

## METIIB Paper 1 Cribs 2023

### Question 1

1 (i): This question asks for a description of two production technologies. It is expected that this question will be answered very well by the majority of candidates. This can be entirely described using text or may be a combination of text and diagrams.

A basic answer will give a general layout of the process in a diagram, or note the key processing step in 1-2 sentences. A good answer will give an overview of all the steps, even if only very brief for some of these steps. An excellent answer will show a very good understanding of all the steps within the process.

For SLS, the key steps that are anticipated are: Components are manufactured in a layer-by-layer process, building the component from the bottom up. With each layer, the fine powder is spread across the bed of the additive manufacturing tool using a roller. The roller is separated from the bed with a small, controlled gap size to ensure a controlled layer is deposited. A laser is scanned across this powder surface to sinter together particles in the regions where the component is to be created. Where the powder is not sintered, it remains and acts as a support material that can be removed later through cleaning/post-processing. This is repeated until the component is finished.

For injection moulding, the key steps that are anticipated are: The coarse LDPE granules would be fed into a hopper, heated to their melting point and forced under pressure through a heated barrel using a screw injection process. The molten plastic is then forced through a nozzle to the customised mould cavity. The cavity is within two sides of a mould that have been clamped together under high pressure. These moulds are also heated to ensure the flow of the molten plastic reaches all parts of the mould before any solidification. There is a sprue, or waste material where the nozzle feeds the mould and often runners, which are waste materials between features that are also needed to enable flow of material to those features. Once the material has reached all parts of the mould, and sufficient time is given for the material to solidify, the clamp is released, the mould separated, and ejector pins push out the parts.

1(ii): It is important to note that this question is asking how the choice would be made, not what choice would be made. A basic answer can give a clear positive reason about one technology and negative about another to show a basic understanding. A good answer would go through the additional information needed before a decision could be made. There are many examples, all of which are not included in the crib, but these would include identifying the volume required. If the firm is going immediately to a very high throughput production, then IM is likely a selected route. If the firm needs to have a low volume produced first to trial the inhaler and further design changes will be happening, they may decide not to invest in moulds and instead would ask for prototypes, which could be made by SLS. Both the component design and feature size should be mentioned as important information to find out. SLS can create much finer resolution than normal injection moulding, and AM can create more complex 3D structures. There may also be comments about needing a better understanding of the surface finish. An excellent answer will show a clear understanding of 4 considerations.

1(iii): A basic answer will note challenges that have been occurred before when adopting biobased polymers. This includes challenges in achieving the right scale and ensuring a high enough supply volume, as the market is still ramping up. There are examples of other polymers where this is the case that can be drawn upon. It is also valid to discuss the cost may be prohibitive because of the energy intensive nature of the production and limited scale. More advanced answers will consider the challenges when switching to new polymers and new suppliers more generally. While biopolymers are chemically identical, other properties are highly likely to vary, such as the presence of different types of contamination, a very different molecular weight distribution, both of which will lead to different thermal responses and mechanical properties, which will impact the flow in injection moulding, the sintering behaviour and the ultimate mechanical properties. Mitigation will likely focus around the analysis of the new materials and monitoring of the quality of each batch, with details provided about the use of DSC, UV-Vis, rheology, or GPC and how each would relate to specific issues being tracked.

1(iv): In this case, it is likely an LCA is required and that is anticipated to be the focus of a strong answer. It is important that the answer conveys that this is a structured, standardised approach that covers the entire life cycle of the product, considering the energy, materials, emissions and waste at the stage

where the biosource is produced and harvested, as well as the conversion to the polymer itself. It is important that a strong answer notes that the material use, re-use and waste disposal is also considered in a strong answer. The challenges are mostly around defining the boundaries of the LCA, especially those around farming more generally of the biosource, and around the country-specific variables when carrying out LCA. Also, the LCA will give a wide range of impacts, which will be very challenging to prioritise and compare, and so this may make a final decision challenging. There are many other aspects covered across multiple lectures that students can draw upon.

## Question 2

2 (i) This question asks for a description of particular production technologies. This process has been discussed in the course in an academic lecture and industry lecture and so it is anticipated that it will be answered very well by the majority of candidates using a range of information provided. This can be entirely described using text or may be a combination of text and diagrams.

