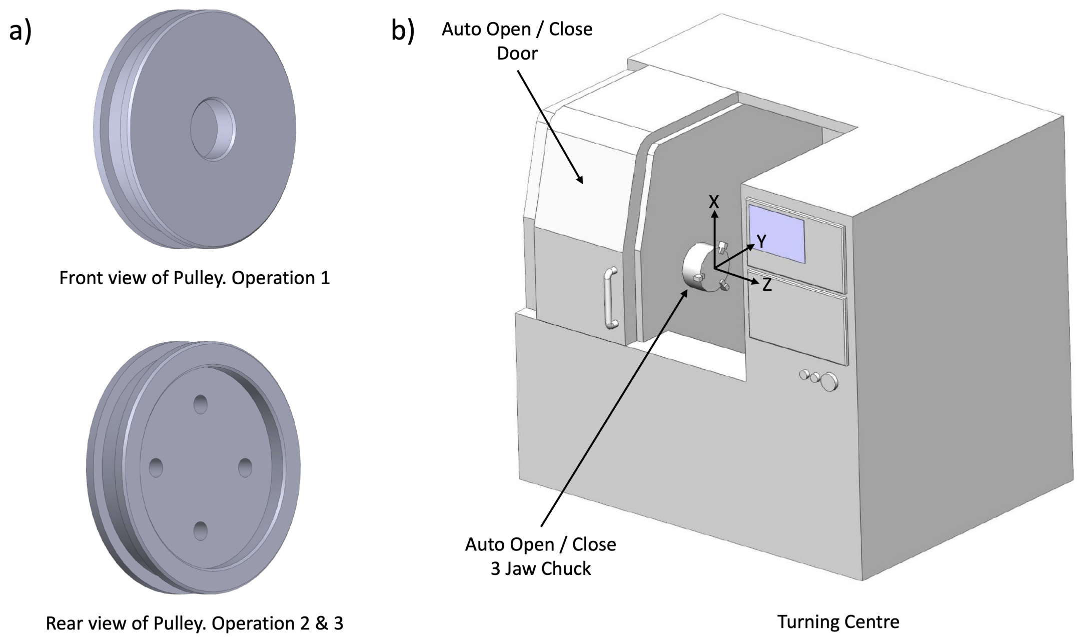

Fig. 1: a) pulley component b) turning centre.

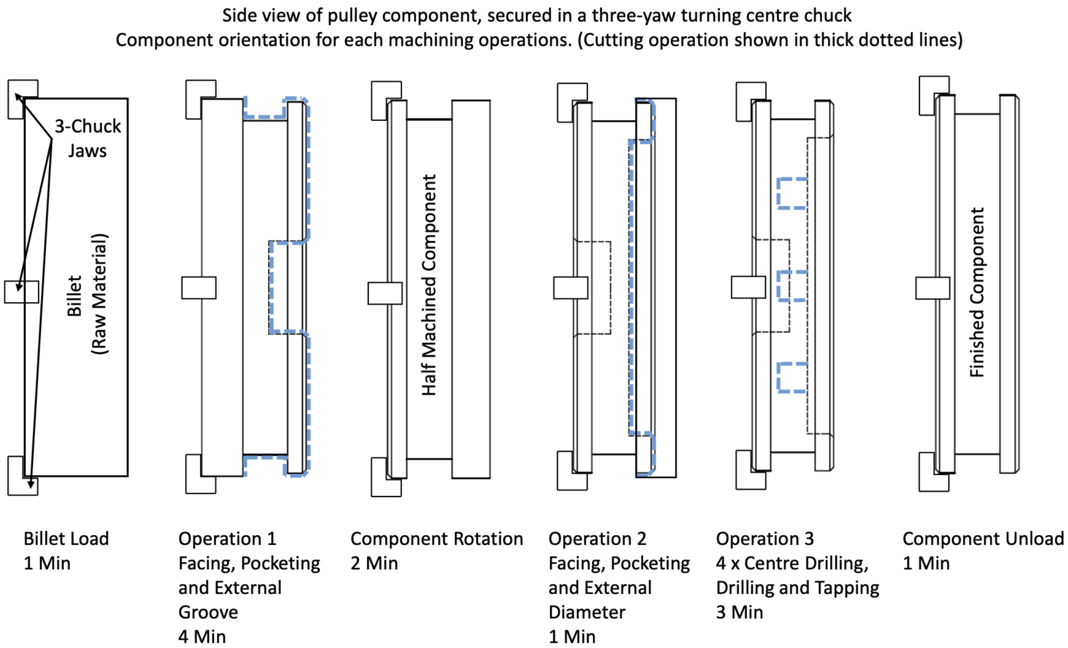

Fig. 2: Part handling, material orientation, machining operations and cycle times.

