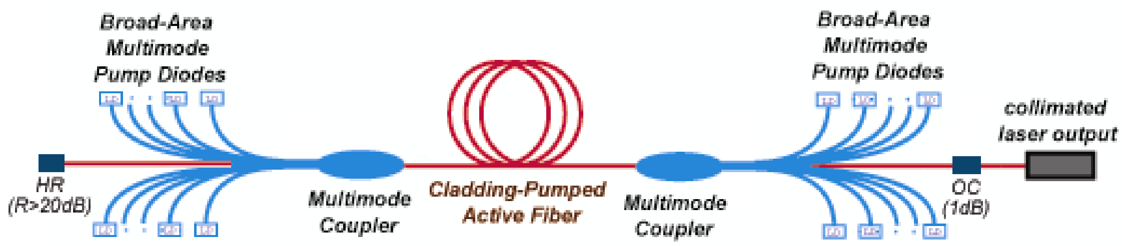
METIIB Paper 1 Question 1

Crib.

a) A laser (Light Amplification by Stimulated Emission of Radiation) operates by producing a highly coherent, monochromatic, and directional light beam through three fundamental principles: stimulated emission, population inversion, and optical feedback. Stimulated emission occurs when an incoming photon interacts with an excited atom, prompting it to release another photon identical in phase, wavelength, and direction. This process amplifies light, forming the basis of laser operation. Achieving population inversion is essential, where more atoms or molecules are in an excited state than in the ground state. This is accomplished by supplying energy through a pumping mechanism, such as optical pumping, electrical discharge, or chemical reactions, which excites atoms to higher energy levels. Optical feedback occurs within a resonator cavity, typically composed of two mirrors—one fully reflective and one partially reflective. Photons generated by stimulated emission reflect back and forth, stimulating further emissions and amplifying light. When the gain (light amplification) exceeds system losses, the laser emits a highly concentrated beam. This light is characterised by its coherence, narrow wavelength, and minimal divergence, making it suitable for applications like manufacturing, medical procedures, and communication. Lasers exemplify precision and efficiency in manipulating light for diverse scientific and industrial uses.

Note: graphics may also be provided.

b)
(i) A fiber laser operates by generating and amplifying light within an optical fiber doped with rare-earth elements, such as ytterbium, erbium, or thulium. The basic principles include stimulated emission, optical pumping, and waveguiding within the fiber core.
Optical pumping involves delivering energy into the doped fiber via a high-power pump source, typically diode lasers. This energy excites electrons in the dopant atoms to higher energy levels. When these electrons return to lower energy states, they release photons, initiating the process of stimulated emission, where an incoming photon induces the emission of additional photons with identical phase, wavelength, and direction.
The optical fiber acts as both the gain medium and a waveguide, confining light to the core through total internal reflection. Mirrors at either end of the fiber form an optical resonator, enhancing the feedback necessary for amplification. One mirror is partially reflective, allowing a portion of the amplified light to escape as the laser beam.
Fiber lasers are highly efficient, compact, and capable of producing high-quality beams with minimal divergence. They are robust, with high energy conversion efficiency, and adaptability to emit various wavelengths.



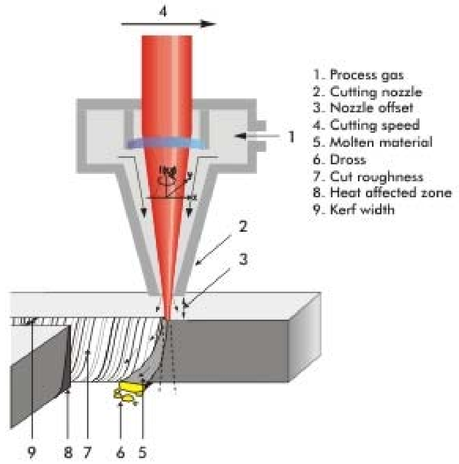
(ii)

Many examples were presented in the lectures

Options could include

Cutting
Fiber lasers deliver precision cutting by concentrating high-energy beams to melt or vaporise material along a defined path. They provide clean, accurate edges with minimal thermal distortion.

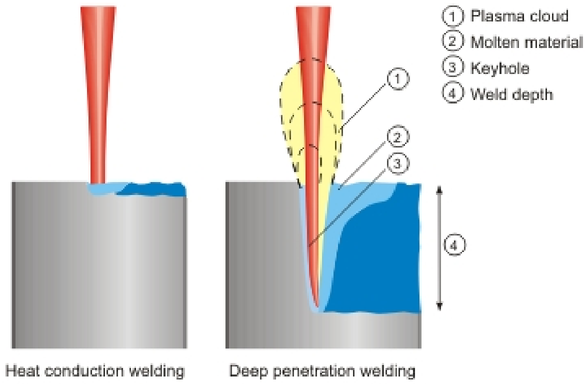
- **Applications**: Sheet metal fabrication, automotive parts manufacturing, and electronic enclosures.



Traditional Process Displaced: Mechanical cutting (e.g., saws, punches) and plasma cutting often produce rough edges, significant heat-affected zones, and material waste.
New Capability: Fiber laser cutting delivers clean, precise edges with minimal thermal distortion, enabling complex patterns and fine details in metals, plastics, and composites.

Welding
Fiber lasers create high-strength welds by fusing materials using concentrated heat. Their precision minimises heat-affected zones and distortion.

- **Applications**: Automotive chassis welding, aerospace component assembly, and medical device fabrication.



Traditional Process Displaced: Arc welding and TIG/MIG welding involve significant heat input, causing distortion and larger heat-affected zones, especially in delicate components.
New Capability: Fiber laser welding creates high-strength, precise welds with minimal distortion, suitable for thin materials and high-speed automation.

Surface Treatment
Fiber lasers modify surfaces through heat, enabling processes like hardening, texturing, or coating removal. This enhances material properties or surface aesthetics.