The anticipated steps that will be described include: The application of photoresist by spin coating of the 12" wafer. Some details are expected to explain what a photoresist's function is. The photolithography exposure step should then be described, including the fact it is UV and exposure is through a mask to give preferential hardening of the polymer and a pattern after resist development. The ion implantation or doping will likely be noted, with a brief reference to n-type and p-type behaviour. The fabrication of multiple layers of functional structures should then be noted. This is quite complex and so a strong answer will give at least 4 steps during the etching, photoresist removal, the creation of a gate dielectric, a gate electrode, an insulator, metal contacts, interconnects, etc. A strong answer will also link to the role of plasma in delivering the resolution needed in both deposition and etching techniques. Sorting, dicing and packaging are also acceptable areas to note. A basic answer will note briefly the key steps whereas a strong answer will describe them sufficiently to show a clear understanding about how the structure is built up.

(ii) A basic answer will correctly identify the processing steps, whereas a stronger answer will provide a clear description as to how these steps can limit resolution. There are many potential examples, including: The lens arrangement of the projection lens is crucial to delivering high precision photolithography. The alignment of multiple layers is limited by the tools and their positioning resolution, which again will impact the overall device resolution. The wavelength used is also a limitation, with a diffraction limit.

(iii) There are many potential answers for this, but again a basic answer will correctly give a brief, but accurate indication of the material innovations, whereas a strong answer will provide a clear description and show correct understanding of the problem and solution. Examples from the past include the move to strained silicon allowed an increase in drive currents, increased mobility, and reduced energy consumption. This was important as it addressed the problem of too much heat building up in the small (in this case 90nm) transistors. Other examples that can be discussed would be the introduction of high-k dielectric materials combined with metal gates, if considering existing device technologies, or the introduction of nanowire transistors or a shift away from silicon, if considering future technologies. In all cases a clear understanding should be conveyed about why they would help. This is a challenging question and so significant detail is not expected.

(iv) This question expects the student to draw upon multiple lectures and understand how the challenges of one production technology would apply to a separate product. A basic answer would note the difficulty of ablating materials with a sufficiently high precision, due to the wavelength and diffraction limit. A more advanced answer would consider the interactions of a laser with a solid surface and discuss aspects of heating, micro-cracks, recast layers, surface debris, surface ripples, etc. and also discuss the challenge of tuning the influence of the ablation on a fixed number of layers, while leaving other layers untouched. This is also a challenging question and so only a limited selection of considerations and a brief description is expected for full marks.

(v) This question focuses on roll-to-roll manufacture and ultra-precision manufacturing. This is anticipated to be quite straightforward for candidates and so three clear descriptions showing a good understanding of roll-to-roll processing is expected for full marks. It is anticipated that tension control of a flexible polymer will be described, and the potential ripples that could form in the material if this is not correct. In addition, roll synchronisation will likely be noted, positioning or alignment of the film on the roller, especially with respect to the patterning or deposition tool, will be extremely challenging, and the control of registration of devices that may be needed to avoid poor alignment. These are basic answers discussed in the lecture, but in addition, a strong answer would draw upon microchip manufacturing processes and think about how these processes must change to be roll-to-roll. This could include noting how spin coating will not be possible and so controlled layer deposition will be extremely challenging, it is

uncertain how photolithography will interfere with the underlying material, which is no longer silicon, and vacuum deposition techniques may be very challenging with this approach.

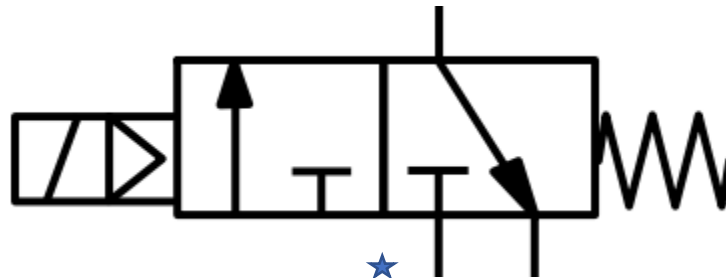
### Question 3

a) The compressed air source can have a number of impurities that have to be removed as well local control of the air pressure that is going to be used within the application.