(a) You are required to design an automated cell meeting the needs of the machining company and only requiring the intervention of an operator every four hours:

(i) Draw the floor layout for the new automated cell making use of the two turning centres. The design should include any additional hardware, such as robots, end effectors, conveyors, kitting trays and safety systems. Describe the overall operation of the system, listing the specific features of the hardware components that make them suitable for the task. [40%]

(ii) Describe how the automated cell would achieve an improved production rate, considering the cycle times of the different machining operations, capabilities of the different turning centres and operator availability. [30%]

(b) Draw a diagram of the different elements required to manage the operation of the automated cell. Include any additional hardware required and describe control functions required on the different pieces of hardware used in your solution. [30%]

4 A logistics company has partially installed a divert station on a tote roller conveyor, allowing plastic totes with an appropriate bar code identifier to be diverted. A plan view of the divert station in two states can be seen in Fig. 3. The station's divert function is achieved through the operation of a tote stopper, interrogation of a tote bar code via a bar code reader and the operation of the tote pusher. To ensure robust operations, it is proposed that a pneumatic actuation solution is used for the tote stopper and pusher. ISO standard symbols for components used in these types of applications can be seen in Fig. 4. The pneumatic actuation solution design needs to meet the following operational criteria:

- the tote stopper must normally be in the up position to stop the totes, even if the air and/or electrical supplies fail;
- the speed of the tote pusher needs to be set independently, either when diverting totes or retracting to its normal retract position;
- and the spacing of incoming totes is controlled independently and prior to the divert station. This ensures that tote collisions do not occur at the entry of the diverter.

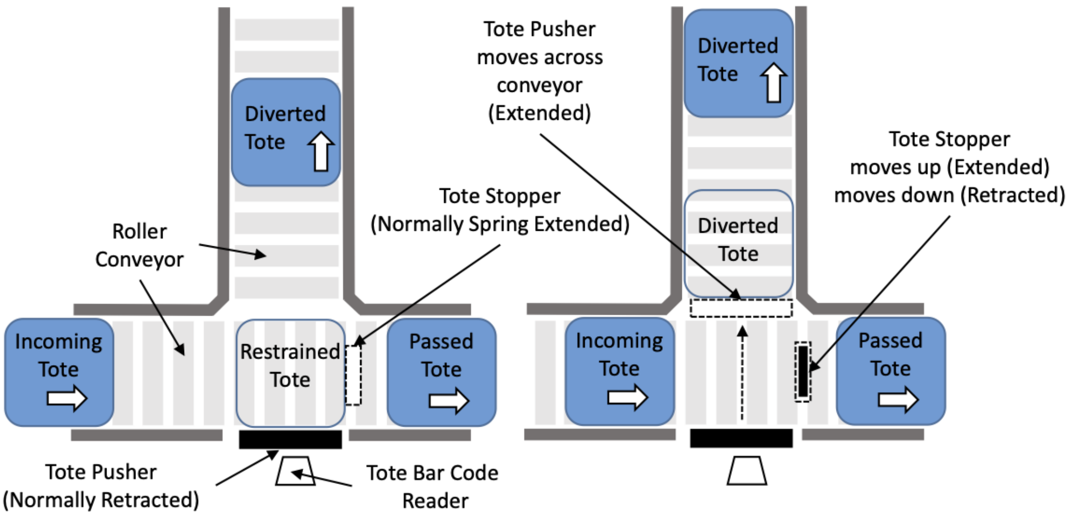

Fig. 3: Plan view divert station

(a) Design a pneumatic circuit that will enable the operation of both the tote stopper and the tote pusher from a Programmable Logic Controller (PLC). The design should fulfil the operational criteria and include all components such as pistons, control valves and regulators. Label each component and describe why it has been used. List any assumptions you make. [50%]

(b) List infrastructural equipment the logistics company must install to enable the operation of the pneumatic hardware. Describe the function of the equipment and why it is critical to the diverter's operation. [25%]

(c) To ensure the PLC can robustly control the operation of the diverter, list the different sensors that you would recommend, stating how it is to be used in each case. List any assumptions you make in your sensor selection. [25%]