- **Applications**: Wear-resistant tool hardening, solar panel texturing, and rust or paint removal.

Traditional Process Displaced: Flame or induction hardening, mechanical texturing, and chemical treatments are less precise and can damage substrates or produce inconsistent results.
New Capability: Fiber lasers offer localised surface hardening, precision texturing, and controlled material removal, enhancing wear resistance or functional surface properties.

Engraving/Marking
Fiber lasers etch permanent, high-contrast marks on materials, such as text, logos, or serial numbers, with excellent precision.

- **Applications**: Branding and identification on medical instruments, electronic components, and luxury goods.

Traditional Process Displaced: Mechanical engraving and chemical etching are slower, less precise, and environmentally harmful due to the use of toxic chemicals.
New Capability: Fiber laser engraving delivers highly detailed, permanent marks on a wide range of materials, including metals and plastics, with excellent speed and precision.

Additive Manufacturing (3D Printing)
Fiber lasers melt metallic powders layer by layer to build complex 3D structures with high precision.

- **Applications**: Aerospace components, medical implants, and custom tooling.

Traditional Process Displaced: Subtractive manufacturing, such as CNC machining, generates waste and limits design freedom for complex geometries.

New Capability: Enables the production of intricate, lightweight, and customised parts, such as lattice structures and complex internal channels, not possible with traditional methods.

Drilling

Fiber lasers create precise, high-aspect-ratio holes in materials by vaporising localized areas.

- **Applications**: Gas turbine blades, fuel injection nozzles, and microelectronics.

Traditional Process Displaced: Mechanical drilling struggles with high-aspect-ratio holes, hard materials, and heat-sensitive substrates, often leading to tool wear or damage.
New Capability: Fiber laser drilling provides precise, clean holes in challenging materials like ceramics and composites without contact or wear.

Cladding

Fiber lasers deposit material onto a substrate by melting a feedstock, creating a protective or wear-resistant layer.

- **Applications**: Surface repair of worn machine parts, corrosion-resistant coatings, and pipeline restoration.

Traditional Process Displaced: Arc welding or thermal spraying results in excessive heat input, poor bonding, and limited precision.
New Capability: Fiber laser cladding achieves localised, high-quality material deposition with minimal waste and reduced thermal distortion, ideal for repairs and wear-resistant coatings.

Cleaning

Fiber lasers remove contaminants, coatings, or rust from surfaces through controlled ablation, without damaging the substrate.

- **Applications**: Paint stripping, mould cleaning, and oxide removal in welding preparation.

Traditional Process Displaced: Abrasive blasting and chemical cleaning are labor-intensive, environmentally harmful, and risk damaging delicate surfaces.
New Capability: Laser cleaning offers non-contact, eco-friendly, and precise surface preparation or contaminant removal, suitable for sensitive materials.

Micro-Machining

Fiber lasers perform precise machining tasks on micro-scale components by ablating or vaporising material with minimal thermal impact.

- **Applications**: Semiconductor fabrication, medical stents, and MEMS (micro-electromechanical systems) production.

Traditional Process Displaced: Mechanical or chemical micro-machining methods lack precision and can damage thin or delicate materials due to heat or mechanical stress.

New Capability: Fiber laser micro-machining ensures minimal thermal impact and exceptional precision, enabling intricate designs in semiconductors, medical devices, and MEMS.

c)
Benefits:

- **Energy Efficiency**: Fiber lasers have a higher electrical-to-optical efficiency (~35%) compared to CO_2 lasers (~10-20%), reducing operating costs.
- **Material Compatibility**: Fiber lasers are more effective for cutting reflective metals, such as aluminium and copper, without beam instability.
- **Lower Maintenance**: Fiber lasers lack complex optics or gas flow systems, making them more robust and easier to maintain.
- **Compact Design**: The smaller footprint of fiber lasers simplifies integration into production lines.

Challenges:

- **Higher Initial Investment**: The upfront cost of fiber lasers is generally higher than CO_2 systems.
- **Learning Curve**: Operators may require training to adapt to fiber laser systems and their specific parameters.
- **Specific Applications**: For some non-metal materials (e.g., certain plastics), CO_2 lasers may still perform better due to their longer wavelength.

Conclusion: Transitioning to fiber lasers is justified if the company prioritises operational efficiency, versatility, and the ability to process reflective materials. The long-term savings in energy and maintenance costs, along with improved performance, outweigh the initial investment and challenges.

d) Environmental benefits
Energy Savings:

- Fiber lasers' high efficiency lowers energy consumption, reducing greenhouse gas emissions associated with electricity generation.
- **Example**: Automotive manufacturers report significant energy savings when transitioning to fiber lasers for cutting and welding.

Reduced Waste:

- Fiber lasers produce minimal material waste due to their precision, minimising the need for rework or excess material.
- **Example**: Precision cutting of metals results in cleaner cuts and less scrap compared to traditional methods.

Elimination of Hazardous Chemicals:

- Fiber lasers eliminate the need for consumable gases or abrasive chemicals in cleaning, cutting, or marking processes.
- **Example**: Laser cleaning replaces chemical cleaning agents, reducing environmental contamination.

e) Challenges
Resource-Intensive Manufacturing:

- The production of fiber lasers involves rare-earth elements (e.g., ytterbium) with environmentally damaging extraction processes.
- **Mitigation**: Develop recycling programs for end-of-life laser systems to recover rare materials and reduce reliance on new mining operations.

Electronic Waste:

- Fiber lasers contribute to electronic waste at the end of their lifecycle.
- **Mitigation**: Extend the lifespan of fiber lasers through modular designs for easy repair and upgrades, and promote responsible recycling initiatives.

Question 2

Crib

a)

Silicon Wafer Production:

- High-purity silicon is extracted from quartz sand and refined into ingots through processes like the Czochralski method.
- The ingots are sliced into thin wafers and polished for a smooth surface.

