Version SWP/05

MET 3

MANUFACTURING ENGINEERING TRIPOS PART IIB

Tuesday 25th April 2023 9.00 to 12:10

Paper 1

*Answer not more than **four** questions.*

Answer each question in a separate booklet.

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number not your name on the cover sheet on each booklet.

STATIONERY REQUIREMENTS

8 page answer booklet x 4
Rough work pad

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Engineering data books
Probability Tables for Question 6

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

Question 1

A firm ('Plastic Inc.') designs and fabricates plastic components for medical device manufacturers. Plastic Inc. has been asked to create a new component for an inhaler, made from low density polyethylene (LDPE). Plastic Inc. stock LDPE as a fine powder for selective laser sintering additive manufacturing, and LDPE as a coarse powder for use in injection moulding.

(a) Describe the steps in the manufacturing process for selective laser sintering additive manufacturing and injection moulding.

[15%]

(b) Describe how you would choose which of these two manufacturing processes to use for the inhaler component.

[25%]

(c) The medical device manufacturer has decided to make their product more sustainable. They have requested that Plastic Inc. make the components using a new bio-based LDPE. What manufacturing challenges do you anticipate may occur with a move to a new bio-based polymer? Refer to the two manufacturing processes where appropriate and include any suggested mitigation actions in your answer.

[30%]

(d) Explain how you would evaluate if the move to a bio-based polymer improves product sustainability. Include any challenges you anticipate you will face when making this evaluation.

[30%]

Question 2

(a) Describe the process of creating an array of transistors on a silicon wafer using photolithography.

[25%]

(b) Give and explain any two examples of how processing steps can limit the achievable pattern resolution when manufacturing arrays of transistors.

[20%]

(c) Give and explain any two examples of materials innovations that have helped (or will help) the microchip industry to follow Moore's Law, i.e., doubling the number of transistors on a microchip every two years.

[20%]

(d) Explain why it would be difficult to replace photolithography with direct patterning with a laser when manufacturing microchips on silicon.

[20%]

(e) Describe any 3 challenges that would be faced if moving to roll-to-roll manufacture of arrays of transistors on thin plastic films for flexible electronics.

[15%]

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Question 3

(a) You are designing a pneumatic application.

(i) List the pneumatic components required to control the quality of a compressed air source. Explain the function of each component.

[10%]

(ii) Draw the ISO pneumatic symbol for a 3/2 Way, Pilot Operated, Solenoid Valve, Normally Closed. Explain the basic symbology of the valve.

[10%]

(b) A robot multi end effector tool stand is shown in Fig. 1a. Each tool is fitted with the tool side of an end effector changer. An enlarged view of the robot end effector changer mechanism, including the robot side of the changer, is shown in Fig. 1b. The changer will be controlled by a robot performing assembly operations. This will require the double acting cylinder within the robot side of the changer shown in Fig. 1c to be controlled by a 5/2 way pneumatic solenoid valve shown in Fig. 1d and multiple sensors.

(i) State key features that would be required to ensure this system functions safely and effectively.

[15%]

(ii) Sketch and clearly label a pneumatic circuit diagram for connecting the 5/2 way pneumatic solenoid valve to the double acting cylinder.

[25%]

(iii) Identify the key logical tasks and input / output mapping to support the logical operations of this system.

[20%]

(iv) Draw a flowchart that depicts the high-level logic required to control the operation of the robot end effector changer when performing an end effector pick up and put down. Describe any assumptions you have made.

[20%]