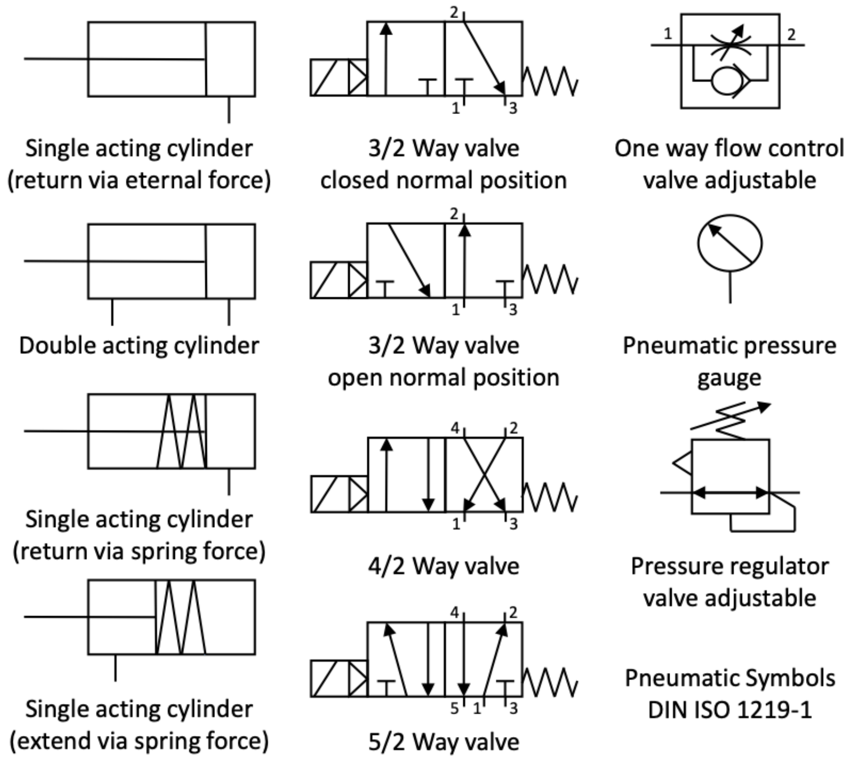

Fig. 4: ISO pneumatic symbols

5 A manufacturer has approached a consulting firm for a process improvement project that requires the development of a discrete event simulation model of their plant.

(a) The consulting firm has identified the activities, precedence relationships and estimated activity times for the project as shown in Table 1. The firm has agreed to complete the project in 40 days at a fixed price of £50,000. If the project is not completed within 40 days, the contract specifies a penalty cost of £3,000 for each day of delay. The estimated daily cost to the firm including wages and expenses for this project is £1,000. Calculate the probability that this project will be profitable for the consulting firm. [30%]

Activity	**Predecessor Activities**	**Time estimate (days)**		
		Minimum	**Most Likely**	**Maximum**
A: Define the project aim and scope	-	1	3	5
B: Identify project stakeholders	-	1	2	3
C: Develop process map	A, B	4	6	8
D: Collect and analyse data	A, B	6	8	16
E: Construct simulation model	C, D	6	7	8
F: Verify model	E	1	2	3
G: Validate model	E	5	7	9
H: Design and conduct experiments	F, G	4	5	6
I: Draw conclusions	H	2	3	4
J: Produce report and presentation	I	1	3	5

Table 1

(b) The analyst assigned to this project is interested in identifying the bottleneck of the manufacturing process. The analyst has run five replications of the simulation model using Arena for a run-length of six months. The simulation outputs for the average waiting times (in minutes) at three critical operations of the plant are shown in Table 2.

(i) Reflecting on the outputs shown in Table 2, discuss the importance of *replications* in a discrete event simulation model. [20%]

 (cont.

(ii) Estimate the minimum number of replications that should be run so that the half-width is within 10% of the average waiting times across replications at each operation. [20%]

Replication	Average waiting times (minutes)		
	Kitting	**Assembly**	**Testing**
1	13.89	20.53	17.98
2	17.65	15.19	26.87
3	14.65	30.26	25.65
4	20.34	18.62	19.67
5	14.98	29.98	22.65
Average across replications	16.30	22.92	22.56
Half-width	3.31	8.51	4.71

Table 2

(iii) The manufacturing plant currently uses a conveyor to transport parts from Kitting to Assembly. Fig. 5 shows part of the simulation model that describes this logic. The manufacturer wants to understand the impact of replacing the conveyor with four Automated Guided Vehicles (AGV) that can each carry one part at a time and move twice as fast as the conveyor. Sketch and describe the logic the analyst should use in Arena to model the AGVs. State any assumptions you have made. [30%]