Photolithography:

- A photosensitive resist is applied to the wafer.
- Light, often from extreme ultraviolet (EUV) sources, is projected through masks to transfer intricate patterns of transistors and circuits onto the wafer.

Etching:

- Unwanted material is removed using chemical or plasma etching, creating the desired circuit patterns.

Doping:

- Ion implantation introduces impurities into specific regions of the silicon, altering its electrical properties.

Deposition:

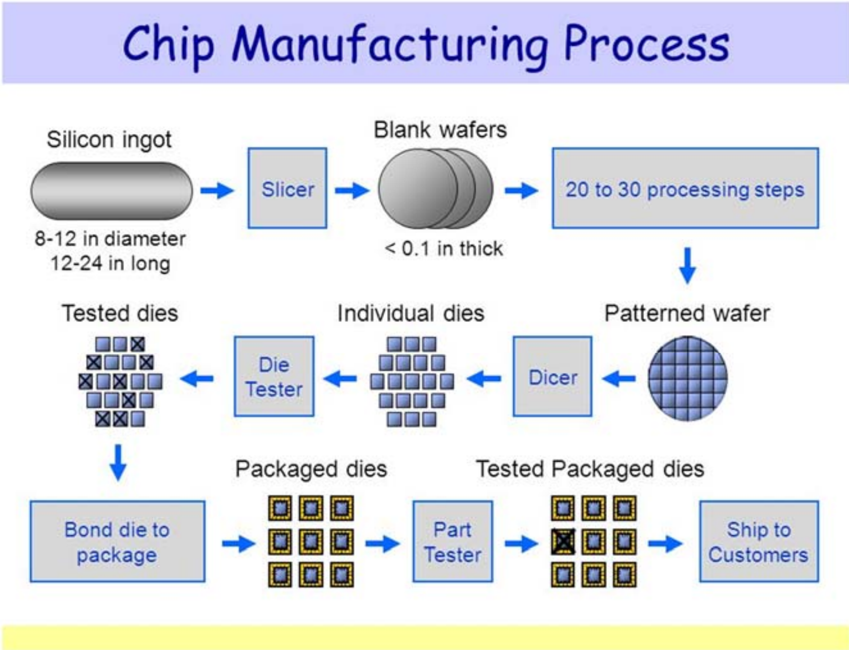
- Layers of conductive or insulating materials are deposited using techniques like chemical vapor deposition (CVD) or physical vapor deposition (PVD).

Metal Interconnects:

- Metal layers (e.g., copper) are added to connect transistors, forming functional circuits.

Packaging:

- Completed chips are encapsulated for protection and connectivity, ensuring functionality in electronic devices.



b)

1. **Physical Limitations**:
   - **Quantum Effects**: At sub-5nm scales, quantum tunnelling occurs, causing current leakage and reducing reliability.
   - **Heat Dissipation**: Smaller transistors generate significant heat, challenging thermal management.
2. **Economic Barriers**:
   - **Rising Costs**: The cost of advanced fabs exceeds billions of dollars, making further scaling economically unsustainable.
   - **Diminishing Returns**: The benefits of adding more transistors per chip are reducing compared to the investment required.
3. **Technological Challenges**:
   - **EUV Lithography Complexity**: Achieving precise patterns at atomic scales requires advanced and expensive equipment.
   - **Material Limitations**: Silicon itself reaches performance limits, demanding new materials for further progress.

c) Emerging Solutions

Chip Architecture:

- **Description**: Chips are stacked vertically instead of being spread out in a single plane, increasing transistor density without reducing feature size.
- **Applications**: Used in high-performance processors and memory devices, such as 3D NAND flash.
- **Evaluation**: Improves efficiency and reduces latency but presents thermal challenges due to the compact design.

Alternative Materials:

- **Description**: Materials like graphene, carbon nanotubes, and silicon carbide (SiC) offer higher electron mobility and better thermal conductivity than silicon.
- **Applications**: Graphene is being explored for ultrafast transistors, while SiC is used in power electronics for electric vehicles.
- **Evaluation**: Promises higher performance but faces challenges in large-scale manufacturing and integration with current technology.

Alternative processes:
A number where discussed across the module

- **Printed electronics. Process Description:** Uses organic semiconductors or printable materials to create flexible, lightweight devices. Processes include inkjet printing and roll-to-roll manufacturing.
    - **Advantage**: Enables the production of bendable, stretchable electronics for applications like wearables and foldable displays.
    - **Challenges**: Lower performance compared to silicon chips for high-speed applications.
    - **Environmental Impact**: Reduced material waste due to additive **manufacturing techniques**
- **CNTs. Process Description:** Uses carbon nanotubes to create transistors with higher electron mobility and lower power consumption than silicon.
    - **Advantage:** Enables faster, more efficient processors with minimal heat generation.
    - **Challenges:** Difficulties in large-scale manufacturing and uniformity of nanotube alignment**.**
    - **Environmental Impact:** Potentially lower energy consumption in operation but concerns about the environmental cost of nanotube synthesis**.**

d) Environmental Impacts and Mitigation Strategies

Environmental Impacts:
<u>High Energy Consumption:</u>

- Fabrication facilities (fabs) are energy-intensive, requiring clean rooms, lithography, and cooling systems.
- **Example**: A single fab can consume electricity equivalent to a small city.

<u>Water Usage:</u>

- Large volumes of ultrapure water (UPW) are needed for wafer cleaning, leading to significant resource consumption.

<u>Chemical Waste:</u>

- Toxic chemicals used in etching, doping, and cleaning can contaminate water and air if not managed properly.

<u>Rare Materials:</u>

- Mining rare materials like cobalt and rare-earth elements for chip manufacturing harms ecosystems and involves significant carbon emissions.