Component	Function
Filter	Different types of filters are used depending on the application. 1) Water Filter / Trap to capture water moisture that has been generated through the compressing process. 2) Particle filter to capture any contaminants in the air. It is possible to get dual purpose filters that will perform both these functions.
Pressure Regulator	Controls and stabilises the air pressure downstream of the regulator.
Pressure Gauge	Pressure gauge is used to allow the operator to see the pressure in the system when adjusting the pressure regulator.

Often the three components listed above are provided in a single unit called a combination mist separator and regulator. It is also common to have additional components at this point, isolation / dump valves and oil lubricators

a ii)



3/2 Way, Pilot Operated, Solenoid Valve, Normally Closed

Symbology of the valve:

- The Main body of the valve has two boxes, indicating that it's a 2 way valve
- The box on the right shows the state of the valve in its at rest operating position.
- The box on the right also shows the internal mechanism has 3 Ports.
- The spring on the right shows that the valve is sprung return to its at rest state.
- The left-hand side of the valve indicates a / for solenoid operated and the |> for pilot operated.
- The star shows the main air feed connection. Normally this would be labelled with a 1 or P and in the at rest state this blocked off (Normally Closed)

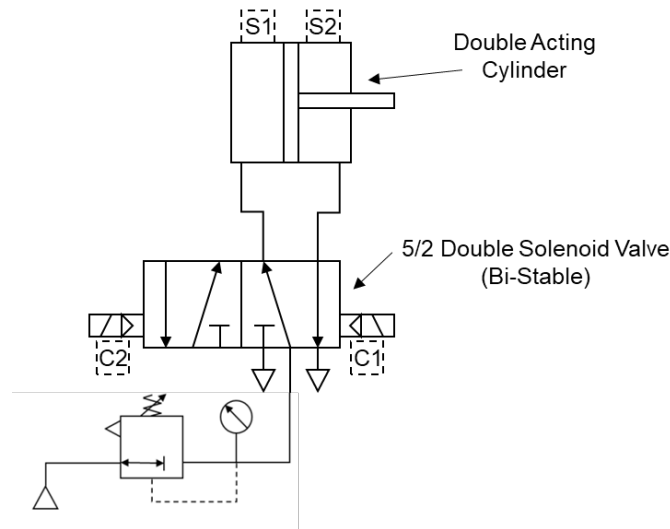
bi)

To allow the end effector changer to be operated easily during different assembly tasks, its operation should be controlled via the robot controller. This will allow the robot to change end effectors directly within its own operations. To achieve this the 5/2 way valve would be connected to the robot IO outputs. The sensors on the double acting cylinder would be connected to the robot IO inputs. Each end effector would have its electrical connector configured with an ID using 3 binary bits (wrap back). These bit combinations would allow the robot IO inputs to determine which end effector is connected to the robot. The robot end effector stand would have additional sensors added to allow the robot IO inputs to determine if an end effector is present in a particular storage position.

**CRITICAL:**

- **End Effector 1 (EE1) is stored in Location 1 (L1), EE2 in L2, EE3 in L3 and EE4 in L4. This fundamental rule that should be followed by the automation solution and any manual operations.**
- **End Effectors are stored at the end of operations. (When robot starts up, no End Effector should be on the robot and all End Effectors should be stored on the tool stand)**
- **Note this solution doesn't have sensed information to determine which EE is positioned in a particular storage location. (This would require additional AutoID technology. e.g. RFID.)**
- **Abnormal Conditions (5/2 Way valve / changer design accommodates air failure)**
- **Abnormal Conditions (Wiring / IO failure should be captured in logic checks)**
- **Error conditions would be resolved manually with care!**

bii)



Circuit diagram for connecting the 5/2 way pneumatic solenoid valve to the double acting cylinder. The diagram also shows a pressure a regulator with integral pressure gauge and compressed air sources and vents.

biii)

Hardware	Robot IP's	Robot OP's
5/2 Way valve (Coil)		C1 (Request Cylinder Extend)
5/2 Way Valve (Coil)		C2 (Request Cylinder Retract)