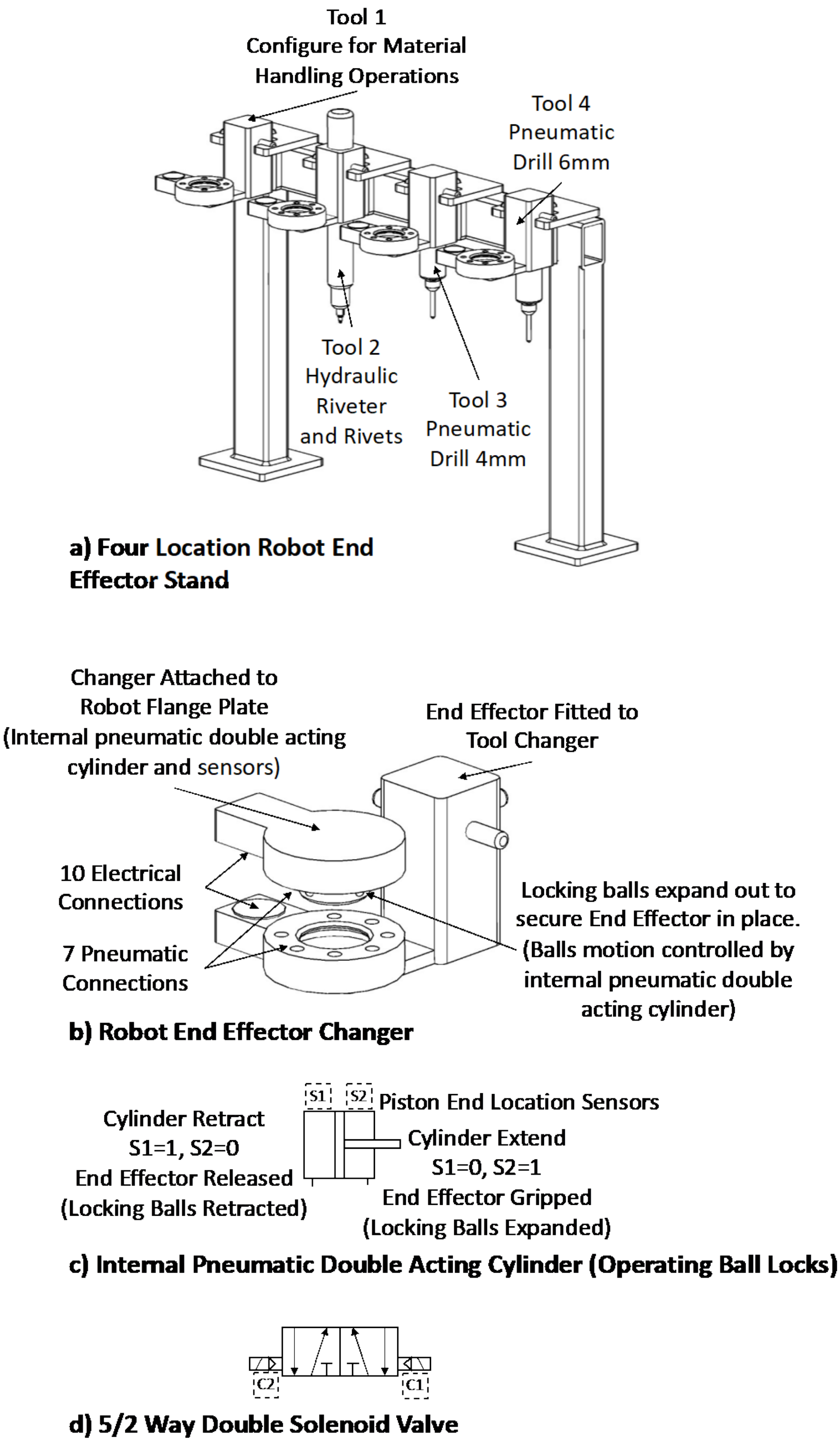

Fig. 1: End Effector Tool Changer

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Question 4

Climate change concerns, fears about future resource availability and desire for recyclability are some of the environmental factors that are often involved in decisions about choice of materials for products. The balance of factors can vary significantly between different materials and products, and many further environmental and other considerations may be the determining criteria.

With specific reference to (1) multilayer flexible plastic packaging for cheese and (2) Li-ion batteries for electric vehicles, discuss the following:

(a) Why is recycling being promoted? To what extent are claims of the benefits of recycling justified?

[20%]

(b) What are the main obstacles to recycling?

[20%]

(c) Discuss the viability and desirability of substitution of a material for another material that is more easily recycled.

[20%]

(d) Re-use rather than single-use is generally regarded as environmentally desirable. What are the considerations for these products of moving towards such a model?

[20%]

(e) What other steps could be taken to reduce the environmental impact of any aspect of the lifecycle of these products? Explain the importance of your suggestions.