Mitigation Strategies:
<u>Energy Efficiency:</u>

- Transition fabs to renewable energy sources and improve energy efficiency in equipment.
- **Example**: Semiconductor companies like TSMC are investing in solar and wind energy.

<u>Recycling and Material Recovery:</u>

- Implement recycling programs for rare materials from end-of-life chips to reduce reliance on mining.

<u>Water Recycling:</u>

- Use advanced filtration systems to recycle UPW within fabs, reducing overall water consumption.

<u>Green Manufacturing Techniques:</u>

- Develop low-impact manufacturing methods, such as replacing hazardous chemicals with safer alternatives.

Question 3

Crib

ai)

Components	
Robot	Industrial anthropomorphic robot
Robot Gripper	Pneumatic pincer gripper
Vision / Laser System	3D Vision, incorporating a laser
Component Flipper	Flip Station
Roller Conveyor	Gravity / Power fed roller conveyor (For Totes)
Tote Shaker	Shaker (For Totes)
Kitting Tray Locator	Location points (For Kitting Trays)
PLC	PLC
ANDON Lights	Status (For Operators)
Guard / Guard Tunnel	Guard
Safety System / Door Interlocks	Safety System
Proximity Sensors	Sensors – Several inductive proximity sensors (For status information)

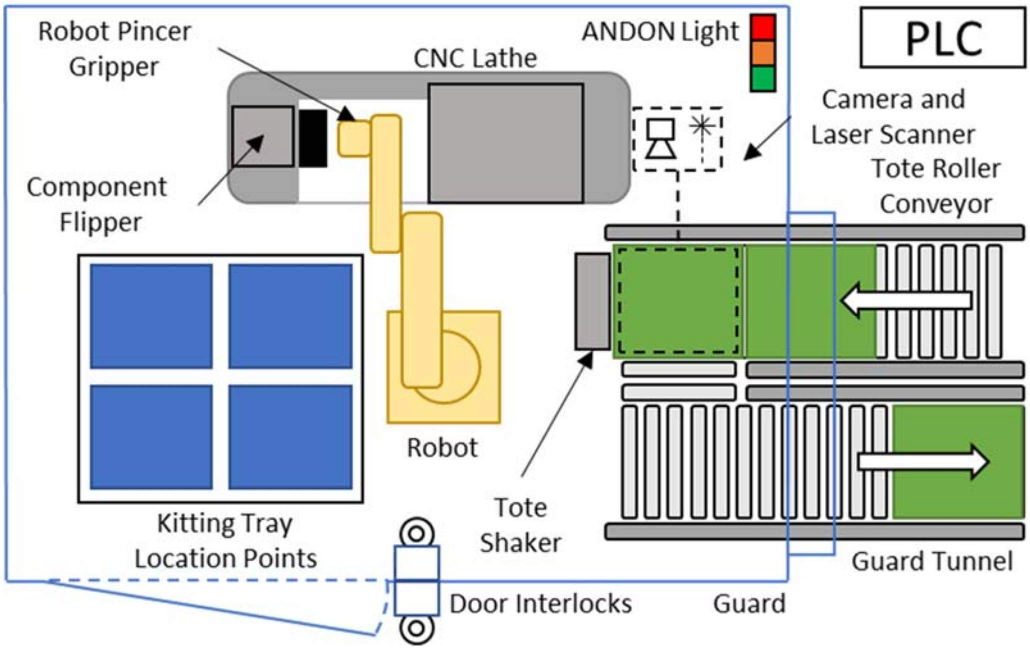
aii)

Components	**Capabilities**
Robot	Industrial anthropomorphic robot – This configuration of robot will have the appropriate flexibility to access the chuck within the CNC lathe as well as to pick jumbled castings from the inbound totes.
Robot Gripper	Pneumatic pincer gripper – This gripper type will be best suited for handling the rough surfaces of the castings as well as finished components covered in cutting fluid. It may be possible to incorporate features of the casting / component into the gripper plates, aiding component alignment. To improve the performance of the pick place operation (Flipping component) it would be beneficial to have a dual headed gripper.
Vision / Laser System	3D Vision, incorporating a laser scanner – This is often an optional capability on the robot controller to allow bin picking applications to be

	run. The vision solution will singulate a reachable component within the bin, providing the robot with the pick locations.
Component Flipper	Flip Station – Once the component has been removed from the CNC machine, it needs to be placed so that the robot can regrip the part in a new orientation.
Roller Conveyor	Gravity / Power fed roller conveyor – This is a cost-effective solution for feeding totes. Tote location is not critical as a vision solution will be used for picking parts. The conveyor is in a u formation allowing inbound totes to be loaded for a full shift of operations and empty totes collected from the outbound area. Both the inbound and outbound conveyor feeds can be serviced without entering the guard.
Tote Shaker	Shaker – The tote shaker works with the 3D vision system. When no castings can be picked by the robot, the tote can be vibrated, potentially making a casting accessible. (Potential damage to castings should be considered.)
Kitting Tray Locator	Locator – The kitting trays are loaded and unloaded by the operator at the beginning / end of shift. Kitting trays are placed on alignment pins, providing registered locations for part placements into kitting tray wells.
PLC	PLC – A programable logic controller will be used to sequence the operation of the CNC lathe, Robot, and tote conveyor.
ANDON Lights	Status – The status of the system will be displayed to the operator via an ANDON light system. (Red - System stopped and needs attention, Yellow - System running and needs attention, Green - System running)
Guard / Guard Tunnel	Guard – The design of this system requires workers and operators to be protected by a guard. Kitting trays can be replenished when the system is in a safe state and the guard door has been opened.
Safety System / Door Interlocks	Safety System – The tending solution has an integral safety system, ensuring the system will only operate under safe conditions with all guards closed. Fortress key locks are used on guard doors controlling access to dangerous areas. When the guards are in an unsafe state the CNC lathe and robot will not be able to operate. Certain key states can allow maintenance processes to be carried out.
Proximity Sensors	Sensors – Several inductive proximity sensors will be required to determine the state of the system to ensure robust control. Detect the presence of: a) tote at the end of roller conveyor. b) 4 x Kitting trays on location points. c) Component in flip station.