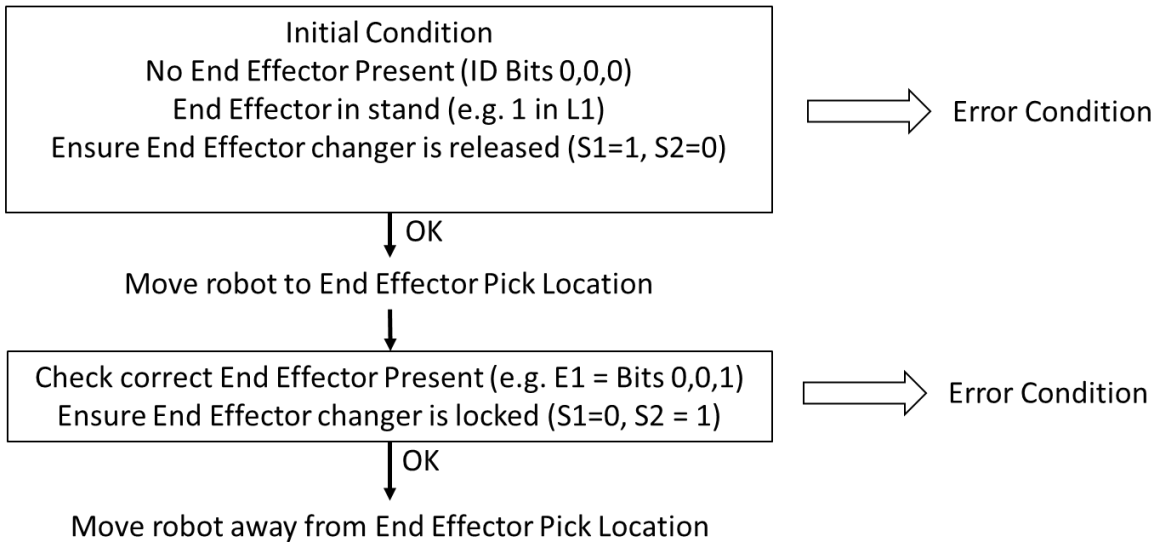
Double Acting Cylinder (Sensor)	S1 (Cylinder Retracted) End Effector Released	
Double Acting Cylinder (Sensor)	S2 (Cylinder Extended) End Effector Gripped	
End Effector 1 Attached (Spare)	ID 1 (3 Bits 0,0,1)	
End Effector 2 Attached (Riveter)	ID 2 (3 Bits 0,1,0)	
End Effector 3 Attached (4mmDrill)	ID 3 (3 Bits 0,1,1)	
End Effector 4 Attached (6mmDrill)	ID 4 (3 Bits 1,1,0)	
End Effector Stand Pos 1 (Sensor)	End Effector present Pos 1	
End Effector Stand Pos 2 (Sensor)	End Effector present Pos 2	
End Effector Stand Pos 3 (Sensor)	End Effector present Pos 3	
End Effector Stand Pos 4 (Sensor)	End Effector present Pos 4	

The logic implemented on the robot controller would perform the following tasks:

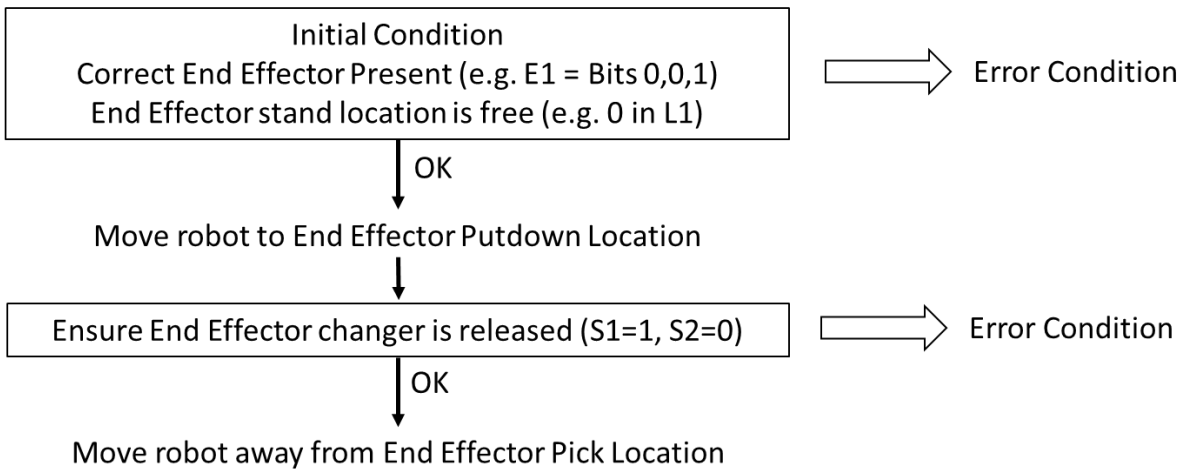
Pickup an End Effector	Ensure an End Effector is not currently attached to the robot Ensure an End Effector is available in the stand location Ensure the End Effector changer is released before approaching Ensure the correct End Effector is picked (Not been manually swapped) Ensure the End Effector changer is locked before withdrawing
Putdown an End Effector	Ensure the correct End Effector is attached to the robot Ensure the End Effector stand location is free Ensure the End Effector changer is released before withdrawing
Don't drop or crash an End Effector during abnormal conditions.	Ensure the end effector is secure during air / electrical faults. Ensure that changes during start-up / shutdown and manual processes can be catered for. (Uncertain conditions should result in an error!)

biV)

### End Effector Pick Up



### End Effector Putdown





#### Question 4

(a) Plastic packaging for cheese	(b) Li-ion battery
<p>(i) Recycling should reduce the amount of plastic waste entering the natural environment. If plastics are recycled to produce new material, there is substitution and the demand for virgin polymers is reduced. Most polymers derive from fossil fuel feedstock; reduction in demand reduces pressure on a finite resource. In many cases, and normally with mechanical recycling, the carbon footprint for recycling polymers is considerably lower than that required for virgin polymer production. Recycling would be beneficial in nearly all cases, but for economic reasons the amount of polymer recycling is currently limited, and largely confined to specific products (mainly bottles and rigid containers). Claims of current benefits are therefore over-stated. Thermal recycling, energy-from-waste, is sometimes included as a recycling technology, which is questionable.</p>	<p>(i) Lithium is very resource-intensive and environmentally damaging to mine and process, and current production is not keeping pace with demand. If recycled lithium can substitute for virgin material the pressure on the existing supply chain will be reduced, plus there will be reduction in environmental footprint. Recycling is currently at a low level, only 5% of batteries recycled, mainly because of lack of economically viable technologies. Batteries not recycled currently often end up in landfill, where they are not particularly a hazard, but it is a waste of resources for a highly-engineered product. Even with maximum recycling, recycled lithium will be insufficient to supply near-future needs.</p>
<p>(ii) Plastic films are always difficult to recycle: handling is problematic; the material range found in plastic films is large, and material identification is difficult. With multi-layer films the problems are exacerbated because the materials cannot easily be separated and the properties (and hence the value) of any recycled product made from mixed polymer intake is normally poor.</p>	<p>(ii) Li batteries are complex structures containing many different components and materials. Processing carries fire hazard danger. The lithium is mixed with other materials. Initial separation is currently labour-intensive. Reclamation to produce lithium is technologically difficult; processes are resource-intensive and often involve hazardous materials. The high costs involved currently deter development of recycling facilities.</p>
<p>(iii) A multilayer film could be replaced by single material, with PET being the best compromise. However, PET's properties (in particular permeability to oxygen and water) are inferior; the film thickness must be increased so resulting in a greater carbon footprint. Although a PET film could in theory be recycled, recycling rates for films are currently very low so the likelihood is that such packaging will end up in landfill, or at best be used for energy-from-waste. The increased carbon footprint and low probability of like-for-like recycling means that the wisdom of this solution is questionable, although it is being widely adopted.</p>	<p>(iii) Technological developments may enable development of batteries that fulfil the operational needs but are more easily recyclable: a current candidate material is sodium. There's a lot of research going on. Battery performance is the dominant factor here, so recyclability is unlikely to be at the expense of other properties. There may be opportunities for Li-ion batteries to be designed for greater recyclability, retaining comparable chemical processes but altering aspects of material choice</p>
<p>(iv) Re-usable packaging rather than single-use. Avoids number of products manufactured and reduces amount of plastic waste. Negatives are: multi-use containers contain much more material so have bigger environmental footprint than single-use and must be re-used many times; collection</p>	<p>(iv) Prolong lifetime of complete batteries or of components by such treatments as reconditioning of electrode plates (the subject of much current research). Assurance of quality control of batteries containing re-used parts will be critical to developing a market, and could be assisted by</p>