[20%]

Question 5

Forkly is an intercity courier company, which is venturing into doorstep delivery of groceries. In order to achieve this, Forkly has plans to establish micro-fulfilment centres (MFCs) across London to stock the necessary inventory.

(a) Forkly has decided to divide London into four regions (North, South, East and West), and establish an MFC in each of these regions. However, they have sufficient resources to only open one MFC in each of the next four months. The additional cost (in pounds) per delivery incurred by riders for delivering across regions (e.g., delivering to a customer in North from an MFC in West) is provided in Table 1. The forecasted monthly demand for each region, and the importance of fulfilling demand from each region is provided in Table 2. Determine the optimal sequence in which the MFCs should be established such that the cost of deliveries over the next four months is minimised.

[40%]

From (i) \ To (j)	North	South	East	West
North	0	6	11	9
South	7	0	12	8
East	9	6	0	5
West	8	8	10	0

Table 1: Cost Matrix showing additional costs incurred in delivering from an MFC in region (i) to customers in region (j)

Region	Monthly Demand (Items)	Importance (Weight)
North	11000	1.4
South	9000	1.0
East	15000	0.8
West	4000	1.1

Table 2: Forecasted monthly demand and importance of each region

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(b) The North MFC has currently received six orders as shown in Table 3. Each rider has capacity to carry a maximum of 40 items. The distance matrix showing the distances (in miles) between the customer locations as well as the MFC is shown Table 4. Determine the minimum number of riders required and the optimal allocation of the orders to each rider such that the overall distance travelled by the riders is minimised.

[40%]

Customer ID	Order size
1	16
2	7
3	15
4	9
5	11
6	18

Table 3: Order details

		MFC	Customers					
			1	2	3	4	5	6
MFC		0	5	6	4	6	3	4
Customers	1	5	0	3	4	6	6	5
	2	6	3	0	4	2	4	5
	3	4	4	4	0	5	3	4
	4	6	6	2	5	0	3	2
	5	3	6	4	3	3	0	4
	6	4	5	5	4	2	4	0

Table 4: Distance Matrix

(c) Forkly commits to delivering customer orders within 30 minutes after they leave the MFC. It is known that the riders travel at a fixed speed of 20 miles per hour. Explain how you would optimise the allocation of orders to riders given the time constraint.

[20%]

Question 6

(a) Flexo produces and sells high precision metal parts. They sell one particular item on a regular basis. At the start of the current week, it has 100 units of this item in stock and no deliveries are expected until next week. The manager knows that weekly sales of this item are normally distributed with an average of 80 and a standard deviation of 15.

(i) Calculate the probability that Flexo will not have enough stock to meet sales in the current week.

[10%]

(ii) Calculate the probability that Flexo will sell more than 50 units of stock in the current week.

[10%]

(b) Recently, the rising cost of running their sintering machines attracted management attention. Operators were convinced that the make of the machine in addition to its usage and age were major determinants of the cost. Flexo operates two different makes of sintering machines – Advent and Basilea. It is thought that the average cost of running Advent machines is higher than Basilea. If a systematic difference in costs between the makes existed, this would have implications for decisions regarding the purchasing of new machines. Before taking any action, the company decided to examine the data and use regression analysis to understand the drivers of cost.

Table 5 presents data for 30 machines. The table shows the cost, age (in years) of each machine and the machine usage (number of parts made in the current year). The "Make" column has the value of 1 for the Advent machines and 0 for the Basilea machines. Fig. 2 presents the correlation matrix for the variables, and Figs. 3, 4 and 5 show the results of three different regression analyses.

(i) Write the regression equations for the three models in figs 3, 4 and 5.

[10%]

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(ii) Using the output in Fig. 3 for the regression of cost on the make of the machines, can you reject the hypothesis that the average cost of running Advent machines is £2,000 higher than that of running Basilea (test at 5% significance level)?

[10%]

(iii) Which model should Flexo use for estimating the cost of running their machines? Explain the rationale for your answer.

[20%]

(iv) Do the regression results confirm the company's suspicion about the importance of the machine make? Explain the rationale for your answer.