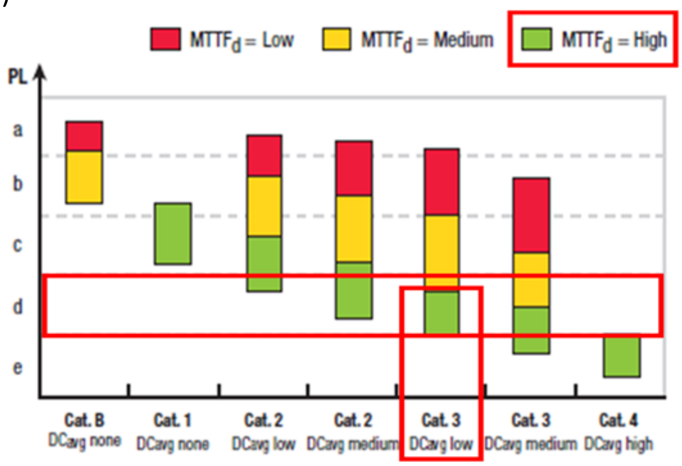
CNC Feature / Mod.	**Details**
Auto Guard Door	Pneumatically operated sliding door to provide access to machine.
Auto Hydraulic Chuck	Hydraulic chuck for gripping and releasing components.
IO Interface / Ctrl.	PLC – CNC Controller interface. Remote cycle operations from PLC. (Program selection, Program start, Status information)
IO Interface / Safety.	Safety System – CNC Controller (Integrated safety operations) Safety System – Robot Controller (Integrated safety operations)

aiii)



The previous diagram shows the floor layout for the part tending solution. This solution provides some level of buffering of inbound totes and outbound kitting trays allowing the system to run fully automatically for a period of time. This solution does limit manual access to the CNC, making it more challenging to run traditional manual processes. This could be addressed if the robot was mounted on a linear rail allowing the kitting tray location and the roller conveyors to be spread further apart. As an industrial robot is being used, a guard system has been included. It may be possible to use a collaborative robot and eliminate the guard, but robot cycle times would be slower and the robot payload would be less.

bi)



A safety system with a Performance Level (d) is required.

Selecting PL lane d, shows that a Cat 3 safety system architecture should be considered, requiring either a low or medium diagnostic coverage and a high mean time to dangerous failure. Cat 4 solution can also be used but it would be above specification requirements.

A Cat 3 safety systems architecture with a low diagnostic coverage and a high mean time to dangerous failure would be an initial suggestion.

The chart pulls together important issues in implementing a safety control system. PL is the performance level required for the final system, (e) being the most challenging. Cat. defines the different system / hardware architectures that can be implemented, with Cat 4 being the most capable. Diagnostic Coverage (DCavg) is the capability of the system to detect failures, with the more capable systems (Cat 4) having a higher diagnostic capability. Mapped across all these factors is mean time to dangerous failure (MTTFd), this considers

the quality of the safety system components and their installation. To achieve the higher PL levels the most capable system architectures are required using good quality components.
Discussion equipment selection and architecture features:
MTTFd = High, Use components that typically fail between 30 and 100 Years. Thus, requiring components to be selected from reputable suppliers and implemented using best practices.
Cat. 3 Architecture with DCavg = Low, This uses dual channel circuits for checking E-Stop switches and dual contactors on safety outputs to eliminate any single points of failure. Auxiliary switches on output contactors are monitored prior to and during power up cycles to ensure safe operations. Duplicate hardware to eliminate single points of failure is required in this solution.

Question 4

Crib

(a) Definition: Double marginalisation is the distortion caused by successive markups of independent firms in a multi-firm logistics network. When each firm in the supply chain (e.g., manufacturer, wholesaler, retailer) applies its own markup to maximise profit, it leads to higher overall price for the customer and causes inefficiencies. The implication that this reduces firm profits and harms consumers is known as the double-marginalization problem.

Example of double marginalisation: A manufacturer produces a gadget at £10 and sells it to a distributor for £20, who marks it up to £40 for retailers. Retailers then add their own markup, selling it to consumers for £60, inflating the selling price due to double marginalization.

Any examples that reasonably explain the overall cost increase to the customer, in the process of markups by individual firms to optimise their proves, shall be awarded marks

(b) Differences between the logistics operations of a high-end consumer electronics manufacturer versus that of a low-priced clothing manufacturer are below:

i. Electronics Manufacturer: Typically deals with high-value, low-volume orders requiring precise demand forecasting and customised configurations
Clothing Company: Manages lower-value, high-volume orders with seasonal demand patterns and a focus on inventory variety to meet fashion trends

ii. Electronics Manufacturer: Emphasizes secure, fast, and damage-resistant shipping due to the high value and fragile products. Often uses air freight for speed
Clothing Company: Prioritizes cost-effective bulk shipping (e.g., sea or rail freight), as clothing is less sensitive to transportation conditions

iii. Electronics Manufacturer: Requires climate-controlled, secure warehouses to protect sensitive components and finished products.

Clothing Company: Focuses on efficient storage for high inventory turnover, often using standard warehouses with easy access to manage frequent restocking

iv. Electronics Manufacturer: Focuses on returns for repairs, refurbishment, or recycling due to the high value of components and strict environmental regulations (e.g., e-waste management). Also requires specialised handling to recover materials like precious metals or dispose hazardous components responsibly.

Clothing company: Primarily deals with returns for size, fit, or fashion preferences, often reselling returned items if in good condition. Reverse logistics is less technical and focuses on restocking, repackaging, or reselling items at a discount (e.g., in outlet stores).