<p>and cleaning also take resources (and can be logistically challenging).</p>	<p>centralised collection and processing; eco-leasing of batteries could be part of this.</p>
<p>(v) A major impact comes from plastic waste in the environment (microplastics and larger-scale plastic waste, in marine and terrestrial locations). The amount of such waste should be reduced if the volume of plastics produced (and particularly single-use plastics) can be reduced, but there will always be some leakage. Microplastics are particularly difficult to avoid or to collect. Such problems would be avoided by using biodegradable plastics that will safely decompose in the natural environment. However, plastics are valued for many applications because of their chemical resistivity and durability, so too much emphasis on end-of-life considerations may impair their usefulness.</p> <p>biodegradable polymers</p> <p>Moving from fossil-based to bio-based feedstock is a long-term goal (though currently only about 2-4% of polymers are made from bio-based feedstock). However, the environmental footprint of bio-based polymers may not be lower than fossil-based polymers, and the price is higher.</p>	<p>(v) Moving closer to a closed-loop product lifecycle for current batteries could have significant environmental benefits, particularly if recycling technologies develop. The main benefit will be to reduce the demand for virgin lithium, so avoiding production environmental burdens. For next-generation batteries the aim should be that they are made from materials that are not resource-intensive or environmentally hazardous to produce, and they should be designed for end-of-life (easy disassembly, and re-use or recycling of parts).</p>

*Best answers will show strong understanding and good knowledge of the environmental implications of the two products. Examples will draw on experience from many different parts of the course together with the student's own observations of industrial practice. The discussion will be critical and insightful.*

*Strong answers will cover the key environmental aspects of the two products and provide adequate discussion with pertinent examples.*

*Basic answers may demonstrate lack of knowledge and understanding of some important factors. Examples may be inappropriate. The discussion may be superficial and inaccurate.*

### Question 5

- Forkly wants to determine the optimal sequence of establishing the MFCs in London such that the cost of deliveries over the next four months is minimised. The appended matrix showing the additional costs incurred in delivering to the customers from various MFCs alongside the demand and the weight of the corresponding region is shown below.

Site to	A	B	C	D	Demand	Weight
From	Cost increase for the delivery riders to move across the MFCs					
A	0	6	11	9	11000	1.4
B	7	0	12	8	9000	1
C	9	6	0	5	15000	0.8
D	8	8	10	0	4000	1.1

Ardalan heuristic can be used to evaluate the required optimal sequence to establish the MFCs. Ardalan heuristic iterates through the following steps:

- Multiply the costs with the demand and the weight of the corresponding region (along the rows)

Site to	A	B	C	D	Demand	Weight
From						
A	0	92400	169400	138600	11000	1.4
B	63000	0	108000	72000	9000	1
C	108000	72000	0	60000	15000	0.8
D	35200	35200	44000	0	4000	1.1

- Add the values across the columns and identify the location corresponding to the minimum total cost after the previous step. This is the optimal location for the first MFC

Site to	A	B	C	D
From				
A	0	92400	169400	138600
B	63000	0	108000	72000
C	108000	72000	0	60000
D	35200	35200	44000	0
Total	206200	199600	321400	270600

3. Replace the values in the other columns with the minimum total cost column values, if they are greater than the minimum total cost column value along the corresponding rows

Site to	A	B	C	D
From				
A	0	92400	92400	92400
B	0	0	0	0
C	72000	72000	0	60000
D	35200	35200	35200	0
		199600		

4. Remove the first location for establishing the MFC and repeat the above steps until the entire sequence is obtained. The Matrices obtained after every iteration and step are shown below