[20%]

(v) What would be your estimate for the running cost of a 5-year old Basilea machine producing 50,000 parts/year? Construct an approximate 95% confidence interval for this estimate.

[20%]

	COST	AGE	MAKE	USAGE
COST	1.000			
AGE	0.942	1.000		
MAKE	0.642	0.643	1.000	
USAGE	0.125	-0.016	0.060	1.000

Fig. 2: Correlation Matrix

Regression Statistics	
Multiple R	0.642
R Square	-0.413
Adjusted R Sq.	-0.392
Standard Error	1.519
Observations	30

	Coefficients	*Standard Error*	*t Stat*	*P-Value*	*Lower 95%*	*Upper 95%*
Intercept	7.780	0.368	21.120	0.000	7.030	8.530
MAKE	2.482	0.560	4.430	0.000	1.340	3.630

Fig. 3: Regression Model 1

Regression Statistics	
Multiple R	0.952
R Square	0.907
Adjusted R Sq.	0.900
Standard Error	0.615
Observations	30

	Coefficients	*Standard Error*	*t Stat*	*P-Value*	*Lower 95%*	*Upper 95%*
Intercept	3.357	0.934	3.810	0.000	1.640	5.470
AGE	0.667	0.041	16.090	0.000	0.580	0.750
USAGE	0.043	0.018	2.390	0.020	0.010	0.080

Fig. 4: Regression Model 2

Regression Statistics	
Multiple R	0.953
R Square	0.908
Adjusted R Sq.	0.898
Standard Error	0.623
Observations	30

	Coefficients	*Standard Error*	*t Stat*	*P-Value*	*Lower 95%*	*Upper 95%*
Intercept	3.631	0.955	3.800	0.000	1.670	5.590
AGE	0.647	0.055	11.770	0.000	0.530	0.760
USAGE	0.042	0.018	2.290	0.030	0.000	0.080
MAKE	0.170	0.301	0.560	0.580	-0.450	0.790

Fig. 5: Regression Model 3

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COST (£000s)	AGE (years)	MAKE (0/1)	USAGE (000s)
6.76	1	0	56.40
10.58	5	1	57.60
6.60	3	0	51.60
10.06	7	1	58.80
11.70	8	1	49.20
7.20	3	0	45.60
9.42	4	0	58.80
9.32	6	1	54.00
6.44	1	0	45.60
12.00	10	1	52.80
10.84	6	1	57.60
7.14	3	0	44.40
8.78	4	0	42.00
5.66	0	0	60.00
10.30	6	0	54.00
8.24	3	1	44.40
6.28	1	0	55.20
13.10	10	1	57.60
6.16	1	0	38.40
9.14	6	0	51.60
6.96	1	0	54.00
10.76	8	0	42.00
10.06	8	1	46.80
6.98	3	0	51.60
10.94	8	1	39.60
9.20	5	1	55.20
8.96	5	0	46.80
8.72	4	0	50.40
9.26	6	1	44.40
8.10	5	1	42.00

Table. 5

END OF PAPER

Probability Tables for Question 6

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{u} e^{-\frac{x^2}{2}}\,dx \quad \text{FOR} \quad 0{\cdot}00 \le u \le 4{\cdot}99.$$