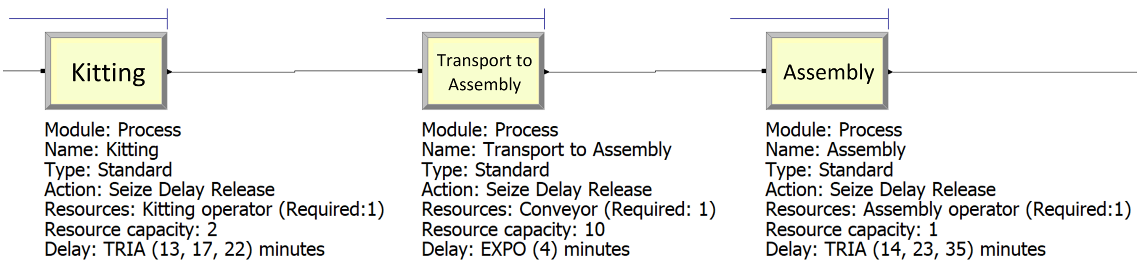

Fig. 5

6 (a) The formula for calculating variance (σ^2) in a data sample is given below. Explain why the formula involves taking the square of the difference between the data points (x_i) and the mean (μ). [5%]

$$\sigma^2 = \frac{\sum_{i=1}^{N} (x_i - \mu)^2}{N - 1}$$

(b) The formula for calculating standard deviation (σ) in a data sample is given below. Explain why the formula involves taking the square root of the variance. [5%]

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \mu)^2}$$

(c) Explain the difference between a *parameter* and a *statistic*? [10%]

(d) Compare and contrast *population standard deviation* and *sample standard deviation*. [10%]

(e) Briefly explain the central limit theorem and why it is useful. [10%]

(f) As part of a new product launch, a manufacturer is planning to perform a survey to explore the product market. The manufacturer would like to survey prospective customers in two regions as well as in three age groups, i.e., six different populations in total. The costs of surveying are different for those six populations, but known to the manufacturer. The manufacturer would like to minimize the costs of surveying 1,000 prospective customers while making sure that each population is properly represented. Therefore, the sample should contain at least 25% from each age group and at least 40% from each region. Explain how you would formulate this problem for an optimiser to minimize the surveying costs that would be incurred by the manufacturer. [15%]

(g) You are working as a data analyst for a medical device producer that is having quality issues on its production line. It is estimated that on average 35% of devices fail quality control. The manufacturer wants to explore whether device quality could be predicted using image based sensors. Based on 165 historical records, you have produced a classifier for this problem which groups predictions into two classes: ‘Fail’ and ‘Ok’. After you run the model you find the following results:

- ‘Fail’ predictions that have actually failed quality checks: 20
- ‘Fail’ predictions that were found to be good quality: 55
- ‘Ok’ predictions that have actually failed quality checks: 15
- ‘Ok’ predictions that were found to be good quality: 75

As this approach may replace manual quality checks on critical devices, it is important that the classifier performs well.

(i) Calculate the *accuracy*, *recall* and *precision* of the model using a *confusion matrix* and comment on model performance using these metrics. Are all these performance metrics equally important in the above context? [25%]

(ii) How would your interpretation of model performance be affected if only 1% of devices fail quality control on average? [20%]

END OF PAPER

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Probability Tables for Question 5