Site to	A	C	D
From			
A	0	92400	92400
B	0	0	0
C	72000	0	60000
D	35200	35200	0
Total	107200	127600	152400

Site to	A	C	D
From			
A	0	0	0
B	0	0	0
C	72000	0	60000
D	35200	35200	0
	107200		

Site to	C	D
From		
A	0	0
B	0	0
C	0	60000
D	35200	0
Total	35200	60000

Site to	C	D
From		
A	0	0
B	0	0
C	0	0
D	35200	0
	35200	

Site to	D
From	
A	0
B	0
C	0
D	0
Total	0

From the above steps, we can see that the optimal sequence for Forkly to establish the MFCs in London is: **B-A-C-D**

- (b) The Clarke Wright algorithm can be used to evaluate the optimal number of riders required for the Forkly orders in North London that minimise the distance travelled.

1. In the first step, the savings in distance across all possible pairs of customers are calculated. The savings compare the rider going to the next customer directly, versus coming to the MFC and then going to the next customer. Distance saving for customer  $i$  and  $j$  are calculated as:

$$S_{ij} = d_{D,1} + d_{D,2} - d_{i,j}$$

Since there are six customers, total possible pairs of customers are:

$$\binom{6}{2} = \frac{6!}{2!(6-2)!} = 15$$

$$\frac{6}{2} = \frac{6!}{2!(6-2)!} = 15$$

2. These 15 pairs of customers with the corresponding savings in descending order are shown below:

Pairs	Savings	Pairs	Savings	Pairs	Savings
(2,4)	14	(1,4)	5	(3,6)	4
(1,2)	8	(2,5)	5	(5,6)	3
(4,6)	8	(2,6)	5	(1,5)	2
(2,3)	6	(3,4)	5	(3,5)	4
(4,5)	6	(1,6)	4	(3,6)	4
(1,3)	5	(3,5)	4	(5,6)	3

3. Assigning the top pair to the first rider, we move down the list adding the orders that fit in the capacity of the rider. The orders that do not fit, and do not overlap with already assigned orders, are assigned to the next rider. This is followed until all customer orders are accounted for. The order pairs allocated to the riders, **with total distance savings being 26** after following the above procedure, are listed below:

Rider 1	Capacity reached	Rider 2	Capacity reached	Rider 3	Capacity Reached
(2,4)	16	(3,5)	26	6	18
(1,2)	32				

4. The route for each rider is, for rider 1: **D-4-2-1-D** for rider 2: **D-3-5-D** and for rider 3: **D-6-D**. The total distance saved, compared to the riders going individually to each customer and back to the MFC is 26 miles.
5. However, the Clarke Wright heuristic uses a greedy approach and therefore the Holmes Parker extension must be used to check any further possible optimisation. To check for this, we branch off first at the customer pair (1,2) instead of (2,4) and follow the same procedure. The order pairs allocated to riders if we ignore the first pair, **with total savings 28**, are shown below:

Rider 1	Capacity reached	Rider 2	Capacity reached	Rider 3	Capacity Reached
(1,2)	23	(4,6)	27		
(2,3)	38	(4,5)	38		

6. The Holmes Parker therefore gives a better solution with fewer riders and also reducing the total distance travelled. The route for rider 1 is **D-1-2-3-D**, and rider 2 is **D-6-4-5-D**.

NOTE : Holmes Parker extension can be continued further, for example by branching at (4,6) instead of (1,2). But the students are expected to show only once branching.

- (c) The students are expected to provide a discussion of 3-4 sentences on evaluating the above problem for two cases of time constraints. (a) If the speed of the vehicles is constant, the distance matrix can simply be converted into time matrix and be solved for minimising time. While moving down the list in Clarke Wright algorithm in the same manner, the pairs can be checked for satisfying the capacity and the time constraints. However, if (b) the speeds of the vehicles are variable, the solution requires sophistication rather than a simple inclusion of an additional constraint. the students are expected to suggest their own methods based on the additional reading or discuss technologies like vehicle/ traffic/ weather tracking and forecasting for flexible logistics.