u	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7703	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
1·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
1·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·90147
1·3	·90320	·90490	·90658	·90824	·90988	·91149	·91309	·91466	·91621	·91774
1·4	·91924	·92073	·92220	·92364	·92507	·92647	·92785	·92922	·93056	·93189
1·5	·93319	·93448	·93574	·93699	·93822	·93943	·94062	·94179	·94295	·94408
1·6	·94520	·94630	·94738	·94845	·94950	·95053	·95154	·95254	·95352	·95449
1·7	·95543	·95637	·95728	·95818	·95907	·95994	·96080	·96164	·96246	·96327
1·8	·96407	·96485	·96562	·96638	·96712	·96784	·96856	·96926	·96995	·97062
1·9	·97128	·97193	·97257	·97320	·97381	·97441	·97500	·97558	·97615	·97670
2·0	·97725	·97778	·97831	·97882	·97932	·97982	·98030	·98077	·98124	·98169
2·1	·98214	·98257	·98300	·98341	·98382	·98422	·98461	·98500	·98537	·98574
2·2	·98610	·98645	·98679	·98713	·98745	·98778	·98809	·98840	·98870	·98899
2·3	·98928	·98956	·98983	$\cdot 9^2 0097$	$\cdot 9^2 0358$	$\cdot 9^2 0613$	$\cdot 9^2 0863$	$\cdot 9^2 1106$	$\cdot 9^2 1344$	$\cdot 9^2 1576$
2·4	$\cdot 9^2 1802$	$\cdot 9^2 2024$	$\cdot 9^2 2240$	$\cdot 9^2 2451$	$\cdot 9^2 2656$	$\cdot 9^2 2857$	$\cdot 9^2 3053$	$\cdot 9^2 3244$	$\cdot 9^2 3431$	$\cdot 9^2 3613$
2·5	$\cdot 9^2 3790$	$\cdot 9^2 3963$	$\cdot 9^2 4132$	$\cdot 9^2 4297$	$\cdot 9^2 4457$	$\cdot 9^2 4614$	$\cdot 9^2 4766$	$\cdot 9^2 4915$	$\cdot 9^2 5060$	$\cdot 9^2 5201$
2·6	$\cdot 9^2 5339$	$\cdot 9^2 5473$	$\cdot 9^2 5604$	$\cdot 9^2 5731$	$\cdot 9^2 5855$	$\cdot 9^2 5975$	$\cdot 9^2 6093$	$\cdot 9^2 6207$	$\cdot 9^2 6319$	$\cdot 9^2 6427$
2·7	$\cdot 9^2 6533$	$\cdot 9^2 6636$	$\cdot 9^2 6736$	$\cdot 9^2 6833$	$\cdot 9^2 6928$	$\cdot 9^2 7020$	$\cdot 9^2 7110$	$\cdot 9^2 7197$	$\cdot 9^2 7282$	$\cdot 9^2 7365$
2·8	$\cdot 9^2 7445$	$\cdot 9^2 7523$	$\cdot 9^2 7599$	$\cdot 9^2 7673$	$\cdot 9^2 7744$	$\cdot 9^2 7814$	$\cdot 9^2 7882$	$\cdot 9^2 7948$	$\cdot 9^2 8012$	$\cdot 9^2 8074$
2·9	$\cdot 9^2 8134$	$\cdot 9^2 8193$	$\cdot 9^2 8250$	$\cdot 9^2 8305$	$\cdot 9^2 8359$	$\cdot 9^2 8411$	$\cdot 9^2 8462$	$\cdot 9^2 8511$	$\cdot 9^2 8559$	$\cdot 9^2 8605$
3·0	$\cdot 9^2 8650$	$\cdot 9^2 8694$	$\cdot 9^2 8736$	$\cdot 9^2 8777$	$\cdot 9^2 8817$	$\cdot 9^2 8856$	$\cdot 9^2 8893$	$\cdot 9^2 8930$	$\cdot 9^2 8965$	$\cdot 9^2 8999$
3·1	$\cdot 9^3 0324$	$\cdot 9^3 0646$	$\cdot 9^3 0957$	$\cdot 9^3 1260$	$\cdot 9^3 1553$	$\cdot 9^3 1836$	$\cdot 9^3 2112$	$\cdot 9^3 2378$	$\cdot 9^3 2636$	$\cdot 9^3 2886$
3·2	$\cdot 9^3 3129$	$\cdot 9^3 3363$	$\cdot 9^3 3590$	$\cdot 9^3 3810$	$\cdot 9^3 4024$	$\cdot 9^3 4230$	$\cdot 9^3 4429$	$\cdot 9^3 4623$	$\cdot 9^3 4810$	$\cdot 9^3 4991$
3·3	$\cdot 9^3 5166$	$\cdot 9^3 5335$	$\cdot 9^3 5499$	$\cdot 9^3 5658$	$\cdot 9^3 5811$	$\cdot 9^3 5959$	$\cdot 9^3 6103$	$\cdot 9^3 6242$	$\cdot 9^3 6376$	$\cdot 9^3 6505$