****Students are expected to state any three differences. Any points that mention reasonable differences between the logistics operations, in line with the lecture slides, shall be awarded marks*****

(c) The logistics network provided in the question is:

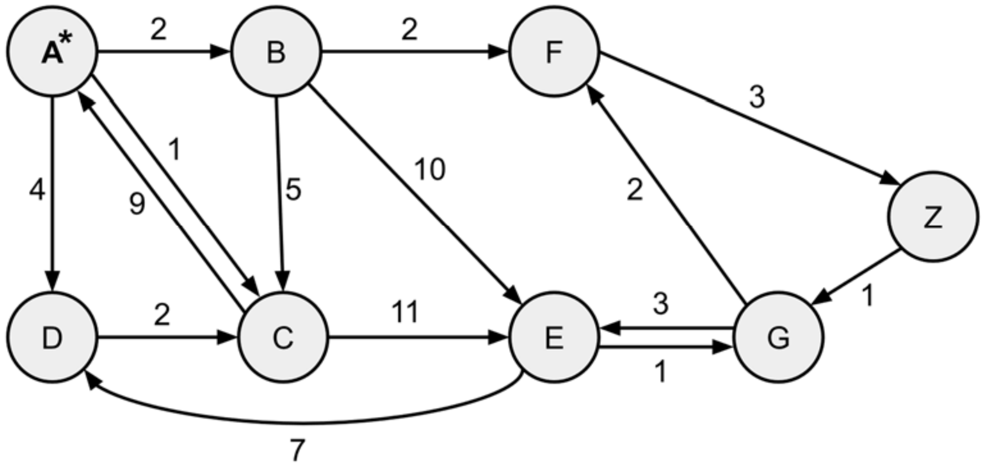


Fig. 1

Where the nodes represent various facilities and the edges representing the paths that exist between the facilities. Starting location is given to be A^* and the objective is to evaluate the shortest distances from A^* to all other locations in the given logistics network using Dijkstra's algorithm.

The below table represents the steps followed in the Dijkstra's algorithm, with the next shortest distance unexplored node highlighted in each step starting with A^*. The superscripts against each of the explored nodes show their corresponding previous nodes and distances.

Explored	Unexplored
	A^{0} B^{∞} C^{∞} D^{∞} E^{∞} F^{∞} G^{∞} Z^{∞}
A^{0}	B^{2A} C^{1A} D^{4A} E^{∞} F^{∞} G^{∞} Z^{∞}
A^{0} C^{1A}	B^{2A} D^{4A} E^{11C} F^{∞} G^{∞} Z^{∞}
A^{0} C^{1A} B^{2A}	D^{4A} E^{10B} F^{2B} G^{∞} Z^{∞}
A^{0} C^{1A} B^{2A} F^{2B}	D^{4A} E^{10B} G^{∞} Z^{3F}
A^{0} C^{1A} B^{2A} F^{2B} Z^{3F}	D^{4A} E^{10B} G^{1Z}
A^{0} C^{1A} B^{2A} F^{2B} Z^{3F} G^{1Z}	D^{4A} E^{3G}
A^{0} C^{1A} B^{2A} F^{2B} Z^{3F} G^{1Z} E^{3G}	D^{4A}
A^{0} C^{1A} B^{2A} F^{2B} Z^{3F} G^{1Z} E^{3G} D^{4A}	

In each of the above rows the nodes that are connected to the current node are *relaxed*, and the corresponding parent node is mentioned against the relaxed distance.

The node with the shortest distance is then chosen as the next parent node. The solution string obtained above is: **A^{0} C^{1A} B^{2A} F^{2B} Z^{3F} G^{1Z} E^{3G} D^{4A}**

We can see that the shortest distances to all other nodes from node A^{*} are:

B: A → B distance = 2
C: A → C distance = 1
D: A → D distance = 4
E: A → B → F → Z → G → E distance = 11
F: A → B → F distance = 4
G: A → B → F → Z → G distance = 8
Z: A → B → F → Z distance = 7

(d) <u>(i)</u>

Let $\{x1, x2, \dots, x12\}$ represent the number of goods transported across each of the warehouse-market combinations mentioned in the matrix. For example, $x1$ represents the number of goods transported from GLA to LDN, $x2$ represents GLA to MAN, and so on.

Let $\{y1, y3, y, 5\}$ and $\{y2, y4, y6\}$ represent the number of low-capacity and high-capacity warehouses, at GLA, LBA, and PLH, respectively.

<u>Part 1.</u>

Here, the company aims to only optimise its *logistics costs*, more specifically the costs related to warehouse location and transportation of goods. To that end, the company tries to meet the customer demand with least possible costs, which in this case is determined by the transportation costs, fixed warehouse costs, and warehouse capacities.

Minimise the total cost Ct given by:

$$Ct = 81 * x1 + 92 * x2 + 101 * x3 + 130 * x4 + 117 * x5 + 77 * x6 + 108 * x7 + 98 * x8 + 102 * x9 + 105 * x10 + 95 * x11 + 119 * x12 + 6000 * y1 + 4500 * y2 + 6500 * y3 + 9000 * y4 + 6750 * y5 + 9750 * y6$$

Subject to:

i. $x1 + x2 + x3 + x4 <= 30 * y1 + 50 * y4$
ii. $x5 + x6 + x7 + x8 <= 30 * y2 + 50 * y5$
iii. $x9 + x10 + x11 + x12 <= 30 * y3 + 50 * y6$
iv. $x1 + x5 + x9 = 20$
v. $x2 + x6 + x10 = 30$
vi. $x3 + x7 + x11 = 20$
vii. $x4 + x8 + x12 = 18$
viii. Non-negative integer values of $\{x1, x2, \dots, x12\}$
ix. Non-negative integer values of $y1, y2, y3, y4, y5, y6$

(ii)

This is an extension of Part 1, where the company aims to optimise its profit determined by the factors mentioned in the question. Here, the factors are transportation cost, fixed warehouse cost, warehouse capacities, and additionally the selling price of the product. From the company's perspective, if the selling price of the product is fixed (as given in the question), the company would rather not sell the product in that market if the logistics cost were greater than the selling price. Else this would result in decline in company profit.