## Question 6

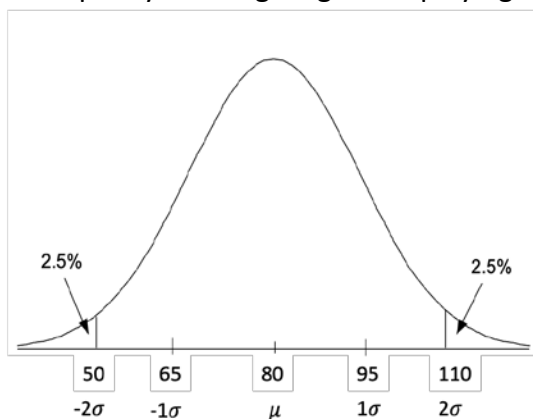
a)

(i) Flexo will not have enough stock to meet sales if demand exceeds 100 units of stock. To calculate the probability that demand will be greater than 100, students need to derive the z-score associated with 100. Standard deviation is given as 15.

Hence,

$$Z = \frac{100 - \mu}{\sigma} = 20/15 = 1.33$$

Using the statistical tables, students can then deduce 0.0918 as the probability of demand exceeding 100 units per week, which is equivalent to 9.2%. Students may also illustrate the concepts by drawing a figure displaying distance from the  $\mu$  as below.



(ii) As illustrated above, 50 units of sales would constitute  $-2\sigma$ , which is 2.5%. Hence the probability of having more than 50 units sold, would be  $1 - 2.5 = 97.5\%$ .

(Statistical table to refer to is given further down)

(b)

(i) Students should not forget to add error term and marks will be penalised if they do.

Model 1:  $\text{COST} = 7.78 + 2.48\text{MAKE} + e$

Model 2:  $\text{COST} = 3.557 + 0.667\text{AGE} + 0.043\text{USAGE} + e$

Model 3:  $\text{COST} = 3.631 + 0.647\text{AGE} + 0.042\text{USAGE} + 0.170\text{MAKE} + e$

(ii) The p-value of MAKE is significant. The coefficient of MAKE is 2.48 (i.e. £2,480), and the confidence intervals are given as £1,340 and £3,630 respectively, at the 5% significance level. £2,000 is within these intervals. Therefore, we cannot reject the hypothesis.

(iii) Model 1 has a low adjusted R square meaning that it has no explanatory power for the variation in cost. Out of the three models, Model 2 (given by Fig 4) has a slightly higher Adjusted R square than Model 3, and two significant independent variables (age and usage).



If a compact model is required, then Model 2 is attractive as it offers approximately same explaining power as Model 3.

(iv) Students should observe that while Make is significant in Model 1, it is not in Model 3. This is because of multicollinearity, as shown by Fig 2. Age and Make are significantly correlated with one another, meaning that the information given by Make is contained by Age and vice versa. Hence adding Make into a model that has Age results in little additional information. Therefore one would either choose Model 2, or attempt to create a further model where Make and Usage are used, but not age, and compare the resulting Adjusted R square.

Students may also mention that Model 1 has a negative R square whilst Make within this model is significant. This is correct because Make is a binary variable – the model, having a single binary variable – is not explaining variance in cost, but testing the significance of the independent variables.

(v) The question gives students three variables (Make, Use and Age) and asks for a prediction on Cost. Students can either use Model 3 with all variables, or ignore Make, and use Model 2.

Using Model 2 we obtain:

$$\text{COST} = 3.557 + 0.667 * \text{AGE} + 0.043 * \text{USAGE} + e$$

$$9.042 = 3.557 + 0.667(5) + 0.043(50)$$

At 95% confidence interval, we would calculate  $9.042 \pm 1.96 \times 0.615$ ,  
hence  $7.836 < \text{COST} < 10.247$

Using Model 3 we obtain:

$$\text{COST} = 3.631 + 0.647 * \text{AGE} + 0.042 * \text{USAGE} + 0.170 * \text{MAKE} + e$$

$$8.966 = 3.631 + 0.647 * 5 + 0.042 * 50 + 0.170 * 0$$

At 95% confidence interval, we would calculate  $8.966 \pm 1.96 \times 0.623$ ,  
hence  $7.744 < \text{COST} < 10.187$

## Z-Score Table

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
3.0	.0014	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010