3·4	$\cdot 9^3 6631$	$\cdot 9^3 6752$	$\cdot 9^3 6869$	$\cdot 9^3 6982$	$\cdot 9^3 7091$	$\cdot 9^3 7197$	$\cdot 9^3 7299$	$\cdot 9^3 7398$	$\cdot 9^3 7493$	$\cdot 9^3 7585$
3·5	$\cdot 9^3 7674$	$\cdot 9^3 7759$	$\cdot 9^3 7842$	$\cdot 9^3 7922$	$\cdot 9^3 7999$	$\cdot 9^3 8074$	$\cdot 9^3 8146$	$\cdot 9^3 8215$	$\cdot 9^3 8282$	$\cdot 9^3 8347$
3·6	$\cdot 9^3 8409$	$\cdot 9^3 8469$	$\cdot 9^3 8527$	$\cdot 9^3 8583$	$\cdot 9^3 8637$	$\cdot 9^3 8689$	$\cdot 9^3 8739$	$\cdot 9^3 8787$	$\cdot 9^3 8834$	$\cdot 9^3 8879$
3·7	$\cdot 9^3 8922$	$\cdot 9^3 8964$	$\cdot 9^4 0039$	$\cdot 9^4 0426$	$\cdot 9^4 0799$	$\cdot 9^4 1158$	$\cdot 9^4 1504$	$\cdot 9^4 1838$	$\cdot 9^4 2159$	$\cdot 9^4 2468$
3·8	$\cdot 9^4 2765$	$\cdot 9^4 3052$	$\cdot 9^4 3327$	$\cdot 9^4 3593$	$\cdot 9^4 3848$	$\cdot 9^4 4094$	$\cdot 9^4 4331$	$\cdot 9^4 4558$	$\cdot 9^4 4777$	$\cdot 9^4 4988$
3·9	$\cdot 9^4 5190$	$\cdot 9^4 5385$	$\cdot 9^4 5573$	$\cdot 9^4 5753$	$\cdot 9^4 5926$	$\cdot 9^4 6092$	$\cdot 9^4 6253$	$\cdot 9^4 6406$	$\cdot 9^4 6554$	$\cdot 9^4 6696$
4·0	$\cdot 9^4 6833$	$\cdot 9^4 6964$	$\cdot 9^4 7090$	$\cdot 9^4 7211$	$\cdot 9^4 7327$	$\cdot 9^4 7439$	$\cdot 9^4 7546$	$\cdot 9^4 7649$	$\cdot 9^4 7748$	$\cdot 9^4 7843$
4·1	$\cdot 9^4 7934$	$\cdot 9^4 8022$	$\cdot 9^4 8106$	$\cdot 9^4 8186$	$\cdot 9^4 8263$	$\cdot 9^4 8338$	$\cdot 9^4 8409$	$\cdot 9^4 8477$	$\cdot 9^4 8542$	$\cdot 9^4 8605$
4·2	$\cdot 9^4 8665$	$\cdot 9^4 8723$	$\cdot 9^4 8778$	$\cdot 9^4 8832$	$\cdot 9^4 8882$	$\cdot 9^4 8931$	$\cdot 9^4 8978$	$\cdot 9^5 0226$	$\cdot 9^5 0655$	$\cdot 9^5 1066$
4·3	$\cdot 9^5 1460$	$\cdot 9^5 1837$	$\cdot 9^5 2199$	$\cdot 9^5 2545$	$\cdot 9^5 2876$	$\cdot 9^5 3193$	$\cdot 9^5 3497$	$\cdot 9^5 3788$	$\cdot 9^5 4066$	$\cdot 9^5 4332$
4·4	$\cdot 9^5 4587$	$\cdot 9^5 4831$	$\cdot 9^5 5065$	$\cdot 9^5 5288$	$\cdot 9^5 5502$	$\cdot 9^5 5706$	$\cdot 9^5 5902$	$\cdot 9^5 6089$	$\cdot 9^5 6268$	$\cdot 9^5 6439$
4·5	$\cdot 9^5 6602$	$\cdot 9^5 6759$	$\cdot 9^5 6908$	$\cdot 9^5 7051$	$\cdot 9^5 7187$	$\cdot 9^5 7318$	$\cdot 9^5 7442$	$\cdot 9^5 7561$	$\cdot 9^5 7675$	$\cdot 9^5 7784$
4·6	$\cdot 9^5 7888$	$\cdot 9^5 7987$	$\cdot 9^5 8081$	$\cdot 9^5 8172$	$\cdot 9^5 8258$	$\cdot 9^5 8340$	$\cdot 9^5 8419$	$\cdot 9^5 8494$	$\cdot 9^5 8566$	$\cdot 9^5 8634$
4·7	$\cdot 9^5 8699$	$\cdot 9^5 8761$	$\cdot 9^5 8821$	$\cdot 9^5 8877$	$\cdot 9^5 8931$	$\cdot 9^5 8983$	$\cdot 9^6 0320$	$\cdot 9^6 0789$	$\cdot 9^6 1235$	$\cdot 9^6 1661$
4·8	$\cdot 9^6 2067$	$\cdot 9^6 2453$	$\cdot 9^6 2822$	$\cdot 9^6 3173$	$\cdot 9^6 3508$	$\cdot 9^6 3827$	$\cdot 9^6 4131$	$\cdot 9^6 4420$	$\cdot 9^6 4696$	$\cdot 9^6 4958$
4·9	$\cdot 9^6 5208$	$\cdot 9^6 5446$	$\cdot 9^6 5673$	$\cdot 9^6 5889$	$\cdot 9^6 6094$	$\cdot 9^6 6289$	$\cdot 9^6 6475$	$\cdot 9^6 6652$	$\cdot 9^6 6821$	$\cdot 9^6 6981$