Assuming the fixed selling price across the operating markets is Cs, the company aims to **maximise** the profit given by:

$$Cr = Cs * (\textstyle\sum_{n=1}^{12} x_n) - Ct,$$

Subject to:

i. $x1 + x2 + x3 + x4 <= 30 * y1 + 50 * y4$
ii. $x5 + x6 + x7 + x8 <= 30 * y2 + 50 * y5$
iii. $x9 + x10 + x11 + x12 <= 30 * y3 + 50 * y6$
iv. $x1 + x5 + x9 <= 20$
v. $x2 + x6 + x10 <= 30$
vi. $x3 + x7 + x11 <= 20$
vii. $x4 + x8 + x12 \leq 18$
viii. Non-negative integer values of $\{x1, x2, \dots, x12\}$
ix. Non-negative integer values of $y1, y2, y3, y4, y5, y6$

Question 5

CRIB

(a)(i) To formulate an optimisation problem we need to identify the following three elements: decision variables, objective function(s), and constraints.
The decision variables are the characteristics that the designer can choose. These should allow enough flexibility to explore the solution space of a design problem. However, the larger the number of decision variables the larger the solution space and hence it is a more difficult optimisation problem to solve. So, we would like to keep a minimum number of decision variables.
The objective function(s) expresses a quantifiable measure of the 'goodness' of a product or a process which we try to maximise or minimise. As the number of objective functions increases the complexity of the optimisation problem increases exponentially.
The constraints maintain the practicality of the optimum solutions. The larger the number of constraints the higher the complexity of the optimisation problem. We try to keep the number of constraints to the minimum. The equality constraints are more difficult to satisfy rather than the inequality. Equality constraints can be transformed to inequality or as penalty terms.
(a)(ii) Assume there are two objectives in an optimisation problem. The first approach is to keep one objective as a constraint and the other as an objective function. This approach simplifies the optimisation problem but requires running the optimisation multiple times for different values of the constraint to obtain the Pareto front.
The second approach is to represent the two objectives in one objective function using weighted sums. This approach simplifies the optimisation problem but requires running the optimisation multiple times for different values of the weights to obtain the Pareto front.
The third approach is to consider two objective functions simultaneously. There is the need for a suitable multi-objective optimisation algorithm, but the Pareto front can be obtained directly.
(a)(iii) As heuristic methods do not guarantee an optimal solution, and solutions differ between different runs, a suitable number of runs (suggested min. 25) should be determined and the average best solution and its standard deviation between multiple runs should be obtained. If the starting point affects convergence, multiple starting points should be experimented with. Other performance metrics could include the time taken to reach convergence or the final solution after a pre-specified CPU time. A small sample with known optimality can also be experimented with.
Advanced answers may also mention performance of heuristic multi-objective algorithms, where solution diversity and objective search space coverage is also taken into account.

(b)(i) There are 10 projects, $x_i, i = 1{,}10$, with corresponding cost, $c_i, i = 1{,}10$, and utility value, $v_i, i = 1{,}10$.
The decision variables x_i are binary variables deciding whether the project i is chosen, =1 if the project is chosen, =0 otherwise.

(1) The problem can be formulated as a multi-objective optimisation problem maximising utility value and minimising cost.

$$\max V = \sum_{i=1}^{10} v_i x_i$$

$$\min C = \sum_{i=1}^{10} c_i x_i$$

(2) Or it can be formulated as a single-objective optimisation problem maximising utility value and constraining maximum cost.

$$\max V = \sum_{i=1}^{10} v_i x_i$$

$$s.t. \sum_{i=1}^{10} c_i x_i \leq budget$$

(3) Or it can be formulated as a multi-objective optimisation problem maximising utility value, maximising number of chosen projects, and constraining maximum cost.

$$\max V = \sum_{i=1}^{10} v_i x_i$$

$$\max X = \sum_{i=1}^{10} x_i$$

$$s.t. \sum_{i=1}^{10} c_i x_i \leq budget$$

(4) Or it can be formulated as a single-objective optimisation problem maximising number of chosen projects and constraining maximum cost and minimum utility value.

$$\max X = \sum_{i=1}^{10} x_i$$

$$\mathrm{s.t.} \sum_{i=1}^{10} v_i x_i \geq minimum\ utility\ value$$

$$s.t. \sum_{i=1}^{10} c_i x_i \leq budget$$

(5) Or it can be formulated as a single-objective optimisation problem maximising utility value and constraining maximum cost and minimum number of chosen projects.

$$\max V = \sum_{i=1}^{10} v_i x_i$$

$$\mathrm{s.t.} \sum_{i=1}^{10} x_i \geq minimum\ number\ of\ projects$$

$$s.t. \sum_{i=1}^{10} c_i x_i \leq budget$$

b(ii) For example, Projects 1 and 2 cannot be chosen together. At least one of Project 3, 4, and 5 must be chosen.

$$\max V = \sum_{i=1}^{10} v_i x_i$$

$$s.t. \sum_{i=1}^{10} c_i x_i \leq budget$$

$$x_1 + x_2 \leq 1$$

$$x_3 + x_4 + x_5 \geq 1$$

b(iii) For example, Project 7 cannot be chosen unless Project 6 is chosen.

$$x_6 \geq x_7$$

b(iv) For example, Projects 1, 5, and 6 should achieve a minimum utility value.

$$s.t. \sum_{i=1,5,6} v_i x_i \geq minimum\ utility\ value$$

b(v) Tabu Search, Genetic Algorithms.