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot 9^{2}0097$	$\cdot 9^{2}0358$	$\cdot 9^{2}0613$	$\cdot 9^{2}0863$	$\cdot 9^{2}1106$	$\cdot 9^{2}1344$	$\cdot 9^{2}1576$
2·4	$\cdot 9^{2}1802$	$\cdot 9^{2}2024$	$\cdot 9^{2}2240$	$\cdot 9^{2}2451$	$\cdot 9^{2}2656$	$\cdot 9^{2}2857$	$\cdot 9^{2}3053$	$\cdot 9^{2}3244$	$\cdot 9^{2}3431$	$\cdot 9^{2}3613$
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2·6	$\cdot 9^{2}5339$	$\cdot 9^{2}5473$	$\cdot 9^{2}5604$	$\cdot 9^{2}5731$	$\cdot 9^{2}5855$	$\cdot 9^{2}5975$	$\cdot 9^{2}6093$	$\cdot 9^{2}6207$	$\cdot 9^{2}6319$	$\cdot 9^{2}6427$
2·7	$\cdot 9^{2}6533$	$\cdot 9^{2}6636$	$\cdot 9^{2}6736$	$\cdot 9^{2}6833$	$\cdot 9^{2}6928$	$\cdot 9^{2}7020$	$\cdot 9^{2}7110$	$\cdot 9^{2}7197$	$\cdot 9^{2}7282$	$\cdot 9^{2}7365$
2·8	$\cdot 9^{2}7445$	$\cdot 9^{2}7523$	$\cdot 9^{2}7599$	$\cdot 9^{2}7673$	$\cdot 9^{2}7744$	$\cdot 9^{2}7814$	$\cdot 9^{2}7882$	$\cdot 9^{2}7948$	$\cdot 9^{2}8012$	$\cdot 9^{2}8074$
2·9	$\cdot 9^{2}8134$	$\cdot 9^{2}8193$	$\cdot 9^{2}8250$	$\cdot 9^{2}8305$	$\cdot 9^{2}8359$	$\cdot 9^{2}8411$	$\cdot 9^{2}8462$	$\cdot 9^{2}8511$	$\cdot 9^{2}8559$	$\cdot 9^{2}8605$
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4·0	$\cdot 9^{4}6833$	$\cdot 9^{4}6964$	$\cdot 9^{4}7090$	$\cdot 9^{4}7211$	$\cdot 9^{4}7327$	$\cdot 9^{4}7439$	$\cdot 9^{4}7546$	$\cdot 9^{4}7649$	$\cdot 9^{4}7748$	$\cdot 9^{4}7843$
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4·2	$\cdot 9^{4}8665$	$\cdot 9^{4}8723$	$\cdot 9^{4}8778$	$\cdot 9^{4}8832$	$\cdot 9^{4}8882$	$\cdot 9^{4}8931$	$\cdot 9^{4}8978$	$\cdot 9^{5}0226$	$\cdot 9^{5}0655$	$\cdot 9^{5}1066$
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4·8	$\cdot 9^{6}2067$	$\cdot 9^{6}2453$	$\cdot 9^{6}2822$	$\cdot 9^{6}3173$	$\cdot 9^{6}3508$	$\cdot 9^{6}3827$	$\cdot 9^{6}4131$	$\cdot 9^{6}4420$	$\cdot 9^{6}4696$	$\cdot 9^{6}4958$
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Example: $\Phi(3{\cdot}57) = \cdot 9^{3}8215 = 0{\cdot}9998215$.

FRACTILES OF THE t DISTRIBUTION. $t_{1-P} = -t_P$.

P / f	Probability in per cent: 60	70	80	90	95	97·5	99	99·5	99·9	99·95
1	·325	·727	1·376	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	·289	·617	1·061	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	·277	·584	·978	1·638	2·353	3·182	4·541	5·841	10·22	12·94
4	·271	·569	·941	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	·267	·559	·920	1·476	2·015	2·571	3·365	4·032	5·893	6·859
6	·265	·553	·906	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	·263	·549	·896	1·415	1·895	2·365	2·998	3·499	4·785	5·405
8	·262	·546	·889	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	·261	·543	·883	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	·260	·542	·879	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	·260	·540	·876	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	·259	·539	·873	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	·259	·538	·870	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	·258	·537	·868	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	·258	·536	·866	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	·258	·535	·865	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	·257	·534	·863	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	·257	·534	·862	1·330	1·734	2·101	2·552	2·878	3·611	3·922
19	·257	·533	·861	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	·257	·533	·860	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	·257	·532	·859	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	·256	·532	·858	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	·256	·532	·858	1·319	1·714	2·069	2·500	2·807	3·485	3·767
24	·256	·531	·857	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	·256	·531	·856	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	·256	·531	·856	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	·256	·531	·855	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	·256	·530	·855	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	·256	·530	·854	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	·256	·530	·854	1·310	1·697	2·042	2·457	2·750	3·385	3·646
40	·255	·529	·851	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	·255	·528	·849	1·298	1·676	2·009	2·403	2·678	3·262	3·495
60	·254	·527	·848	1·296	1·671	2·000	2·390	2·660	3·232	3·460
80	·254	·527	·846	1·292	1·664	1·990	2·374	2·639	3·195	3·415
100	·254	·526	·845	1·290	1·660	1·984	2·365	2·626	3·174	3·389
200	·254	·525	·843	1·286	1·653	1·972	2·345	2·601	3·131	3·339
500	·253	·525	·842	1·283	1·648	1·965	2·334	2·586	3·106	3·310
∞	·253	·524	·842	1·282	1·645	1·960	2·326	2·576	3·090	3·291
$2(1-P)$	80	60	40	20	10	5	2	1	0·2	0·1

Example: $P\{t < 2{\cdot}086\} = 97{\cdot}5\%$ for $f = 20$.

$P\{|t| > t_P\} = 2(1-P)$. $P\{|t| > 2{\cdot}086\} = 5\%$ for $f = 20$.

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