Example: $\Phi(3{\cdot}57) = \cdot 9^3 8215 = 0{\cdot}9998215$.

FRACTILES OF THE t DISTRIBUTION. $t_{1-P} = -t_P$.

f \ P	Probability in per cent 60	70	80	90	95	97·5	99	99·5	99·9	99·95
1	·325	·727	1·376	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	·289	·617	1·061	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	·277	·584	·978	1·638	2·353	3·182	4·541	5·841	10·22	12·94
4	·271	·569	·941	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	·267	·559	·920	1·476	2·015	2·571	3·365	4·032	5·893	6·859
6	·265	·553	·906	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	·263	·549	·896	1·415	1·895	2·365	2·998	3·499	4·785	5·405
8	·262	·546	·889	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	·261	·543	·883	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	·260	·542	·879	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	·260	·540	·876	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	·259	·539	·873	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	·259	·538	·870	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	·258	·537	·868	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	·258	·536	·866	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	·258	·535	·865	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	·257	·534	·863	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	·257	·534	·862	1·330	1·734	2·101	2·552	2·878	3·611	3·922
19	·257	·533	·861	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	·257	·533	·860	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	·257	·532	·859	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	·256	·532	·858	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	·256	·532	·858	1·319	1·714	2·069	2·500	2·807	3·485	3·767
24	·256	·531	·857	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	·256	·531	·856	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	·256	·531	·856	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	·256	·531	·855	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	·256	·530	·855	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	·256	·530	·854	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	·256	·530	·854	1·310	1·697	2·042	2·457	2·750	3·385	3·646
40	·255	·529	·851	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	·255	·528	·849	1·298	1·676	2·009	2·403	2·678	3·262	3·495
60	·254	·527	·848	1·296	1·671	2·000	2·390	2·660	3·232	3·460
80	·254	·527	·846	1·292	1·664	1·990	2·374	2·639	3·195	3·415
100	·254	·526	·845	1·290	1·660	1·984	2·365	2·626	3·174	3·389
200	·254	·525	·843	1·286	1·653	1·972	2·345	2·601	3·131	3·339
500	·253	·525	·842	1·283	1·648	1·965	2·334	2·586	3·106	3·310
∞	·253	·524	·842	1·282	1·645	1·960	2·326	2·576	3·090	3·291
$2(1-P)$	80	60	40	20	10	5	2	1	0·2	0·1

Example: $P\{t < 2{\cdot}086\} = 97{\cdot}5\%$ for $f = 20$.

$P\{|t| > t_P\} = 2(1-P)$. $P\{|t| > 2{\cdot}086\} = 5\%$ for $f = 20$.

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