B(i)(1) GA

Sample numbers only. Solutions found using Excel GRG Nonlinear method. NOT to be included in the final crib.

Table 1: MR&R project cost and utility value.

Project	1	2	3	4	5
Cost (kGBP)	63	70	42	31	38
Utility value	16	22	12	8	11

Project	6	7	8	9	10
Cost (kGBP)	60	45	58	65	29
Utility value	19	14	18	21	6

B(i)(2): Utility value = 77 (objective function)
Number of projects = 5
Cost = 250 (constraint, 250)
0, 0, 1, 0, 1, 1, 1, 0, 1, 0 (decision variables)

B(i)(4, 5):

Utility value (minimum constraint)	70	72	75	77
Number of projects (objective function)	6	6	5	5
Cost (constraint, 250)	245	250	250	250
Decision variables	0, 0, 1, 1, 1, 1, 1, 0, 0, 1	0, 0, 1, 1, 1, 0, 1, 0, 1, 1	0, 0, 0, 0, 1, 1, 0, 1, 1, 1	0, 0, 1, 0, 1, 1, 1, 0, 1, 0

B(iii): Utility value = 76 (objective function)
Number of projects = 4
Cost = 240 (constraint, 250)
0, 1, 0, 0, 0, 1, 1, 0, 1, 0 (decision variables)
B(iv): Utility value = 75 (objective function)
Number of projects = 5
Cost = 248 (constraint, 250)
0, 1, 1, 1, 0, 1, 1, 0, 0, 0 (decision variables)
B(v): Utility value = 72 (objective function)
Number of projects = 5
Cost = 250 (constraint, 250)
1, 0, 0, 1, 1, 1, 0, 1, 0, 0 (decision variables)
Three projects (1, 5, 6) have a minimum utility value, 20 (constraint) = 46

Question 6

CRIB

a) Critical metals are metals which are of high economic importance and are at risk of supply chain disruption. They are usually specifically defined for individual countries, although many of the metals on these lists often overlap. They are usually used in small amounts in advanced technologies, such as electrical components, energy systems such as batteries or wind turbines, and in advanced materials. Some examples are aluminium, cobalt, copper, gallium, iridium, lithium, magnesium, neodymium, nickel, platinum, or silicon.

b) Technical: Critical metals are often combined with other metals to form an amalgamation, which is difficult and expensive to separate. The separations process can also be resource intensive in itself, using both energy and chemical additives. Even when they are used in their pure form, it is often in an implementation where separating the critical metal from the rest of the product is physically challenging as products are complex structures and contain many different components and materials. Further, there are potential health hazards if the processing is not done safely or in a correct environment. Finally, the concentration of most critical metals in products is fairly low, meaning that even in the case of economic recovery, the amount of metal recovered will be small.

Operational: The entire recycling process for critical metals is often expensive relative to mining new materials. This disincentivizes recycling plants which can separate these critical materials. Further, the logistics of the extraction, collection, transport, and storage of products in order to recycle materials from them must be sorted.

Metal separations can be resource-intensive and environmentally damaging to perform, and even total recovery probably will not keep pace with demand. If critical metals can be economically recovered through recycling and substituted for virgin material, this would reduce the pressure on the existing supply chain. It would also reduce the amount of heavy metals in landfills. If the recycling process were more energy and resource efficient than the mining process, this would also reduce the environmental footprint.

c) To assess the environmental impact, either an LCA or an eco-audit can be used. In this case, an LCA would be a better choice as it evaluates multiple impact categories (human toxicity, eutrophication, abiotic resource depletion, and more) which are relevant for critical metals. Importantly, since this is a comparison between the critical metal and the alternative, two separate LCAs would need to be performed, one for each.

The first step in an LCA is to choose the function unit. Since this is a comparative analysis, a good choice of function unit would be "one completed product" of each type.

A system boundary is used to define what factors should be included for a particular environmental analysis. It should separate those parts of the process which are being examined from those which are not part of the process being examined.

The following factors should be considered: mining of the critical metal vs mining of the alternative, mining of other materials which may be necessary to use either in the product, component material production, product manufacturing processes which may be affected, the weight and volume of the finished product, the operational efficiency of the final product in case the alternative is not a direct replacement, how it affects the density with which the product can be packed for transport, and end-of-life disposal.

The analysis should be used to identify the overall difference in environmental impact, as well as the phases with the greatest difference. It should also be used to identify the specific sub-processes which most significantly contribute to the overall environmental impact.

d) It should do a life cycle analysis for recovery of the critical metal, then compare life cycle analyses for products which use the recovered metal, and products which use the alternative. This will inform the company about the environmental impact. The company will also need to consider the effectiveness of the alternative – alternatives are rarely a direct substitute, so using the alternative will impact the product in some way (cost, weight, design, functionality). It is also important to consider the cost of each option. Recovery may be too expensive, or conversely, using the alternative may require such significant product redesign that recycling is the better choice. The impact on the product's end-of-life is also important – for example, if using the alternative makes the product unrecyclable in some way. The company will also want to take other factors into account, such as the difference in ability to design their products for disassembly, reuse, or repair at the end of life. They also will want to consider how well they can standardize components under each option, and how well the products can be modularized.

Answer standards:

<u>Basic answers</u>: Demonstrate understanding of the underlying technology of recycling and its environmental justification. May demonstrate lack of knowledge and understanding of some important factors. Examples may be inappropriate. The discussion may be superficial and inaccurate.

Strong answers: Good depth of knowledge and understanding. Will cover the key environmental aspects and provide adequate discussion with pertinent examples discussed with accuracy.
Best answers: Detailed descriptions and analysis. Demonstrate understanding of the more subtle tensions in the environmental assessments. Examples will draw on experience from many different parts of the course together with the student's own observations. The discussion will be critical and insightful.